Name: ________________________________

SID: ________________________________

- You may not use any reference materials during this exam.
- Electronic devices, including calculators, cell phones, mp3 players, and laptops are all prohibited.
- You may not use your own scratch paper. The exam has plenty and you can ask for more if needed.
- You may not leave the classroom once the exam has been distributed.
- Communicating with other students in any way is prohibited.

Academic Honesty: I understand that if I cheat on this exam in any way, I will receive the maximum possible penalty, including an F in this course.

Signature: ________________________________

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<th>Problem</th>
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1 Big-Oh notation

For each function $f(n)$ below, write down the simplest function $g(n)$ you can come up with so that $f(n) = \Theta(g(n))$. For example, for $f(n) = 7n^2$, you would write $g(n) = n^2$. (2 points each)

- $f(n) = 7(n - 5)^2$
- $f(n) = n \log n^2 + n \log \log n$
- $f(n) = 2^{8+\log n}$
- $f(n) = n^2/(\log n - \log \log n)$
- $f(n) = \sqrt{n^2 + 7n - 5}$

Solution

- $n^2$
- $n \log n$ (because $\log n^2 = 2 \log n = \Theta(\log n)$)
- $n$ (because $2^{8+\log n} = 2^{8+2\log n} = 256n = \Theta(n)$)
- $n^2/\log n$ (because $\log n - \log \log n = \Theta(\log n)$)
- $n$


2 Recurrences

Solve the following recurrences. Assume $T(n) = O(1)$ for $n < 10$. You need to give only Big-Oh solutions. You are allowed to use the Master Theorem. You do not need to show your work. (5 points each)

- $T(n) = 2T(n/2) + n$
- $T(n) = 7T(n/2) + n$
- $T(n) = T(n/2) + n$
- $T(n) = T(\sqrt{n}) + n$

Solution

- $O(n \log n)$
- $O(n^{\log_2 7})$
- $O(n)$
- $O(n)$
3 Divide and conquer

This problem asks you to design a linear-time divide-and-conquer algorithm to construct a binary tree from a sorted array of $N$ items.

- Write the pseudo-code for your algorithm. Your pseudo-code should be at most 15 lines long. (10 points)
- Write down the recurrence for the running time for your algorithm. (5 points)

Solution

- binary-tree(A)
  
  if $|A| < 2$
    return new node with elements of $A$
  else
    divide $A$ evenly into 2 subarrays $A_1$ and $A_2$
    for $i = 1$ to 2
      $t_i$ btree($A_i$)
    let $r$ = new tree node with $t_1$ and $t_2$ as children
    return $r$

- $T(n) = 2T(n/2) + O(1)$
4 Dynamic programming

A string $s$ is a palindrome if $s = xx^R$ for some string $x$, where $x^R$ is the reverse string of $x$.

The goal of this problem is to write a dynamic program to find the longest palindrome substring of an input string $s$.

• Define, in 10 words or less, what an entry in your table represents. (5 points)

• Write down the rule for filling in your table. (10 points)

• What is the running time of your dynamic program? (2 points)

• What is the space required? (3 points)

Solution

• $T[i,j] = j - i + 1$ iff $s_i \ldots s_j$ is a palindrome (of even length). Otherwise $T[i,j] = 0$.

• For even-length palindromes:

$$T[i,j] = \begin{cases} 
0 & \text{if } i = j \\
0 & \text{if } j = i + 1 \text{ and } s_i \neq s_j \\
2 & \text{if } j = i + 1 \text{ and } s_i = s_j \\
0 & \text{if } j > i + 1 \text{ and } s_i \neq s_j \\
T[i+1,j-1] + 2 & \text{if } j > i + 1 \text{ and } s_i = s_j 
\end{cases}$$

For general palindromes:

$$T[i,j] = \begin{cases} 
1 & \text{if } i = j \\
0 & \text{if } j = i + 1 \text{ and } s_i \neq s_j \\
2 & \text{if } j = i + 1 \text{ and } s_i = s_j \\
0 & \text{if } j > i + 1 \text{ and } s_i \neq s_j \\
T[i+1,j-1] + 2 & \text{if } j > i + 1 \text{ and } s_i = s_j 
\end{cases}$$

• $O(n^2)$

• $O(n)$
5 findKthSmallest in external memory

- What is the I/O complexity of PARTITION in the DAM model? (5 points)
- What is the I/O complexity of FINDKTHSMALLEST in the DAM model? (5 points)

Solution
- $\Theta(1 + N/B)$
- $\Theta(1 + N/B)$