Resilient Priorirty Queue

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Memory Errors

- One or more Memory bits is read differently from last written values
- Causes can be hardware bit corruption, cosmic rays, corruption in path between memory and CPU

Pointer Corruptions

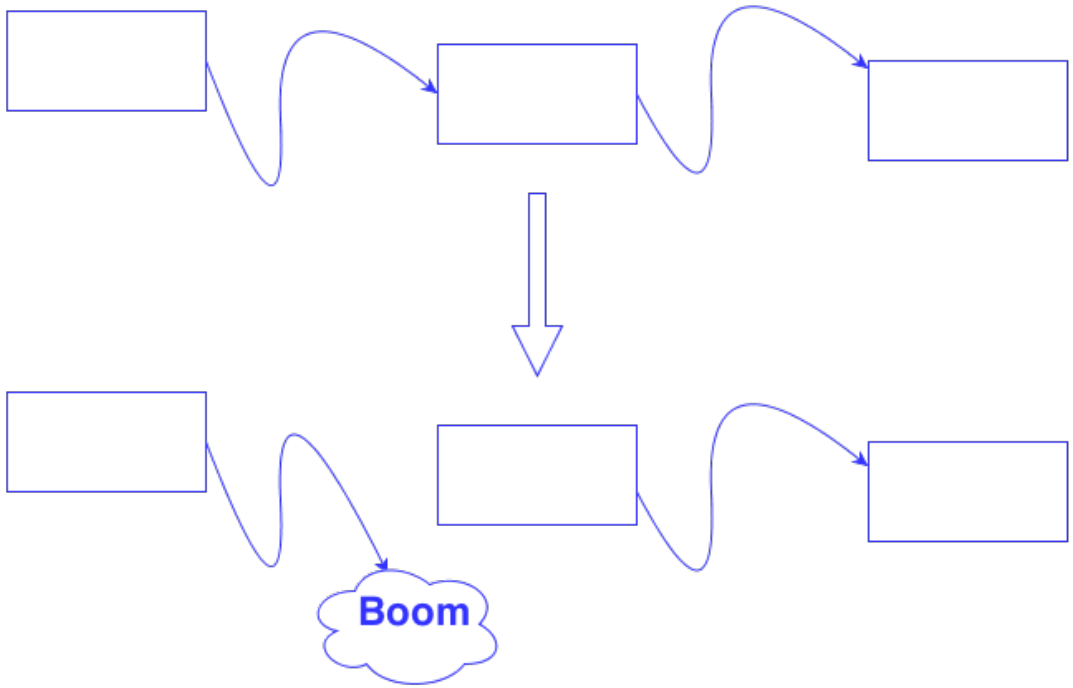

Figure : Even a single memory corruption can be catastrophic. A simple linked list with memory error, entire tail lost due to a single pointer corruption

ECC

- Hardware ECC (Error Correcting Codes) chips can help
- But is expensive in terms of processing time and money
- Even ECC cannot correct every corruption

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Its Not Rare

Mem. size	Mean Time Between Failures
512 MB	2.92 hours
1 GB	1.46 hours
16 GB	5.48 minutes
64 GB	1.37 minutes
1 TB	5.13 seconds

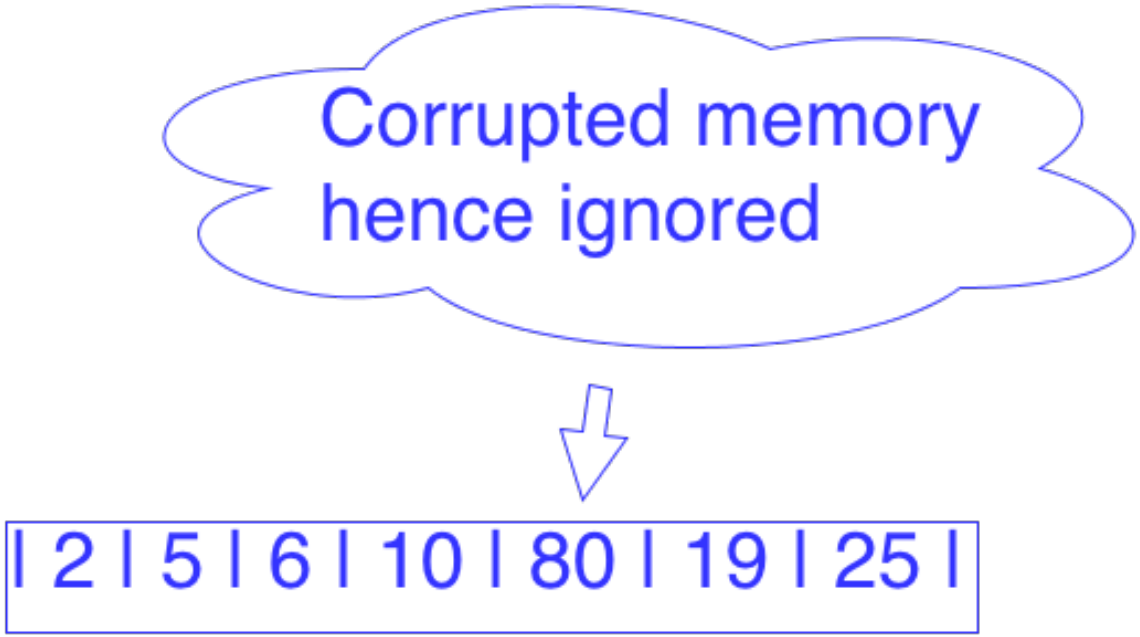
Figure : A field study by Google researchers

Resilient Algorithms

- Make the algorithm and data structure capable of dealing with memory errors
- Design for damage control, minimize the disasters
- Memory model assumes that at maximum δ corruptions throughout the runtime
- δ is an input to the model, known in advance
- Also, assumes that it has some constant amount of relaible memory $P = O(1)$
- Relaible memory cannot get corrupt

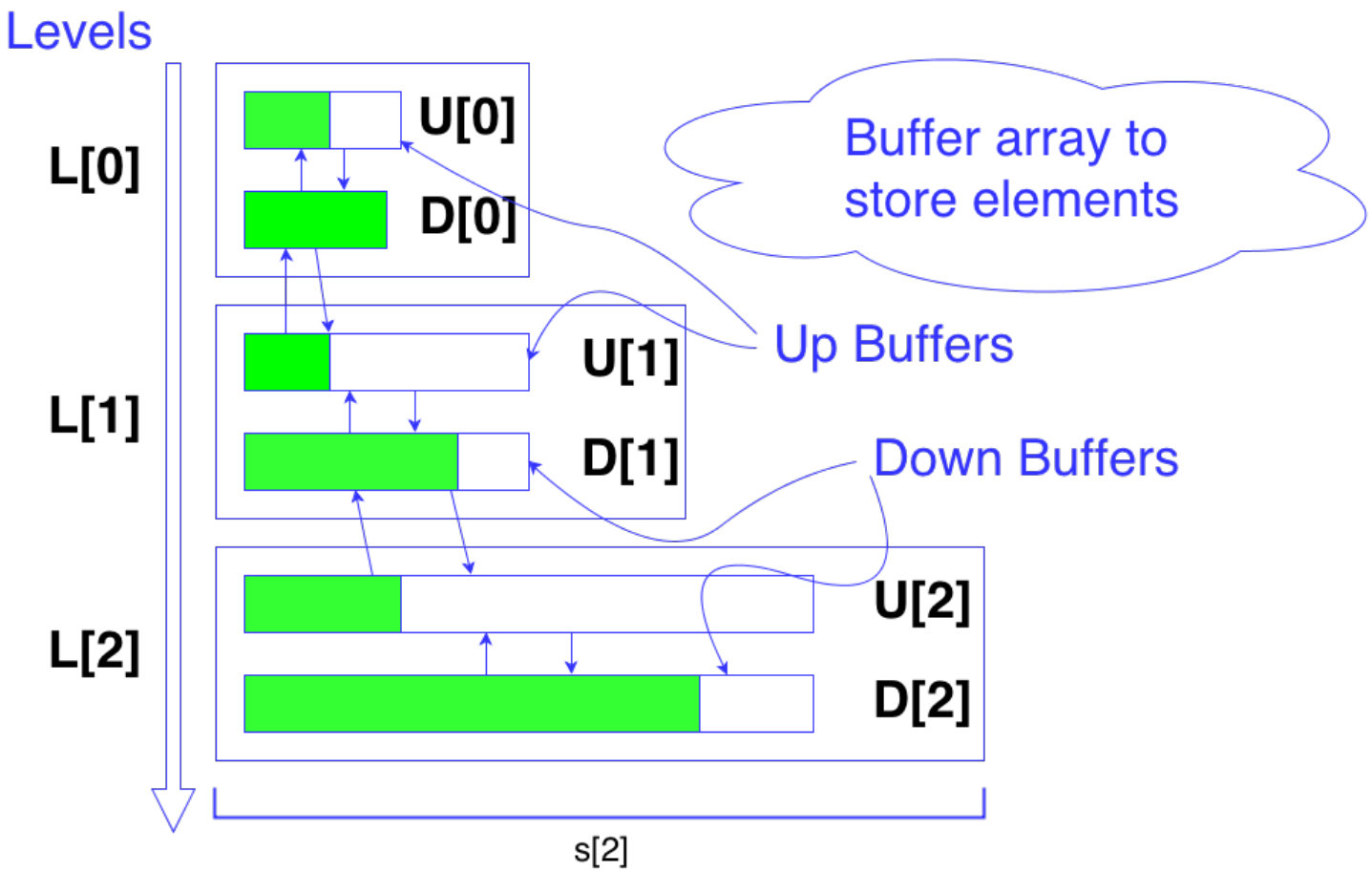
Faithful Ordering

- In resilient sorting, recovery from corruption is expensive
- Hence goal is modified to all un-corrupted items are guaranteed to be correctly sorted
- Order of corrupted items is ignored



Resilient Merging

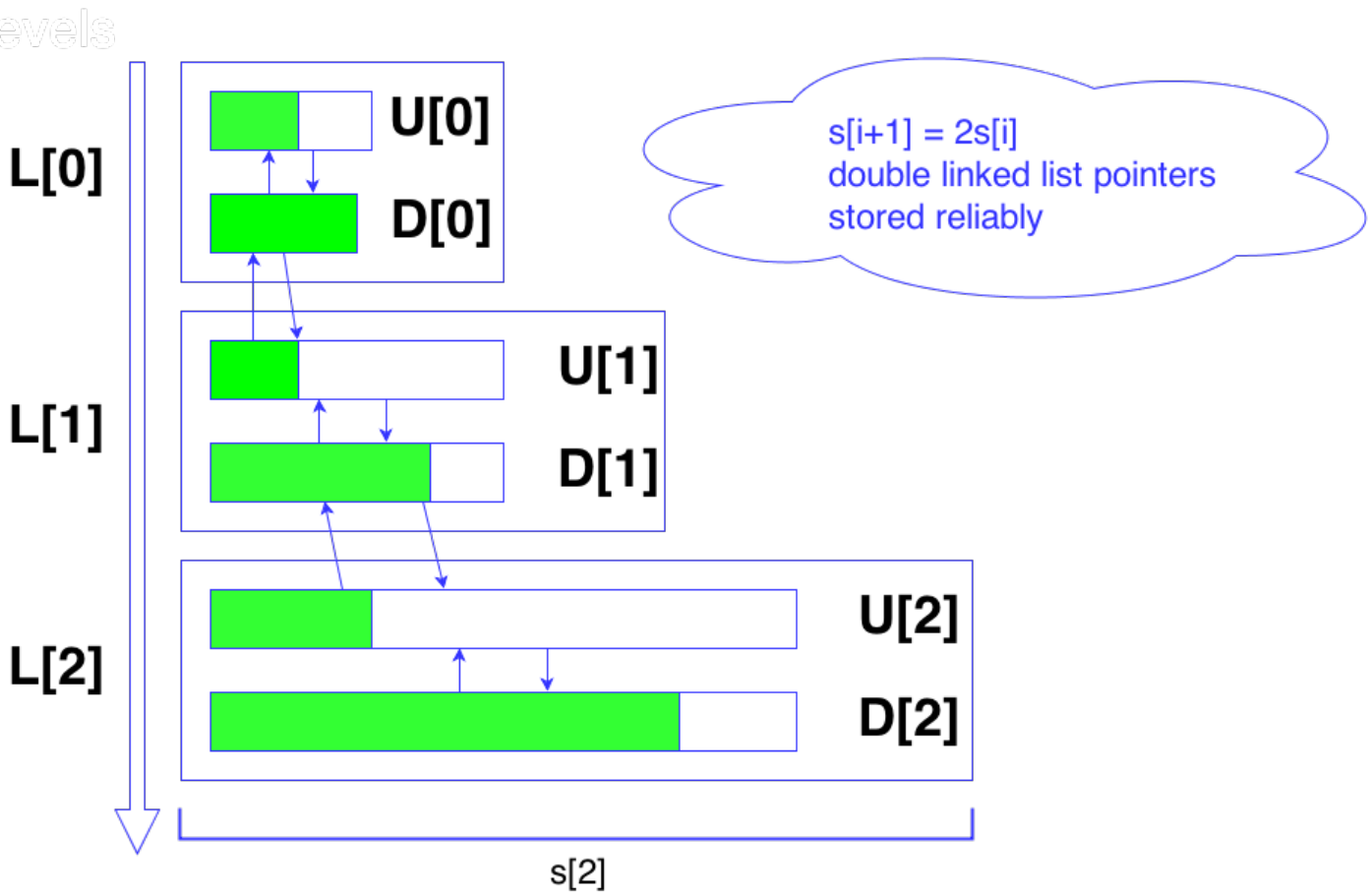
Structure of Resilient Priority Queue



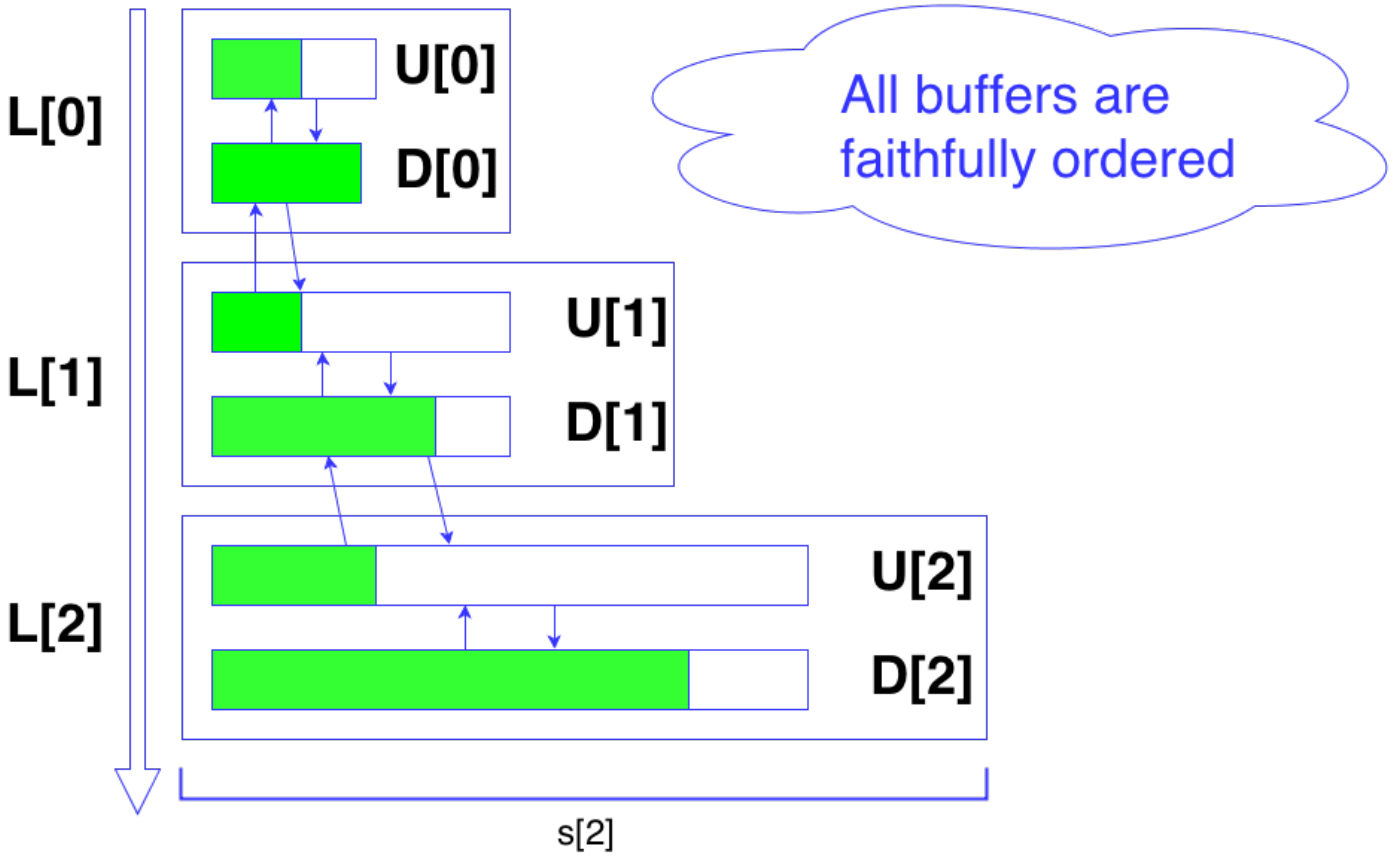
Structure of Resilient Priority Queue

- Intuitively, elements in Up buffer are moving to upper layers
- Elements in Down buffer are moving to lower layers

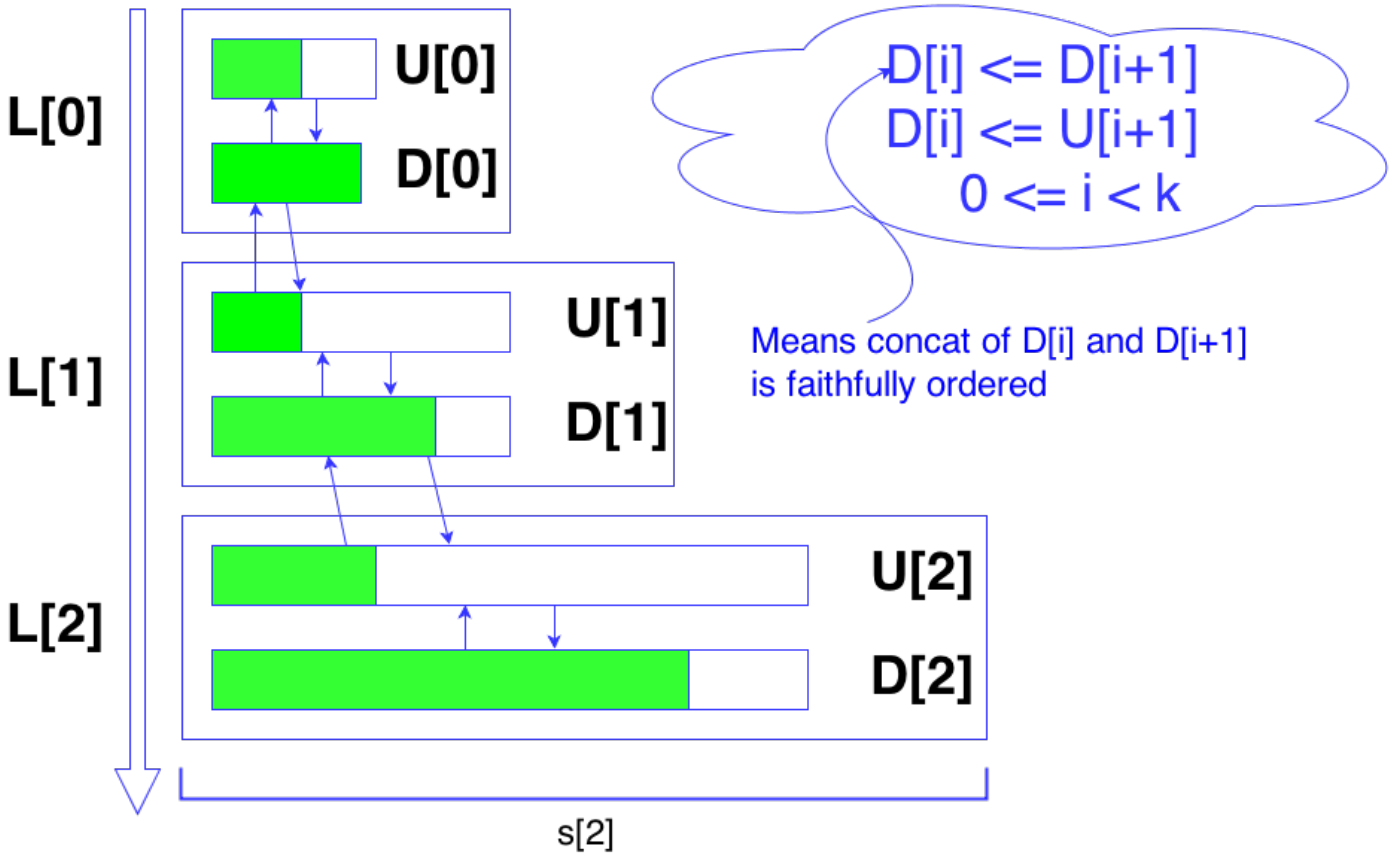
Structure of Resilient Priority Queue



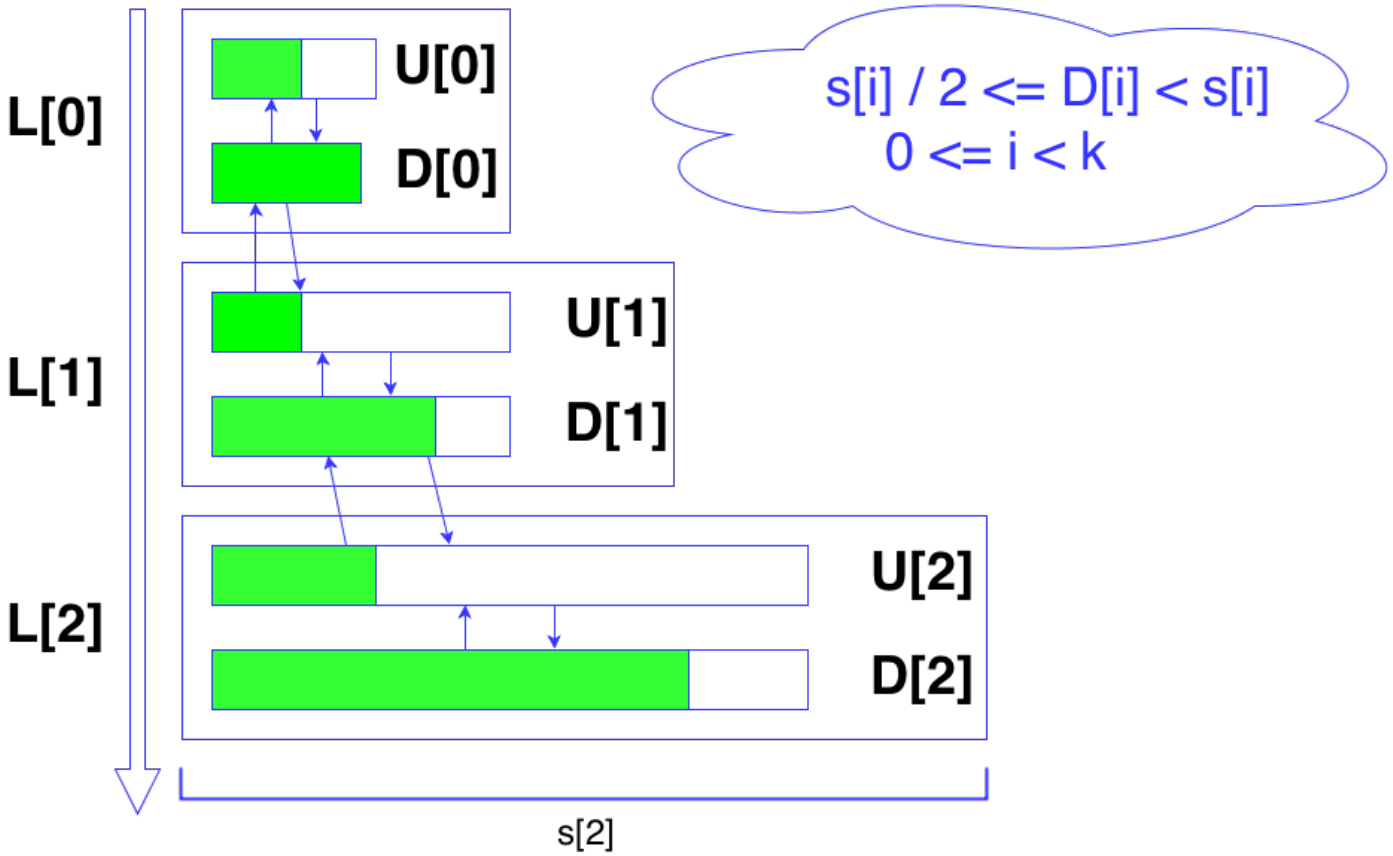
Invariant A


L[0]
L[1]
L[2]
U[0]
D[0]
U[1]
D[1]
U[2]
D[2]
s[2]
All buffers are faithfully ordered

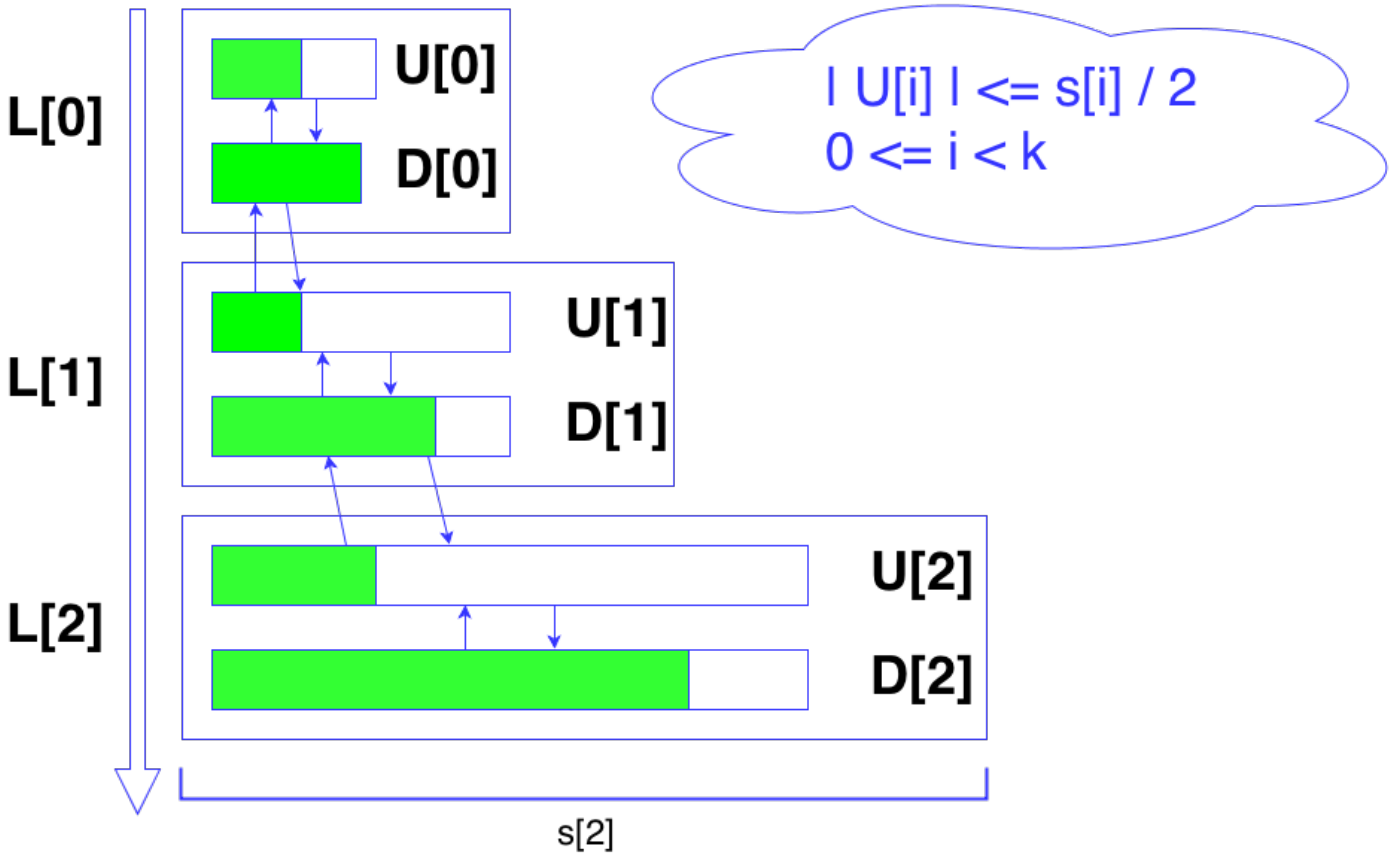
Invariant B


L[0]
L[1]
L[2]
U[0]
D[0]
U[1]
D[1]
U[2]
D[2]
s[2]
D[i] <= D[i+1]
D[i] <= U[i+1]
0 <= i < k
Means concat of D[i] and D[i+1]
is faithfully ordered

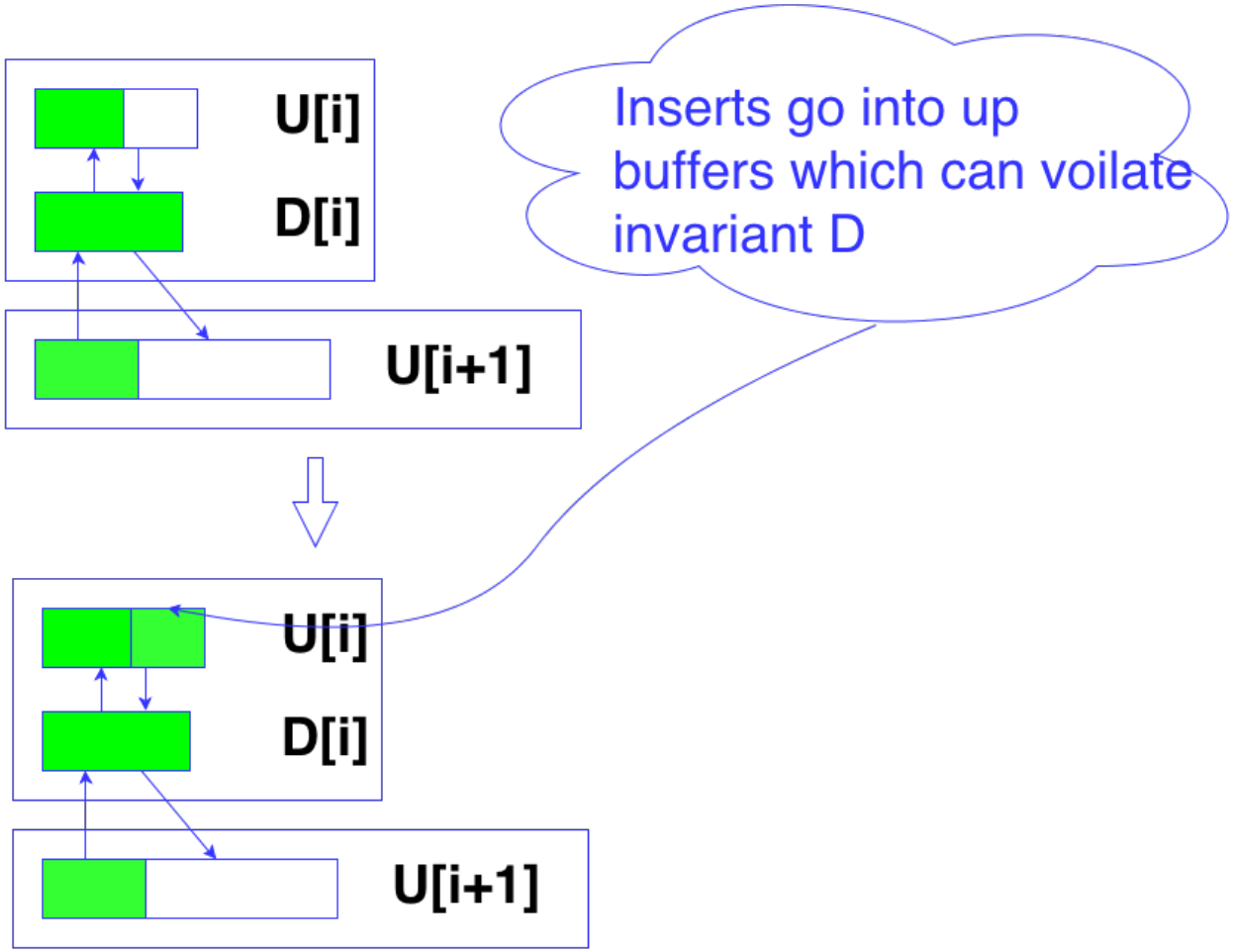
Invariant C - Down at least Half Filled



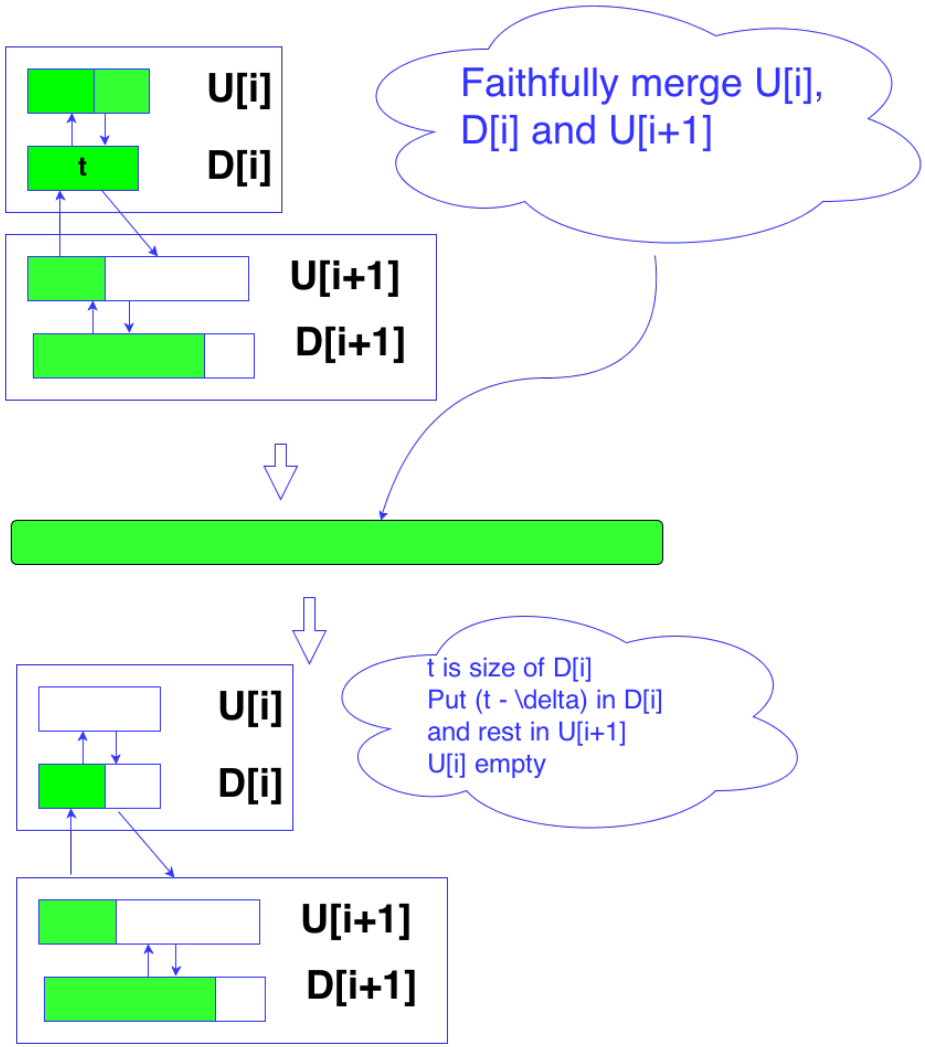
Invariant D - Up at most Half Filled



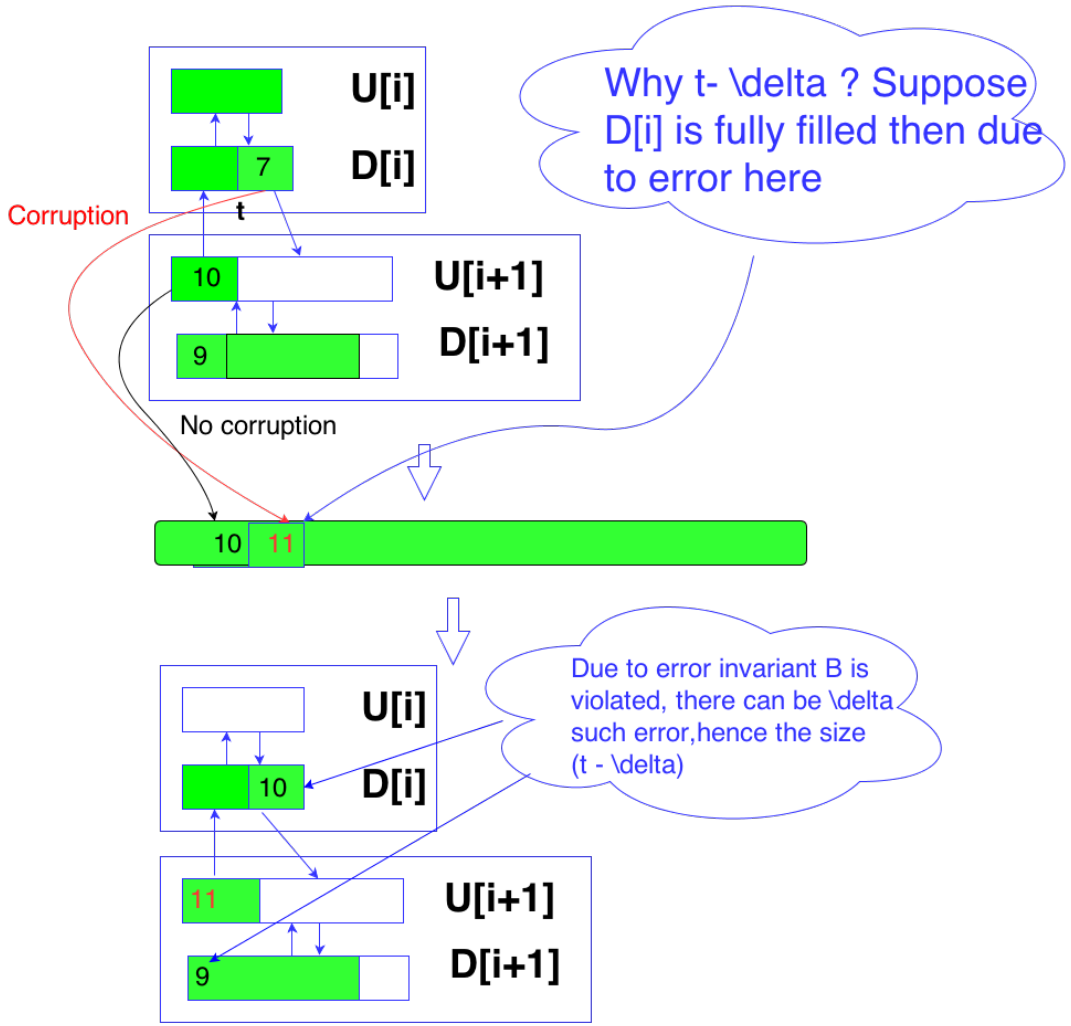
Insert


U[i]
D[i]
U[i+1]
Inserts go into up buffers which can voilate invariant D
U[i]
D[i]
U[i+1]

Insert


U[i]
D[i]
t
U[i+1]
D[i+1]
Faithfully merge U[i], D[i] and U[i+1]
t is size of D[i]
Put (t - \delta) in D[i]
and rest in U[i+1]
U[i] empty
U[i]
D[i]
U[i+1]
D[i+1]

Insert


U[i]
D[i]
7
t
Corruption
10
U[i+1]
D[i+1]
9
No corruption
10
11
Why t- \delta ? Suppose D[i] is fully filled then due to error here
U[i]
D[i]
10
11
U[i+1]
D[i+1]
9
Due to error invariant B is violated, there can be \delta such error,hence the size (t - \delta)

Push and *Pull*

- *Push* and *Pull* primitives used fro *Insert* and *DeleteMin*
- *Push* pushes elements to higher layers when Up at most half invariant is broken
- Similarly, *Pull* pulls elements from higher layer when Down at least half invariant is broken

Algorithm for *Push*

1: **function** PUSH(U_i)
2: $d_i = |D_i|$
3: $M = U_i \cup D_i \cup U_{i+1}$
4: $D_i = M[0..(d_i - \delta)]$
5: $U_i = []$
6: **if** $i = k$ **then**
7: $U_{k+1} = []$
8: $D_{k+1} = M[(d_i - \delta)..]$
9: k = k+1
10: **else**
11: $U_{i+1} = M[(d_i - \delta)..]$
12: **if** $U_{i+1} > s_{i+1}/2$ **then**
13: Push(U_{i+1})
14: **if** $D_i < s_i/2$ **then**
15: Pull(D_i)

Algorithm for *Pull*

1: **function** PULL(D_i)
2: $d_i = |D_i|$
3: $d_{i+1} = |D_{i+1}|$
4: $M = U_i \cup D_i \cup U_{i+1}$
5: $D_i = M[0..s_i]$
6: $D_{i+1} = [s_i..(d_{i+1} + d_i - \delta)]$
7: $U_{i+1} = [d_{i+1} + d_i - \delta..]$
8: **if** $U_{i+1} > s_{i+1}/2$ **then**
9: Push(U_{i+1})
10: **if** $D_{i+1} < s_{i+1}/2$ **then**
11: Pull(D_{i+1})

Insert

- For insertion we maintain a buffer I of size $(\delta + logn + 1)$ and simply append the new element to buffer
- When I is full we faithfully sort it and faithfully merge with U_0
- Call *Push* on U_0 if at most invariant is broken
- For *DeleteMin* return the minimum of the first $\delta + 1$ elements of D_0 and all of I
- if D_0 underflows, call *Pull*

Complexity

- *Insert* and *DeleteMin* takes $O(logn + \delta)$ amortized time
- *Push* and *Pull* on a buffer is called at most once each request
- Intuitively, $\Omega(s_i)$ operations happen between any two call at any level L_i which gives the amortized bounds
- Lower bound is proved to be the same
- Uses $O(n + \delta)$ space

Relation with Cache Oblivious

- This structure was inspired by cache oblivious priority queue by Bender et al in 2002
- There are several data structures which are adpated from their cache oblivious versions
- Even though resilient model does not have memory hierarchy
- One reason could be that cache oblivious structure use chunks of array to gain by locality and employs less pointers
- So they become amenable to be adapted to resilient version because then the small number of pointers can be stored in reliable memory
- Note that there is a tree based cache oblivious priority queue too

References

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