Notes on Grading

- You can write “I don’t know” for any question and receive 25% credit. You can take this option for any numbered problem, but not for part of a problem. For example, you can answer 3.1 and write “I don’t know” for 3.2, but you can’t write part of the solution for 3.2 and then write “I don’t know” for the rest.

- You get a 10% bonus for typing your homework. You are encouraged to use \LaTeX. You must type your entire homework to receive the bonus. The 10% bonus does not apply to problems answered with “I don’t know.”

Important Note

All the problems on this homework ask for dynamic programs.

To answer a problem:

1. Briefly explain the recursive substructure of the problem.
2. Give the rule for filling in the dynamic programming table.
3. Give the asymptotic running time for the dynamic program.
4. Give the asymptotic space required for the dynamic program.

Then pick one of the problems and write out a complete implementation (in pseudo-code) for that algorithm.
1 Optimal Network Transmission Combination

Given a set of \( n \) data transmission request where every request is made up with the following:

- Start Time
- Finish Time
- Amount of data to be transferred

find the subset of requests such that the total amount of data transferred is the maximum while avoiding any collisions in time.

2 Mining for gold

Let’s say we are making a gold collector game. The game has a map \( M \) that is a two-dimensional \( N \times M \) grid. Each coordinate has a certain amount of gold. We would use \( \text{Gold}[i][j] \) to represent the amount of gold at coordinate\((i,j)\). A player starts from coordinate \( M[0][0] \) which is the lower left corner. At each step, the player can either go up or right by one coordinate. The game ends once the player can’t move up or right. The goal of this game is to collect as much gold as possible at the end of game. We want to show the player the maximum amount of gold that can be actually collected when the game ends. Describe an algorithm to calculate the maximum amount of gold that can be collected and analyze its complexity.

3 Maximum product

Describe and analyze an algorithm that, given an array \( A \) of integers, finds the largest product of elements in a contiguous subarray \( A[i .. j] \).

4 String decomposition

Given three strings \( a, b, \) and \( c \), determine whether \( c \) is an interleaving of the letters in \( a \) and \( b \).
Two salesmen

Given a list of cities $X$, define the distance of $X$ to be

$$\sum_{i=0}^{|X|-2} ||X[i+1] - X[i]||$$

In other words, the distance of $X$ is just the total distance you would travel if you started at $X[0]$ and went to $X[1]$, $X[2]$, etc. in order.

Given an ordered list of cities $A$, partition $A$ into two lists $B$ and $C$ so that $\text{distance}(B) + \text{distance}(C)$ is minimized.