Algorithm Seminar: warm up session for art-gallery problem

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This discussion session was more of an warm up session for the family of art gallery problems. We mostly discussed the similar flavor problem on graphs. There are two sets of problems which has that flavor. First one is widely known vertex cover (VC) and another is Dominating Set (DS).

We are more concerned with planar graph here, as it is more relevant in the context of art gallery problem. In vertex cover our goal is to cover all the edges where as in dominating set out goal is to find a subset of vertices such that all the vertices are neighbor of some vertex in that subset. So size of dominating set is less than the size of the size of vertex cover.

In figure 1 , it is shown that the bound $\frac{3n}{4}$ is sometimes necessary.

Then we discussed about the lower bound on the set of DS and VC. If we consider planar graph with n vertices then vertex cover size is $\frac{3n}{4}$. This proof is simple as we can have a consistent coloring of a planar graph with 4 colors (according to then famous 4 color theorem). Then the largest number of vertices in a color class has to be at least $\frac{n}{4}$. If we cover all other vertices then we can cover all the edges. Interestingly there is no *easy* algorithm to achieve that bound. So the questions asked are as following,

- Is there an easy $O(n)$ algorithm to achieve $\frac{3n}{4}$ bound ?
- Is there an easy algorithm for *minimal* vertex cover ?

There is also an interesting algorithm discussed at the end of the session, in context of DS. If number of edges in a graph increases, how does it affect

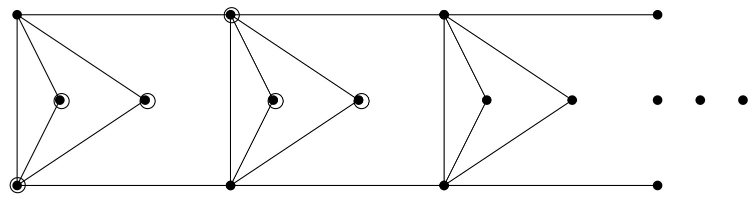

Figure 1: Example of $\frac{3n}{4}$ size vertex cover

size of DS. Let the function $f(n, m)$ where n is the number of nodes and m is the number of edges denote the size of DS, what is the characteristics of f is we change m. It is obvious that if $m < n - 1$ then its a disconnected graph, and if $m = \binom{n}{2}$, then size of DS is 1. But we are not certain of its behavior in the midway.