Name: _____________________________________

SID: _____________________________________

- You may not use any reference materials during this exam.
- Electronic devices, including calculators, cell phones, mp3 players, and laptops are all prohibited.
- You may not use your own scratch paper. The exam has plenty and you can ask for more if needed.
- You may not leave the classroom once the exam has been distributed.
- Communicating with other students in any way is prohibited.

Academic Honesty: I understand that if I cheat on this exam in any way, I will receive the maximum possible penalty, including an F in this course.

Signature: _____________________________________

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<th>Problem</th>
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1 Finding the $k$ smallest elements

(10 points) Write a deterministic algorithm that, given an array, $A$, of $n$ items, returns an array $B$, of the $k$ smallest items in $A$. The items in $B$ do not need to be sorted. What is the asymptotic running time of your algorithm? You may use any algorithm discussed in class as a sub-routine.
2 Root finding

(10 points) Suppose \( f(n) \) is a monotonically increasing function on the positive integers. Give an algorithm to find the smallest integer \( x \) such that \( f(x) > 0 \). Your algorithm should run in \( O(\log x) \) time.
3 Randomized matrix equality testing

Suppose you are given three $n \times n$ matrices, $A$, $B$, and $C$, and you want to test whether $AB = C$.

- (5 points) Suppose $v$ is a vector whose entries are each 0 or 1 with probability $1/2$. Prove that, if $M$ is any non-zero $n \times n$ matrix, then $\Pr[Mv = 0] \leq 1/2$.

- (5 points) Prove that if $AB \neq C$, then for a random vector $v$ as above, $\Pr[ABv = Cv] \leq 1/2$.

- (5 points) Give a randomized algorithm for determining whether $AB = C$. Your algorithm may have a small error probability (i.e. it may occasionally say that $AB = C$ even though they are not equal). However, your algorithm should output the correct answer w.h.p.

- (5 points) What is the running time of your algorithm.
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4 Linear probing

Many hash table implementations store items in the slots of the table, as shown in the following code:

```
insert(x)
  i = hash(x);
  while A[i].occupied
    i = i + 1;
  A[i].occupied = true;
  A[i].value = x;
```

- (5 points) Write the code for the lookup function.
- (10 points) Prove that inserts and lookups run in $O(\log n)$ time w.h.p.