CSE 548 Fall 2012: Homework #1

Due September 24 in class.

**Problem 1**

Prove that, after inserting $n$ items into a hash table with $n$ buckets using a universal hash function, the expected number of items in any bucket is 1.

**Problem 2**

Prove that the time required to lookup an item in a hash table with a random hash function is $O(1)$ w.h.p. Assume that each lookup is for a uniformly random element in the hash table.

**Problem 3**

Prove the following hash function family is universal. Let the universe of keys be

$$U = \{0, 1\}^\ell$$

and suppose the number of buckets in the hash table is $2^b$. We will view keys as column vectors of $\ell$ bits. To select a hash function from the family, pick a random $b \times \ell$ matrix, $A$, and set

$$h(x) = Ax$$

where all the operations are performed mod 2.