Are the sets \( \{0, 1, 2\} \) and \( \{2, 1, 1, 2, 6 - 6\} \) equal?
Are the sets \{0, 1, 2\} and \{2, 1, 1, 2, 6 - 6\} equal?
Yes. Order and repetition don’t matter, and 6 - 6 = 0.
Are the sets \( \{0, 1, 2\} \) and \( \{2, 1, \{0\}\} \) equal?
Are the sets \( \{0, 1, 2\} \) and \( \{2, 1, \{0\}\} \) equal?
No. \( 0 \neq \{0\} \)
Are the sets \( \{dog, cat, mouse\} \) and \( \{\{dog, cat, mouse\}\} \) equal?
Are the sets \(\{\text{dog, cat, mouse}\}\) and \(\{\{\text{dog, cat, mouse}\}\}\) equal? No. The first set has 3 elements: dog, cat, and mouse. The second set has 1 element: the set \(\{\text{dog, cat, mouse}\}\).
Are the sets $\mathbb{Z}$ and $\{-x | x \in \mathbb{Z}\}$ equal?
Are the sets $\mathbb{Z}$ and $\{-x | x \in \mathbb{Z}\}$ equal?
Yes. The negative of every integer is an integer, and every integer has a negative.
Is $5 \in \{5, 6, 7\}$?
Is 5 ∈ \{5, 6, 7\}?
Yes.
Is $5 \in \{\{5, 6, 7\}\}$?
Is $5 \in \{\{5, 6, 7\}\}$?
No. The only element of the set is another set, $\{5, 6, 7\}$. 
If $a = 0$, is $a \in \emptyset$?
If $a = 0$, is $a \in \emptyset$?

No. Nothing is in the empty set.
Is $\emptyset \in \emptyset$?
Is $\emptyset \in \emptyset$?
No. Nothing is in the empty set.
Is $\emptyset \subseteq \emptyset$?
Is $\emptyset \subseteq \emptyset$?
Yes.
Is $\emptyset \subseteq \emptyset$?
Is $\emptyset \subset \emptyset$?
No. If $A \subset B$ then $A \neq B$. 
How many integers are there in the set \( \{10, 11, \text{green}, \{3\}, (7, 8)\} \)?
How many integers are there in the set \{10, 11, \textit{green}, \{3\}, (7, 8)\}?

2. The only integers in this set are 10 and 11. \textit{green} is obviously not an integer. \{3\} is a set, not an integer. (7, 8) is an ordered pair, not an integer.
Is \(\{8, 9\} \subseteq \{\{5, \{8, 9\}\}\}\)?
Is \( \{8, 9\} \subseteq \{\{5, \{8, 9\}\}\}? \\
No. The set \( \{8, 9\} \) is an element of \( \{\{5, \{8, 9\}\}\} \), not a subset of it.
Is \( \{8, 9\} \subseteq \{5, 8, 9\} \)?
Is \( \{8, 9\} \subseteq \{5, 8, 9\} \)?

Yes. Every element of the set \( \{8, 9\} \) is also an element of the set \( \{5, 8, 9\} \).
$|\{1, 1, 1, green, 1\}| =$
\[|\{1, 1, 1, \text{green}, 1\}| = 2.\]

The set has two elements: 1 and green.
If $a = 5$ and $b = 7 - 2$, then $|\{a, b\}| =$
If $a = 5$ and $b = 7 - 2$, then $|\{a, b\}| = 1$. The set has one element: 5
\[ \{|x \in \mathbb{N} \mid 2|x, 7|x, 1 \leq x \leq 10\} = \]
\[ \{|x \in \mathbb{N} \mid 2|\,x, 7|\,x, \, 1 \leq x \leq 10\} \} = 0. \]

There are no natural numbers between 1 and 10 that are divisible by 2 and 7.
If $A \subset B$ and $|B| = 5$, how small or large could $A$ be?
If $A \subset B$ and $|B| = 5$, how small or large could $A$ be? $0 \leq |A| \leq 4$. 
Suppose $a$, $b$, and $c$ are non-zero integers. You do not know which, if any, are equal. How small or large could the set \{a, b, c, a - a\} be?
Suppose $a$, $b$, and $c$ are non-zero integers. You do not know which, if any, are equal. How small or large could the set \{a, b, c, a - a\} be? Note that the set always contains $a - a = 0$. Even if $a = b = c$, we know that $a \neq 0$, so the set must contain at least two distinct elements: $a$ and 0. If $a$, $b$ and $c$ are all different, then the set could contain as many as 4 elements.
\{0, 2, 4\} \cup \{1, 3\} =
\{0, 2, 4\} \cup \{1, 3\} = \{0, 1, 2, 3, 4\}
\{red, blue, green\} \cup \{green, 1, 3\} =
\{red, blue, green\} \cup \{green, 1, 3\} = \{red, blue, green, 1, 3\}
\{1, 2\} \cup \{\} =
\{1, 2\} \cup \{\} = \{1, 2\}
\emptyset \cup \{\text{red}, \text{green}\} =
\emptyset \cup \{red, green\} = \{red, green\}
\[ \mathbb{Q} \cup \mathbb{Z} = \]
\[ \mathbb{Q} \cup \mathbb{Z} = \mathbb{Q}, \text{ since } \mathbb{Z} \subseteq \mathbb{Q}. \]
$$\bigcup_{i=0}^{5} \{2^i, 2^i + 1, \ldots, 2^{i+1} - 1\} =$$
\[ \bigcup_{i=0}^{5} \{2^i, 2^i + 1, \ldots, 2^{i+1} - 1\} = \{1, 2, \ldots, 63\} \]
\[ \bigcup_{i=0}^{\infty} [-2^{-i}, 10 - 2^{i}] = \]
$\bigcup_{i=0}^{\infty}[-2^{-i}, 10 - 2^i] = [-1, 10)$
\(|\bigcup_{i=0}^{5}\{\text{binary strings of length } i \text{ with exactly } \lfloor i/2 \rfloor \text{ zeros}\}| = \)
|\bigcup_{i=0}^{5}\{\text{binary strings of length } i \text{ with exactly } \lfloor i/2 \rfloor \text{ zeros}\}| =

There is 1 string of length 0 with 0 zeros: the empty string.
There is 1 string of length 1 with 0 zeros: “1”.
There are 2 strings of length 2 with 1 zero: “01” and “10”.
There are 3 strings of length 3 with 1 zero: “011”, “101”, “110”.
There are 6 strings of length 4 with 2 zeros: “0011”, “0101”, “0110”, “1001”, “1010”, “1100”.
There are 10 strings of length 5 with 2 zeros: “00111”, “01011”, “01101”, “01110”, “10011”, “10101”, “10110”, “11001”, “11010”, “11100”.
Thus there are $1 + 1 + 2 + 3 + 6 + 10 = 23$ elements in the union.
\{0, 2, 4, 6\} \cap \{2, 6, 8, 10\} =
\{0, 2, 4, 6\} \cap \{2, 6, 8, 10\} = \{2, 6\}
\{1, 2, 3\} \cap \{\{1, 2, 3\}\} =
\{1, 2, 3\} \cap \{\{1, 2, 3\}\} = \emptyset
\[ \mathbb{Z} \cap \mathbb{Q} \cap \mathbb{N} = \]
\( \mathbb{Z} \cap \mathbb{Q} \cap \mathbb{N} = \mathbb{N} \)
\( \{2k \mid k \in \mathbb{N}\} \cap \{3x \mid x \in \mathbb{N}\} = \)
\{2k | k \in \mathbb{N}\} \cap \{3x | x \in \mathbb{N}\} = \{6k | k \in \mathbb{Z}\}
\{-x \mid x \in \mathbb{N}\} \cap \mathbb{N} =
\{ -x | x \in \mathbb{N} \} \cap \mathbb{N} = \{0\}
(\mathbb{Z} \times \mathbb{Q}) \cap (\mathbb{Q} \times \mathbb{Z}) =
$\mathbb{Z} \times \mathbb{Q} \cap (\mathbb{Q} \times \mathbb{Z}) = \mathbb{Z} \times \mathbb{Z}$
\((\mathbb{Z} \times \mathbb{Q}) \cup (\mathbb{Q} \times \mathbb{Z}) =\)
\[(\mathbb{Z} \times \mathbb{Q}) \cup (\mathbb{Q} \times \mathbb{Z}) = \{ (a, b) \in \mathbb{Q} \times \mathbb{Q} | a \in \mathbb{Z} \text{ or } b \in \mathbb{Z} \}\]
$$P(\{1, 2, 3\}) \cap P(\{2, 3, 4\}) =$$
\[ P(\{1, 2, 3\}) \cap P(\{2, 3, 4\}) = \emptyset, \{2\}, \{3\}, \{2, 3\} \]
\[ P(S) \cap P(T) = \]
\[ P(S) \cap P(T) = P(S \cap T). \]

**Proof** First we prove that \( P(S) \cap P(T) \subseteq P(S \cap T) \). So we need to show that every element of \( P(S) \cap P(T) \) is also an element of \( P(S \cap T) \). Well, suppose \( A \) is an element of \( P(S) \cap P(T) \). Then, by definition of intersection and power set, \( A \subseteq S \) and \( A \subseteq T \), and hence \( A \subseteq S \cap T \). Consequently, \( A \in P(S \cap T) \). This establishes that \( P(S) \cap P(T) \subseteq P(S \cap T) \).

Now we argue that \( P(S \cap T) \subseteq P(S) \cap P(T) \). In this case, we must show that every element of \( P(S \cap T) \) is also an element of \( P(S) \cap P(T) \). Consider any element \( A \) of \( P(S \cap T) \). By definition of the power set, \( A \subseteq S \cap T \), and hence \( A \subseteq S \) and \( A \subseteq T \). This implies that \( A \in P(S) \) and \( A \in P(T) \). This implies that \( A \in P(S) \cap P(T) \). Thus we have established that \( P(S \cap T) \subseteq P(S) \cap P(T) \).

Since \( P(S \cap T) \subseteq P(S) \cap P(T) \) and \( P(S) \cap P(T) \subseteq P(S \cap T) \), we must have that \( P(S) \cap P(T) = P(S \cap T) \).