CSE 150 Fall 2008: Problem Set #1

Note: If you don’t know the proper notation for something, just describe it as well as you can.

Problem 1
Write down the truth tables for the following expressions.
1. \((P \land (P \Rightarrow Q)) \Rightarrow Q\)
2. \(P \land (Q \Leftrightarrow R)\)

Problem 2
Define \(P \# Q = \neg(P \lor Q)\). Prove that every boolean expression can be written using just \#. (Hint: reduce \(\neg\) and \(\lor\) to \#.)

Problem 3
Each of the following statements is false. Prove it.
1. \(\exists x \in \mathbb{R}. \forall y \in \mathbb{R}. xy = 1\)
2. \(\forall p \in \mathbb{N}. p \text{ is prime} \Rightarrow 2^p - 1 \text{ is prime}\)
3. \(\exists x \in \{0, 1\}. (x + 1)^2 \neq 2(x^2 + 1)\)

Problem 4
Indicate whether each relation \(R\) is reflexive, symmetric, or transitive:
1. \(S = \{1, 2, 3, \ldots, 10\}, R \subseteq S \times S, R = \text{“is less than the square of”}\)
2. \(R \subseteq \mathbb{R} \times \mathbb{R}, (x, y) \in R \Leftrightarrow y = \sin x\)
3. \(R \subseteq \text{People} \times \text{People}, R = \text{“is the sibling of”}\)

Problem 5
Recall that for a function \(f : A \to B\) and \(X \subseteq A\) and \(Y \subseteq B\), \(f(X) = \{y | y = f(x), x \in X\}\) and \(f^{-1}(Y) = \{x | f(x) \in Y\}\). For each of the following claims, either prove it is true for every function \(f\) and sets \(X_1, X_2 \subseteq A, Y_1, Y_2 \subseteq B\), or provide a counterexample. If it is not true of all functions, determine if it is true for surjections, injections, or bijections, and prove that it is.
1. \(f(X_1 \cup X_2) = f(X_1) \cup f(X_2)\)
2. \(f(X_1 \cap X_2) = f(X_1) \cap f(X_2)\)
3. \(f^{-1}(Y_1 \cup Y_2) = f^{-1}(Y_1) \cup f^{-1}(Y_2)\)
4. \(f^{-1}(Y_1 \cap Y_2) = f^{-1}(Y_1) \cap f^{-1}(Y_2)\)