CSE 613: Parallel Programming

Lecture 4
(Scheduling and Work Stealing)

(inspiration for some slides comes from lectures given by Charles Leiserson)

Rezaul A. Chowdhury

Department of Computer Science
SUNY Stony Brook
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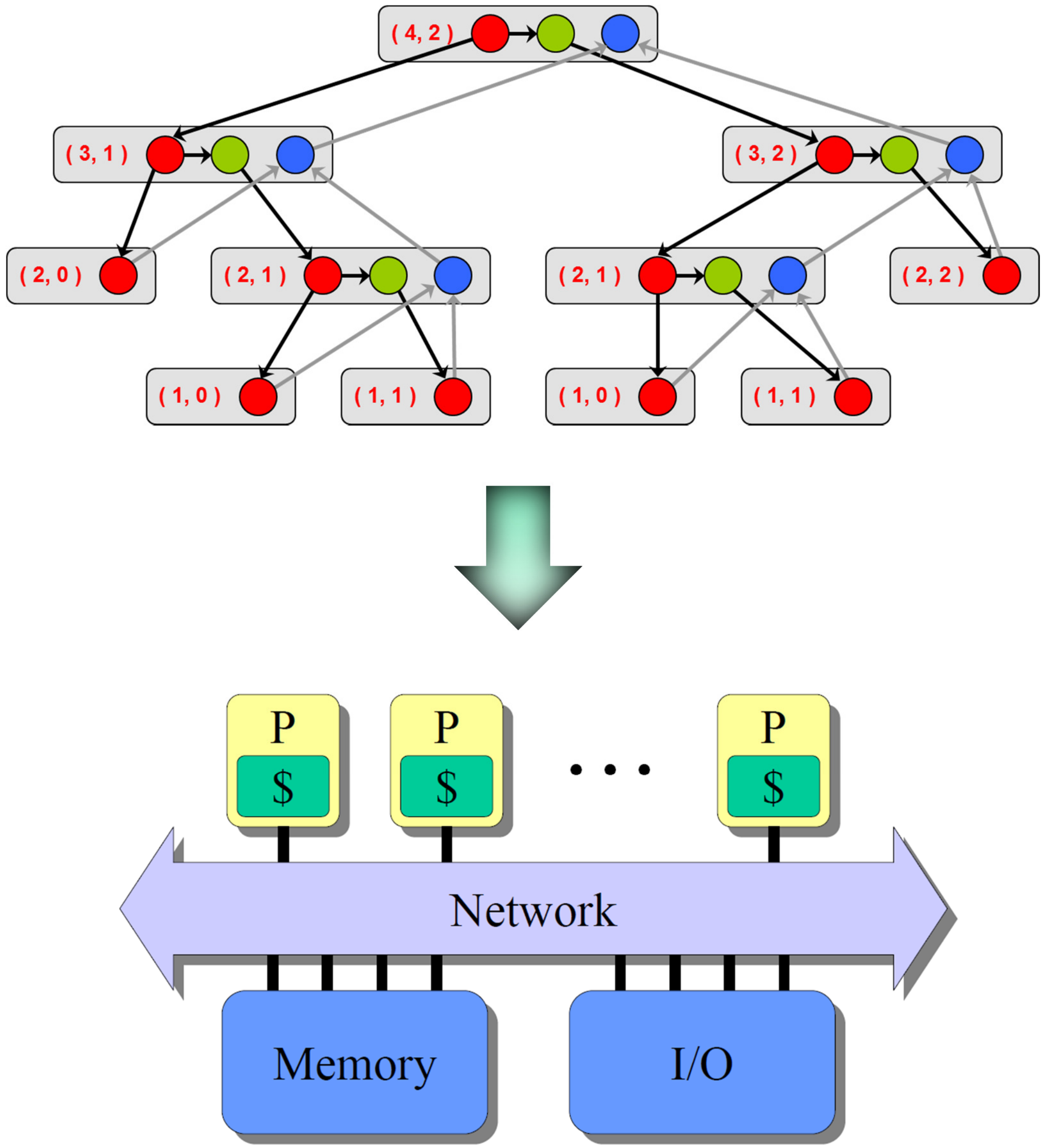
Scheduler

A *runtime/online scheduler* maps tasks to processing elements dynamically at runtime.

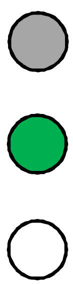
The map is called a *schedule*.

An *offline scheduler* prepares the schedule prior to the actual execution of the program.

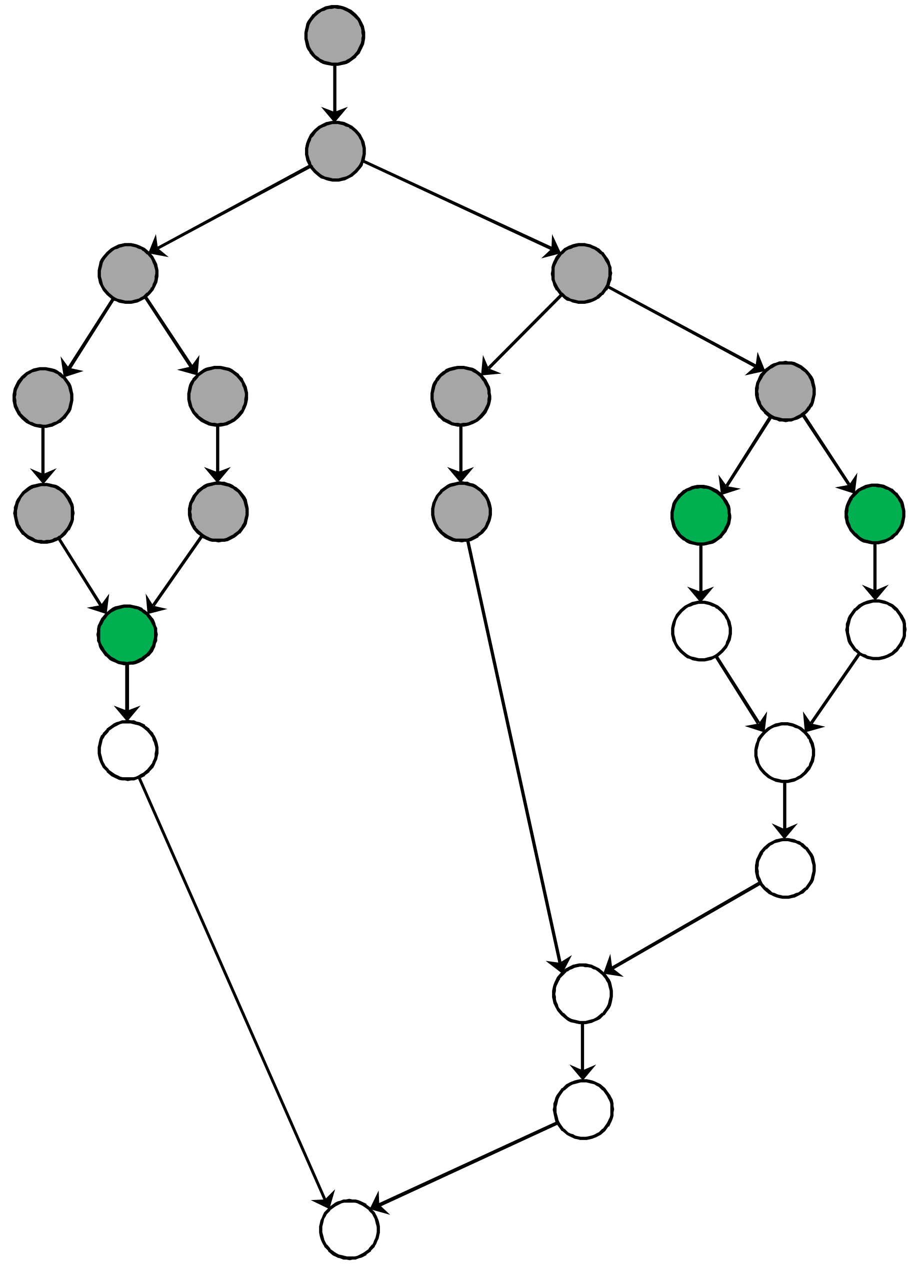


Greedy Scheduling

A strand / task is called *ready* provided all its parents (if any) have already been executed.


executed task
ready to be executed
not yet ready

A *greedy scheduler* tries to perform as much work as possible at every step.



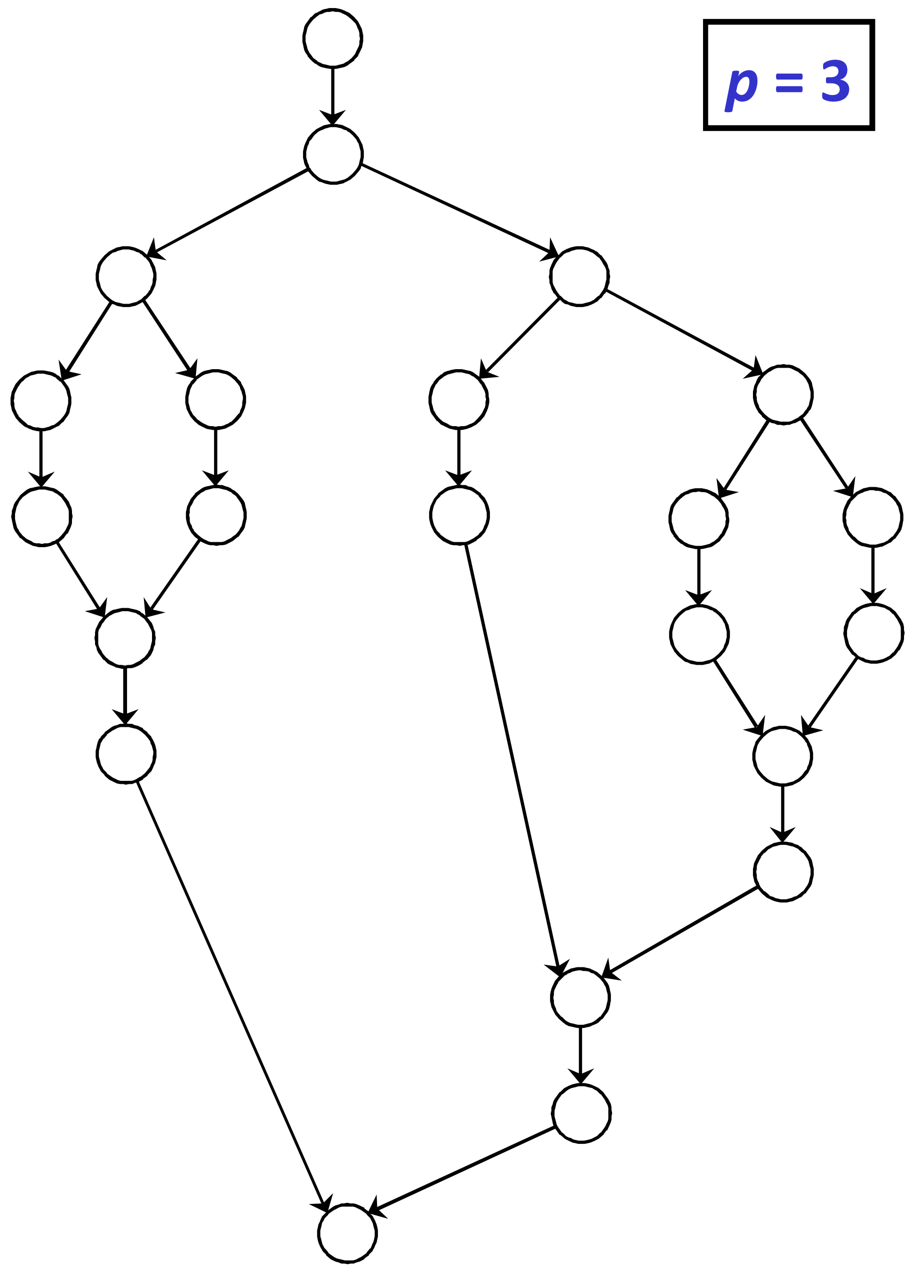
A Centralized Greedy Scheduler

Let p = number of cores

At every step:

– if $\geq p$ tasks are ready: execute any p of them (complete step)

– if $< p$ tasks are ready: execute all of them (incomplete step)



A Centralized Greedy Scheduler

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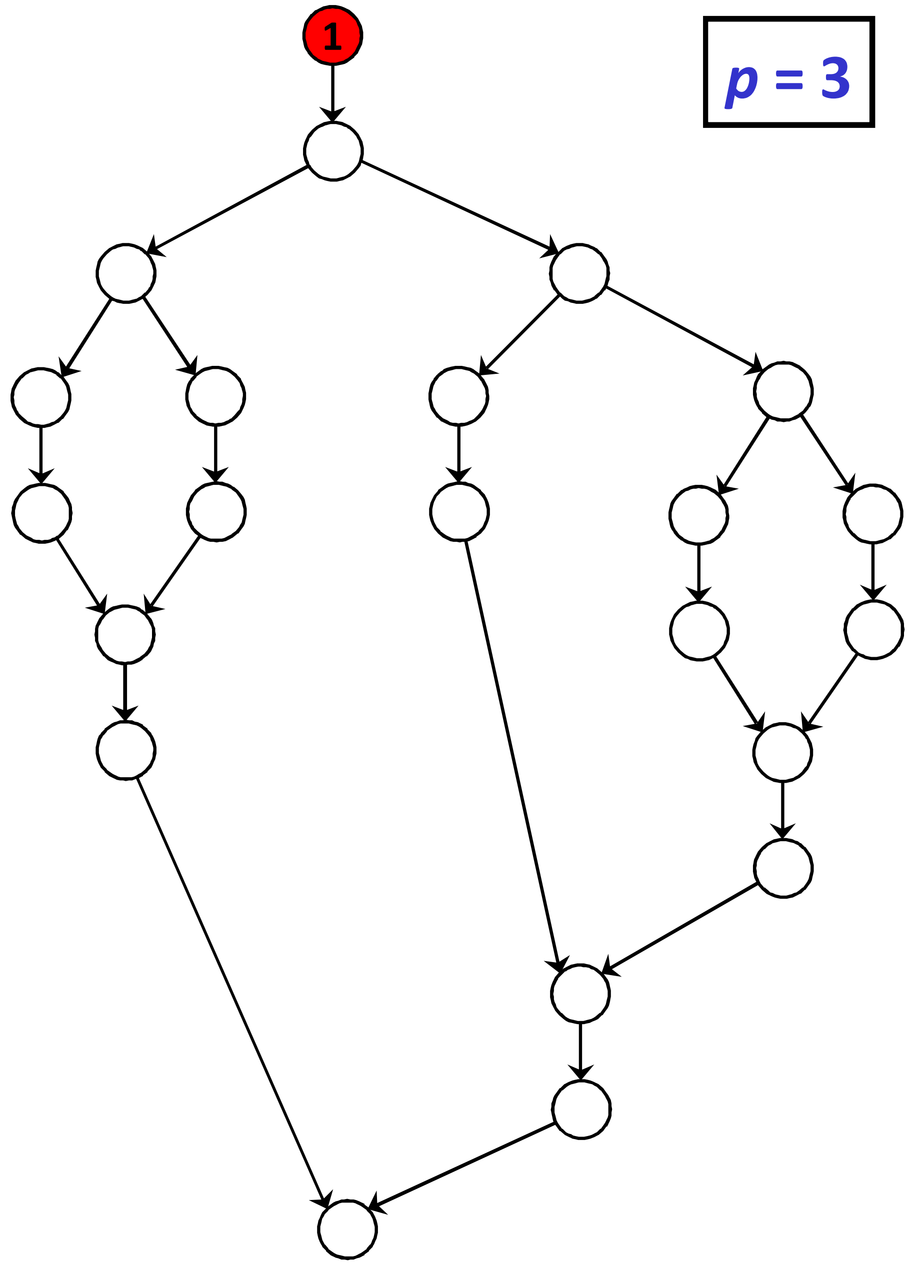

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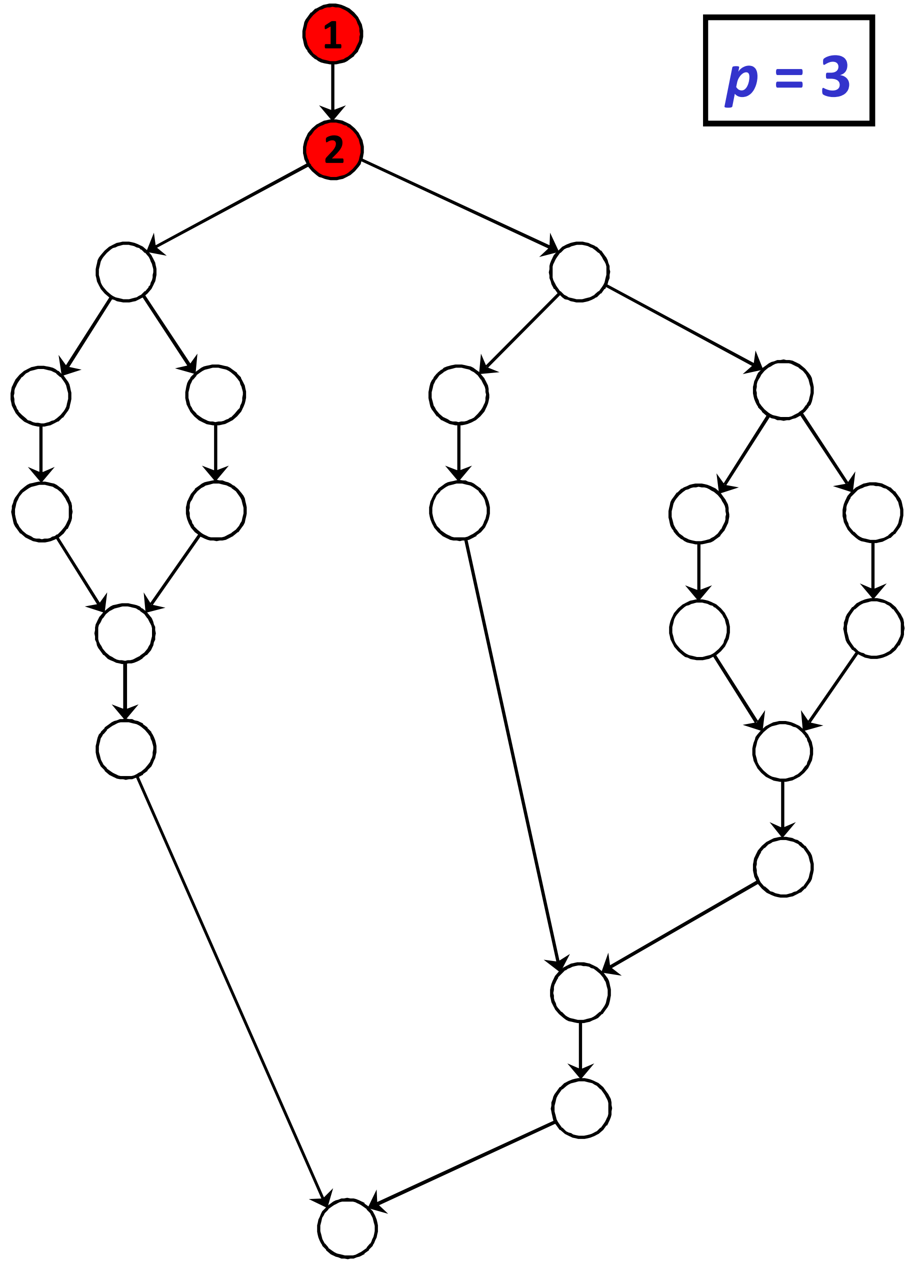

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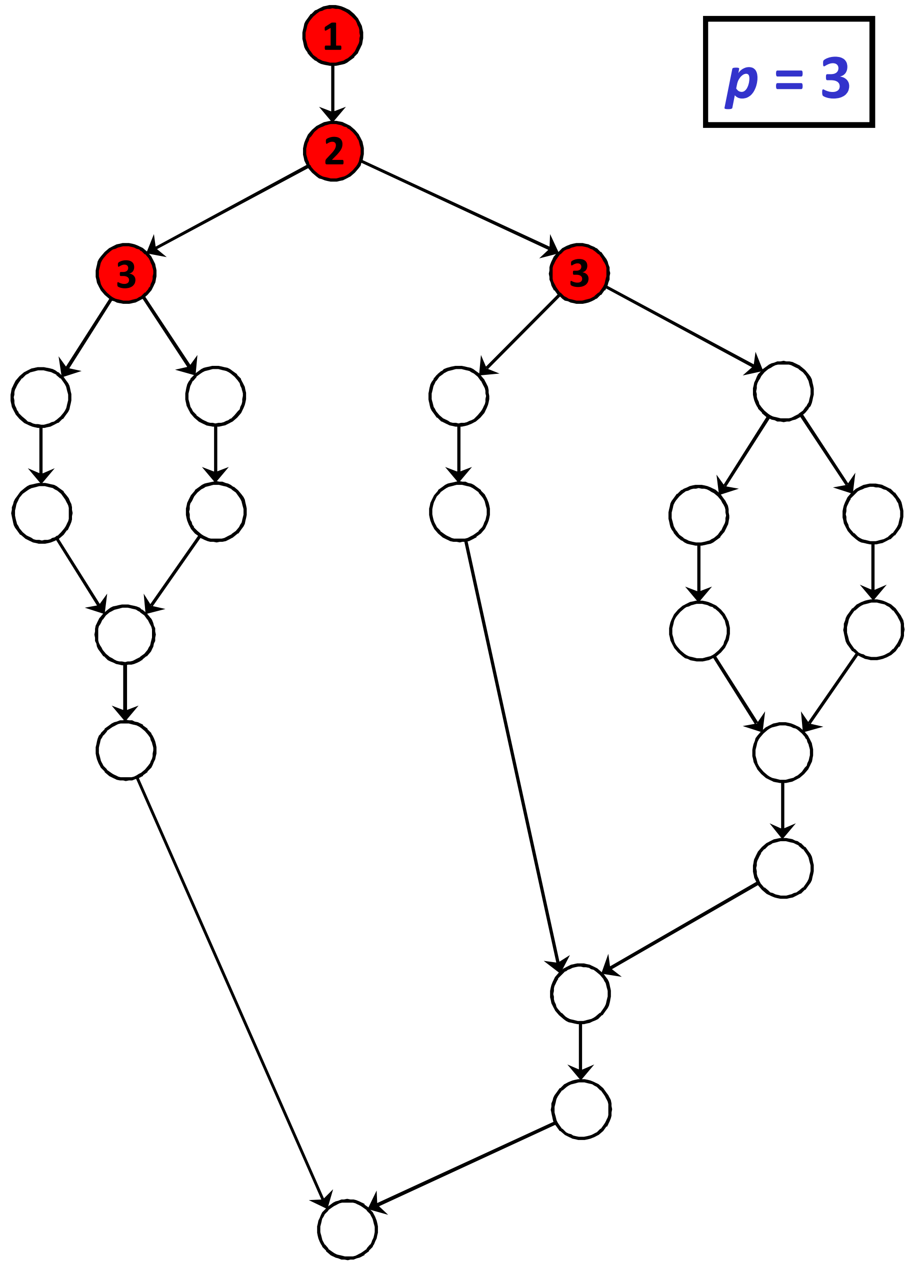

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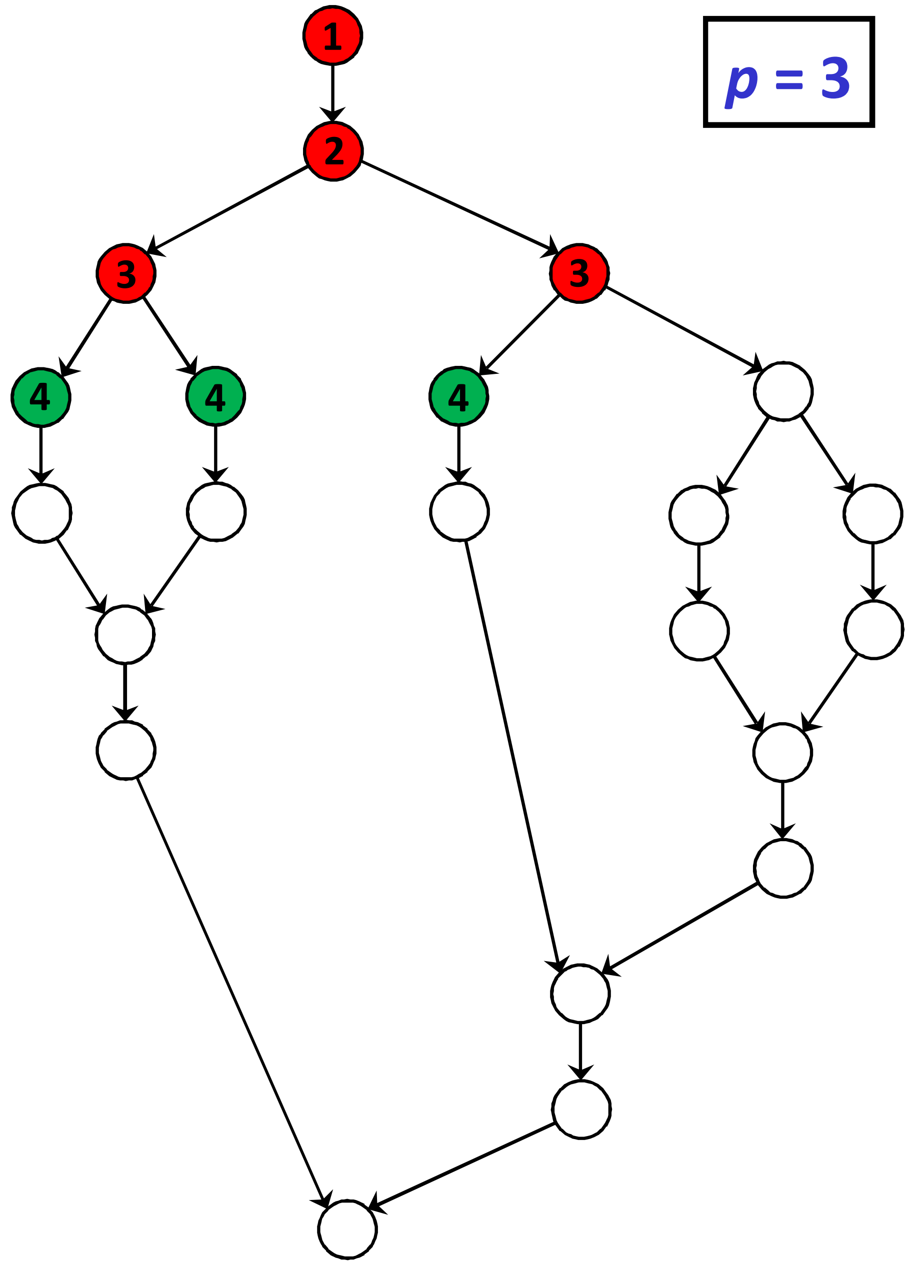

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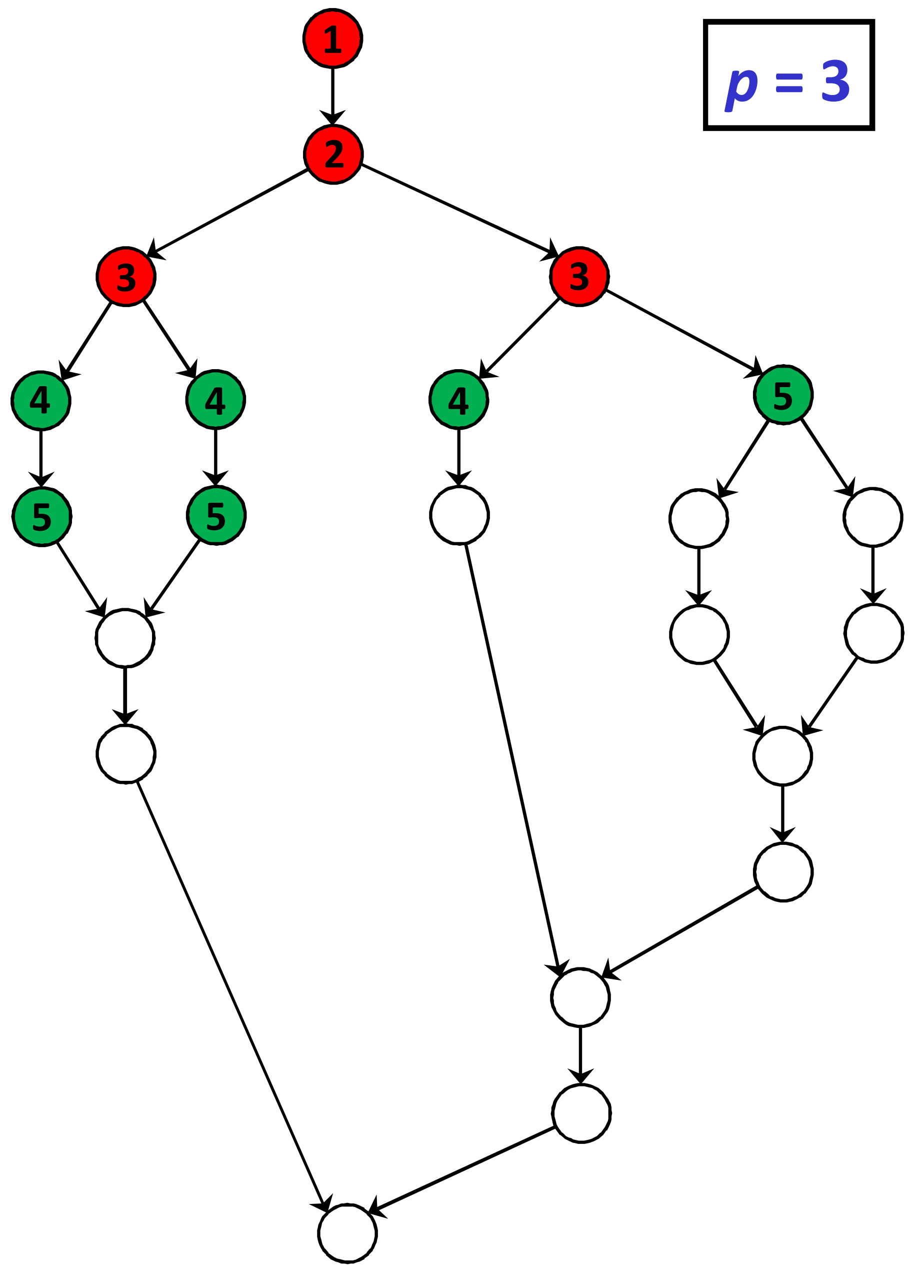

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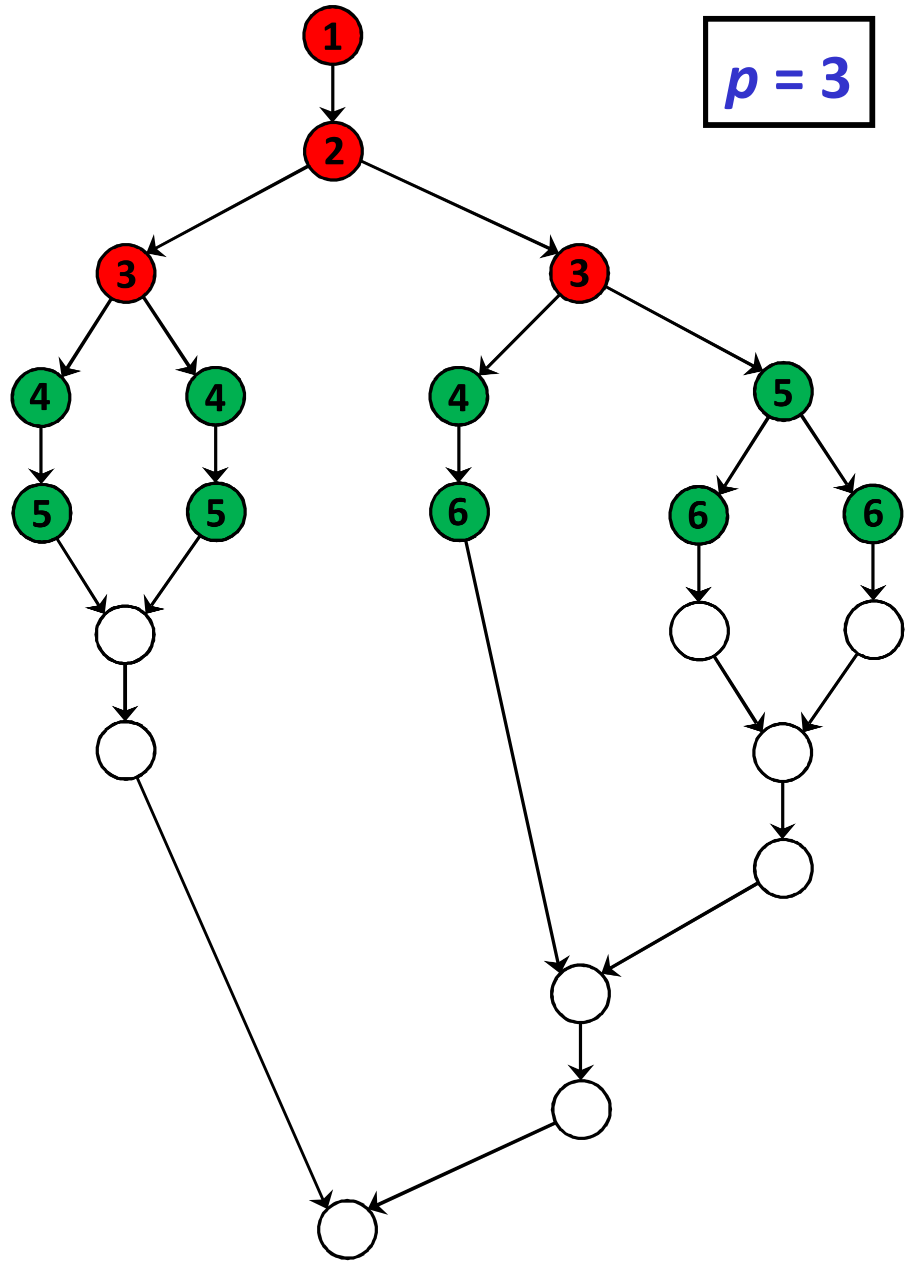

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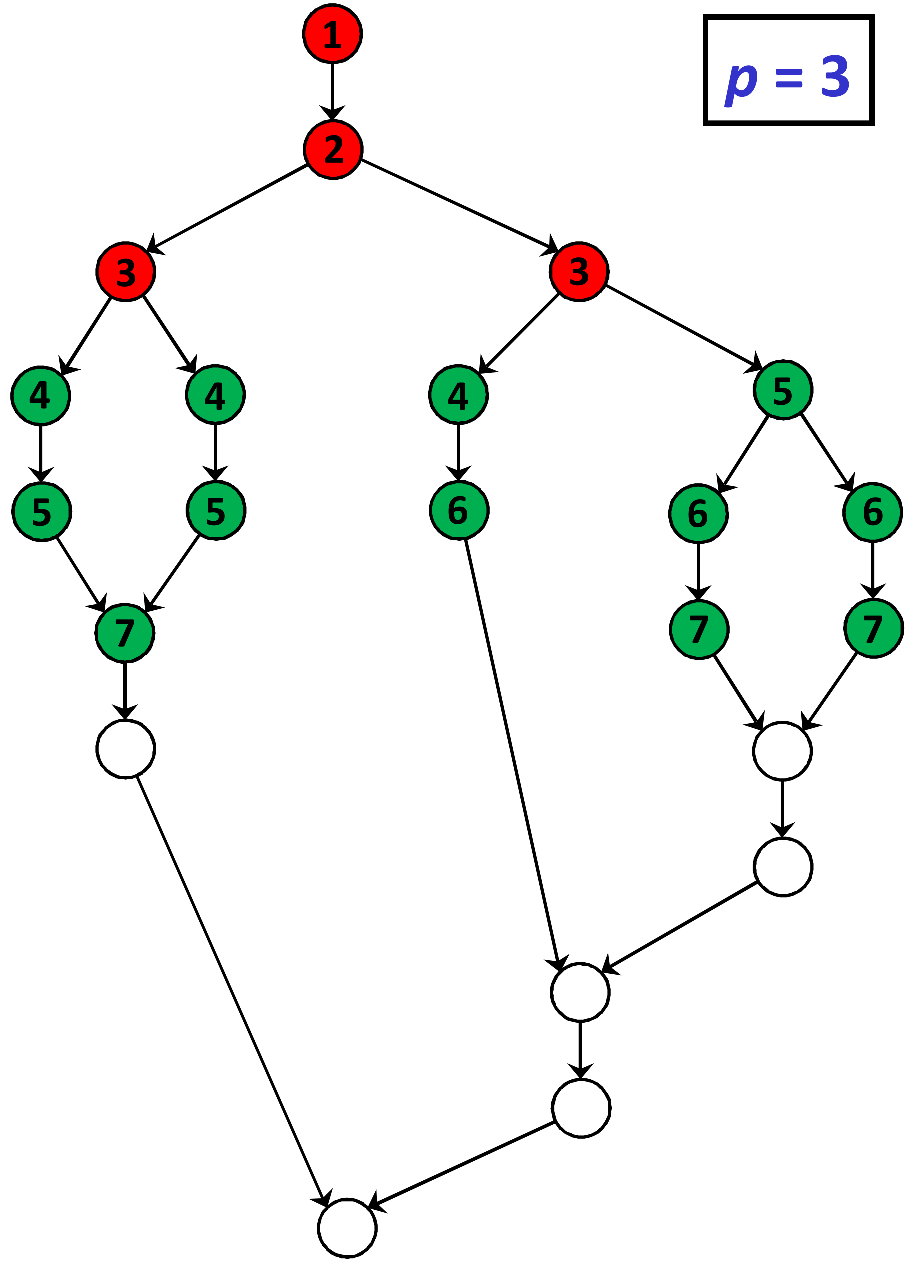

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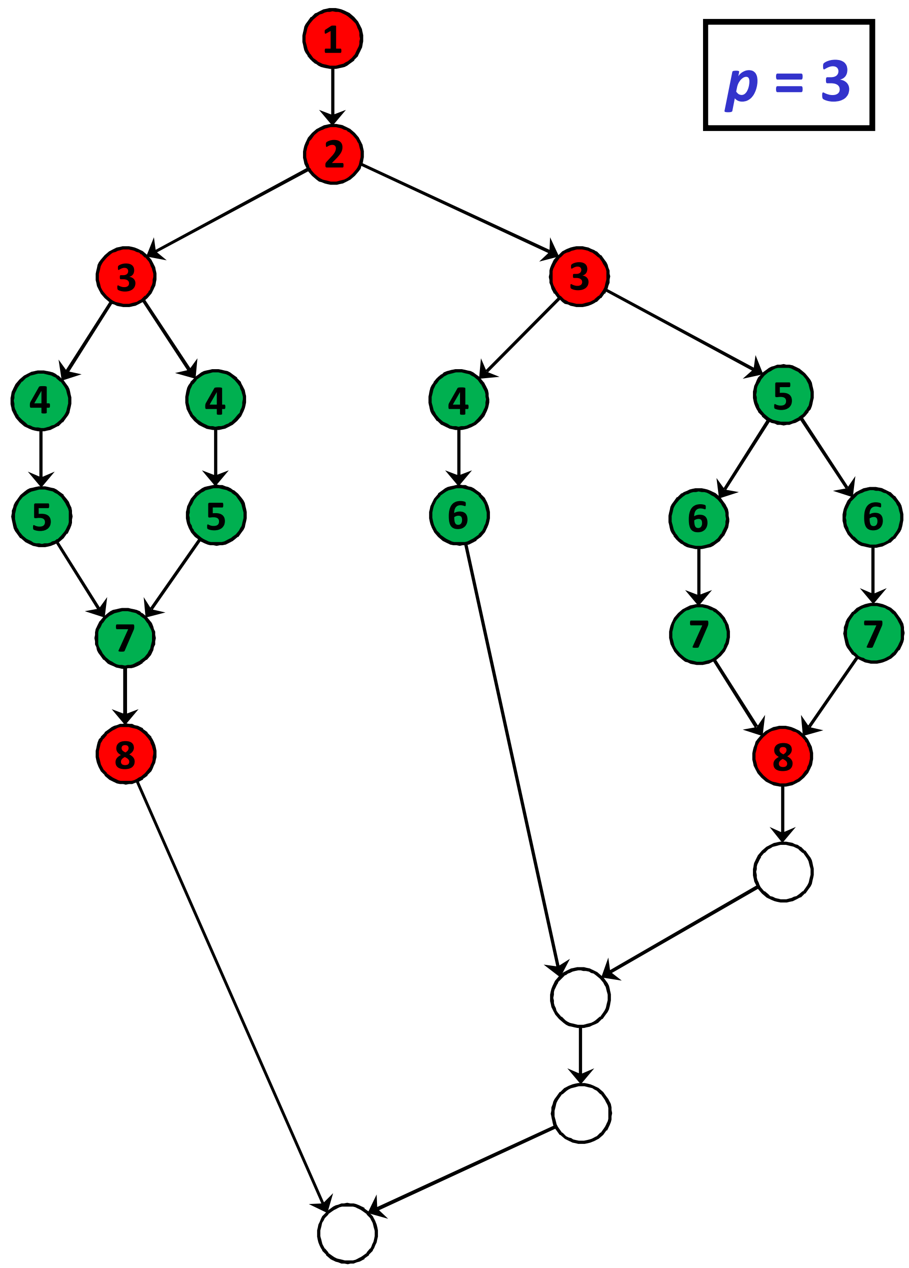

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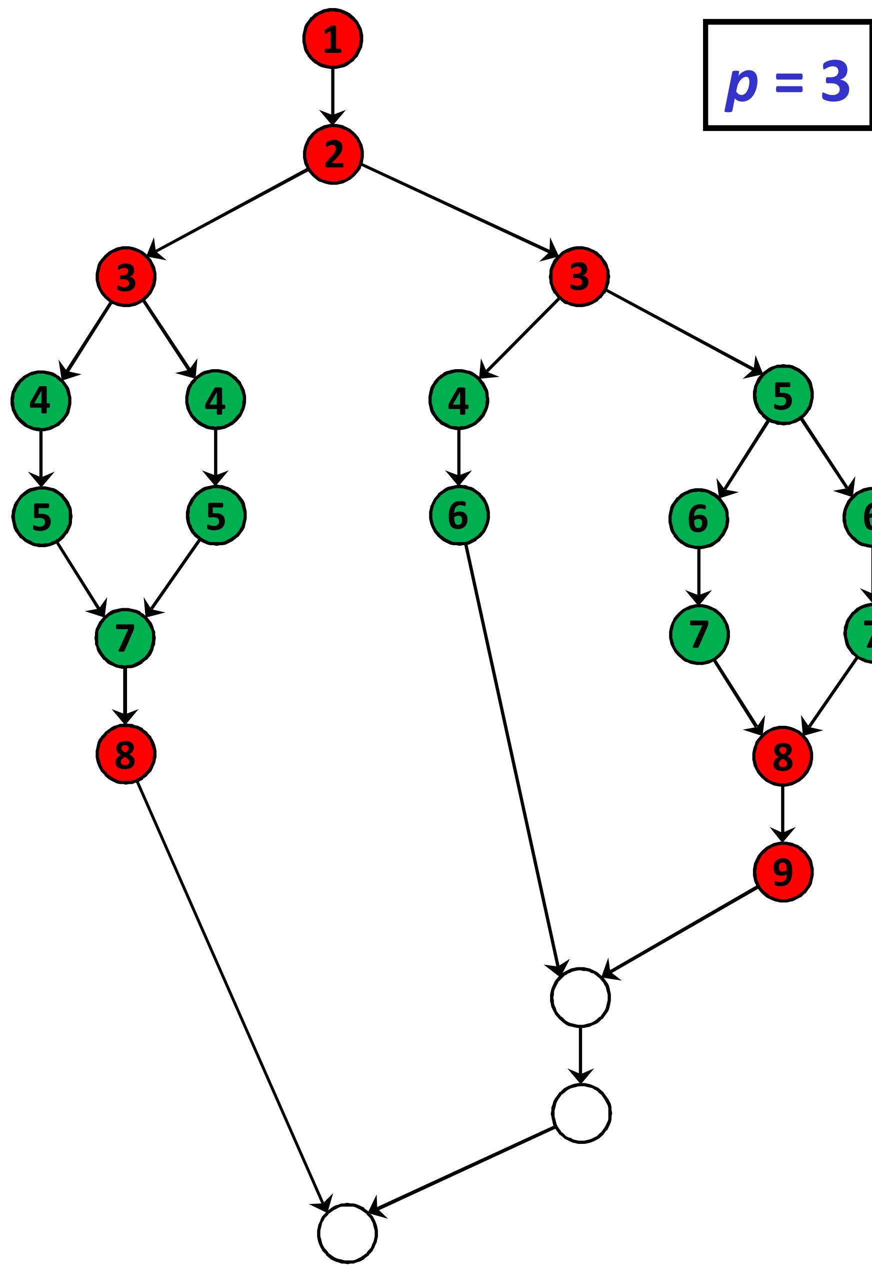

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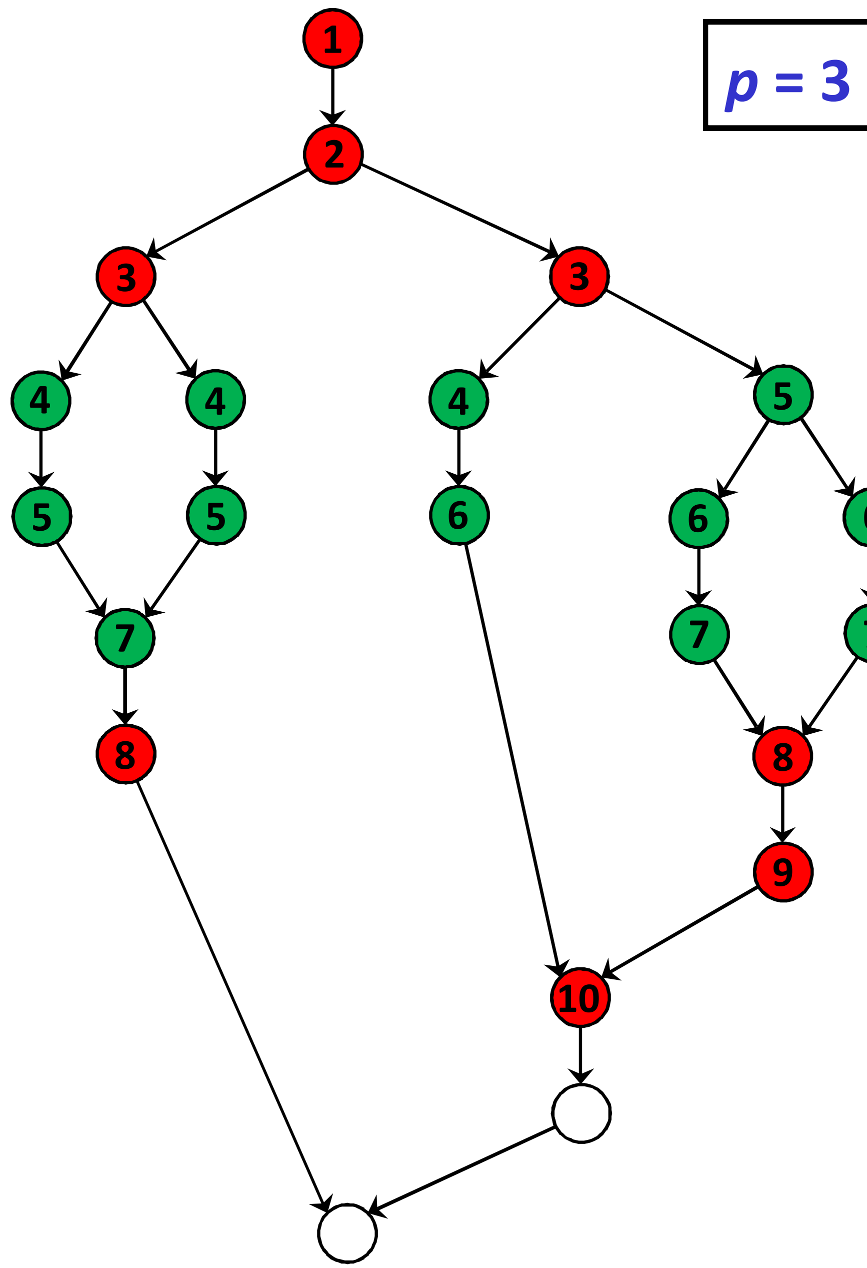

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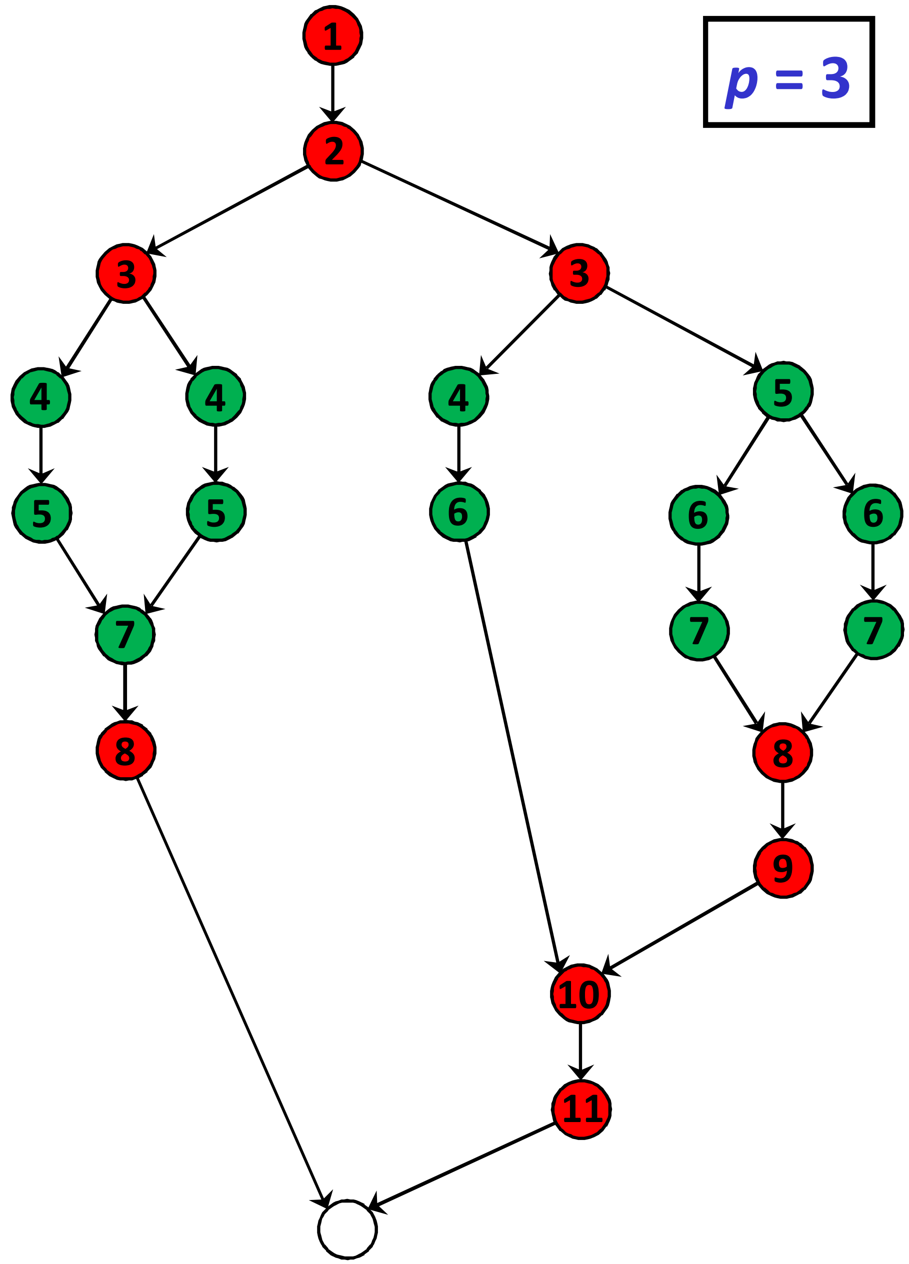

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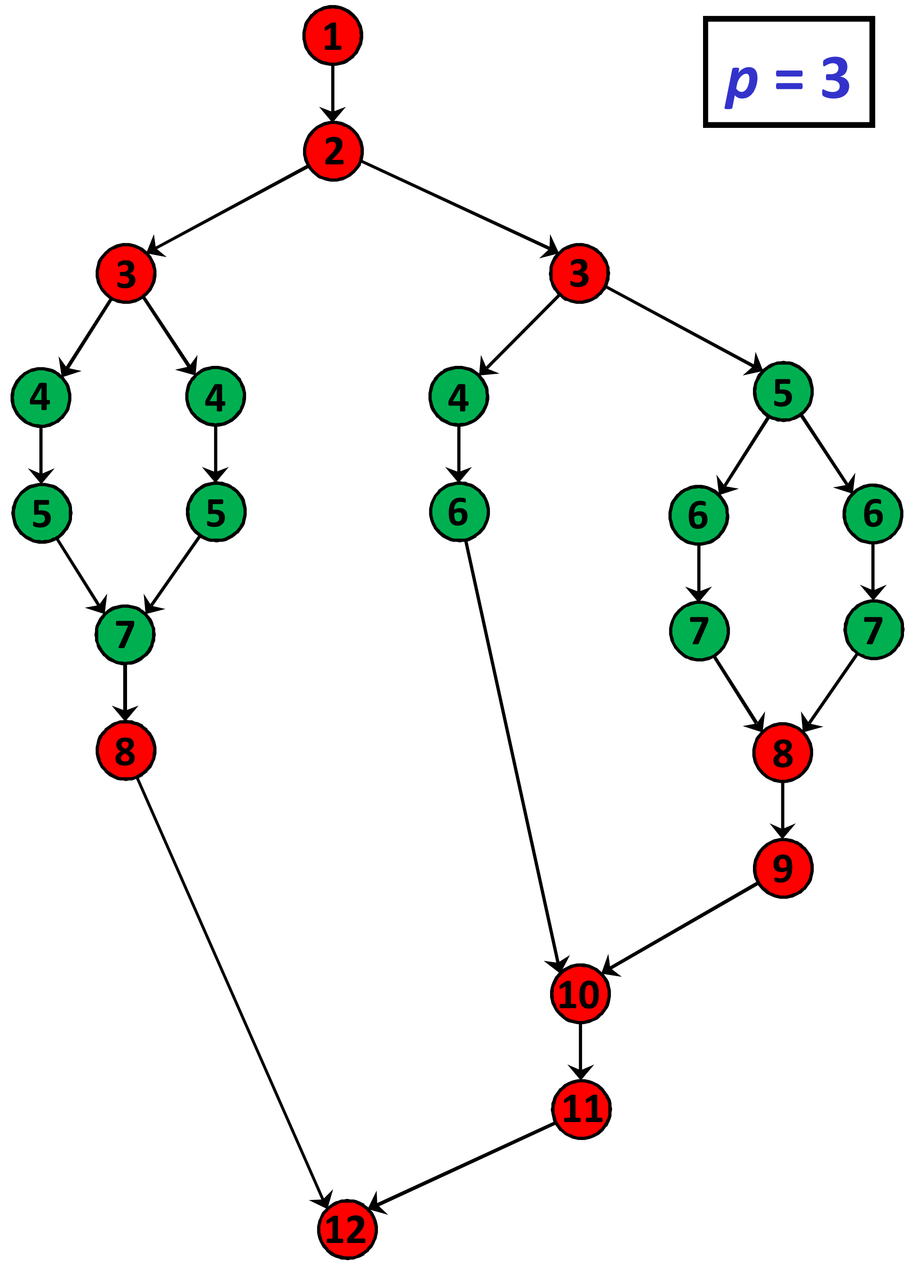

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Greed Scheduling Theorem

Theorem [Graham'68, Brent'74]:

For any greedy scheduler,

$$T_p \leq \frac{T_1}{p} + T_\infty$$

Proof:

T_p = #complete steps

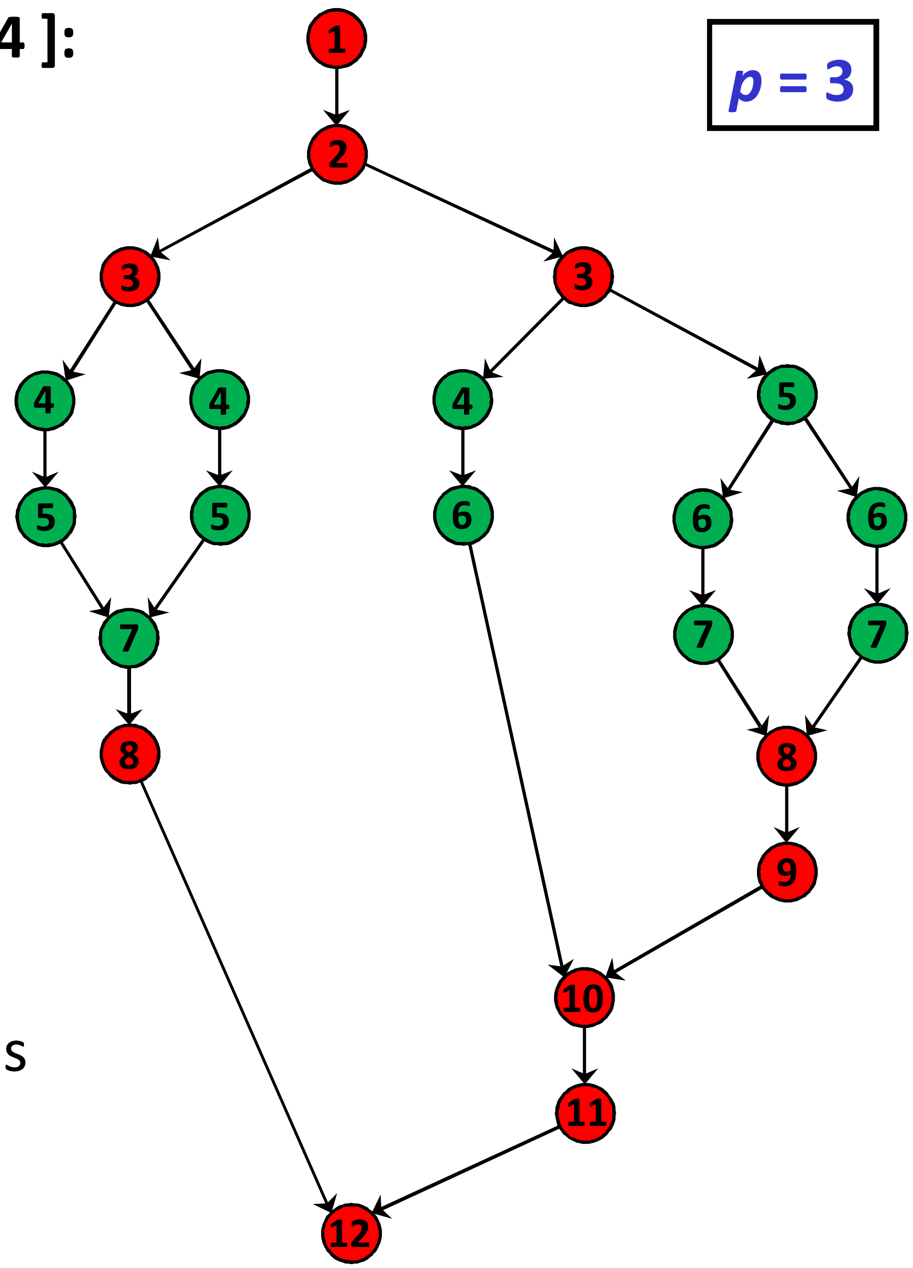
+ #incomplete steps

– Each complete step performs *p* work:

#complete steps $\leq \frac{T_1}{p}$

– Each incomplete step reduces the span by 1:

#incomplete steps $\leq T_\infty$



Optimality of the Greedy Scheduler

Corollary 1: For any greedy scheduler $T_p \leq 2T_p^*$, where T_p^* is the running time due to optimal scheduling on *p* processing elements.

Proof:

Work law: $T_p^* \geq \frac{T_1}{p}$

Span law: $T_p^* \geq T_\infty$

$\therefore$ From Graham-Brent Theorem:

$$T_p \leq \frac{T_1}{p} + T_\infty \leq T_p^* + T_p^* = 2T_p^*$$

Optimality of the Greedy Scheduler

Corollary 2: Any greedy scheduler achieves $S_p \approx p$ (i.e., nearly linear speedup) provided $\frac{T_1}{T_\infty} \gg p$.

Proof:

Given, $\frac{T_1}{T_\infty} \gg p \Rightarrow \frac{T_1}{p} \gg T_\infty$

$\therefore$ From Graham-Brent Theorem:

$$T_p \le \frac{T_1}{p} + T_\infty \approx \frac{T_1}{p}$$

$$\Rightarrow \frac{T_1}{T_p} \approx p \Rightarrow \; S_p \approx p$$

Work-Sharing and Work-Stealing Schedulers

Work-Sharing

- Whenever a processor generates new tasks it tries to distribute some of them to underutilized processors
- Easy to implement through centralized (global) task pool
- The centralized task pool creates scalability problems
- Distributed implementation is also possible (but see below)

Work-Stealing

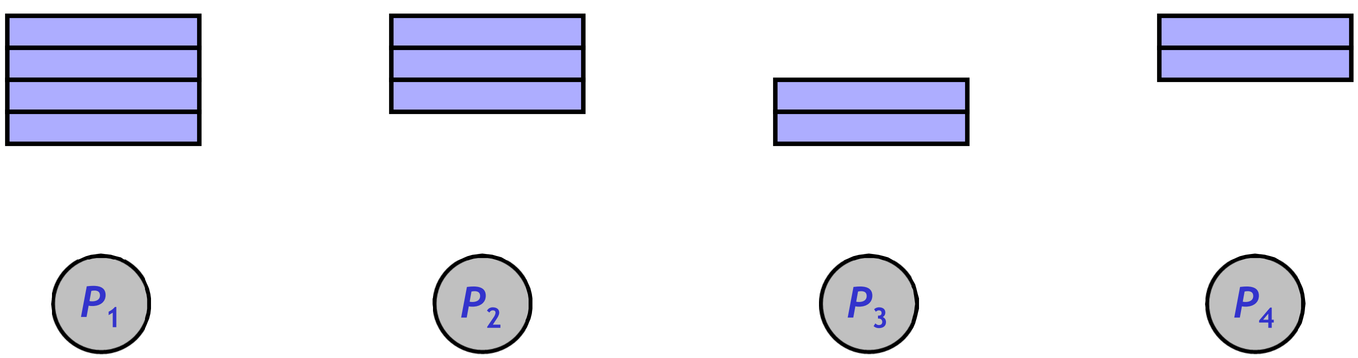
- Whenever a processor runs out of tasks it tries steal tasks from other processors
- Distributed implementation
- Scalable
- Fewer task migrations compared to work-sharing (why?)

Cilk++'s Work-Stealing Scheduler

- A *randomized distributed* scheduler
- Time bounds
  - Provably: $T_p = \frac{T_1}{p} + \mathrm{O}(T_\infty)$ (expected time)
  - Empirically: $T_p \approx \frac{T_1}{p} + T_\infty$
- Space bound: ≤ *p* × serial space bound
- Has provably good *cache performance*

Cilk++'s Work-Stealing Scheduler

- Each core maintains a *work dqueue* of ready threads
- A core manipulates the bottom of its dqueue like a stack
  - Pops ready threads for execution
  - Pushes new/spawned threads
- Whenever a core runs out of ready threads it *steals* one from the top of the dqueue of a *random* core



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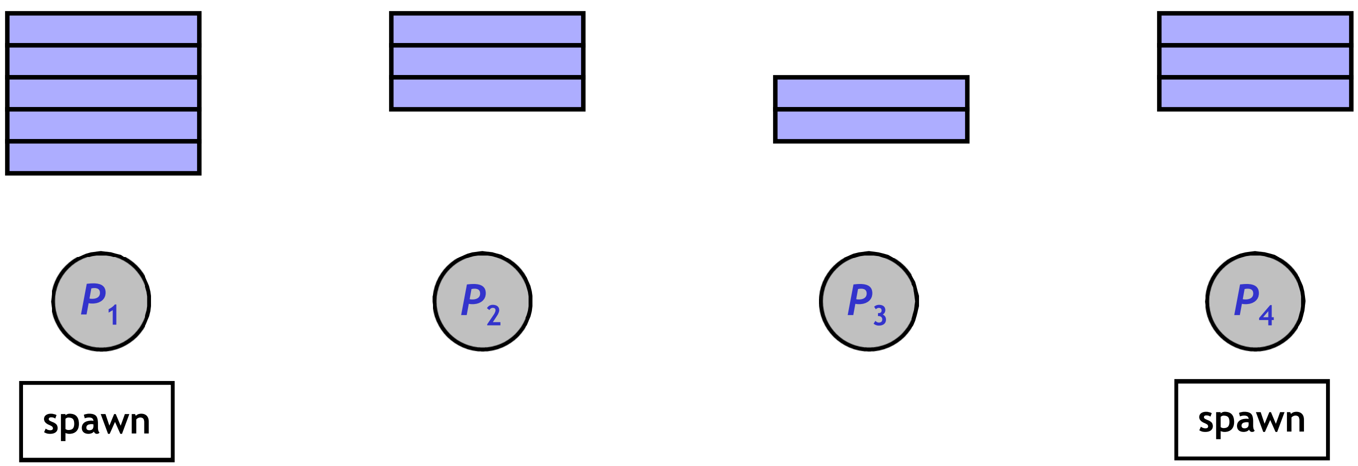

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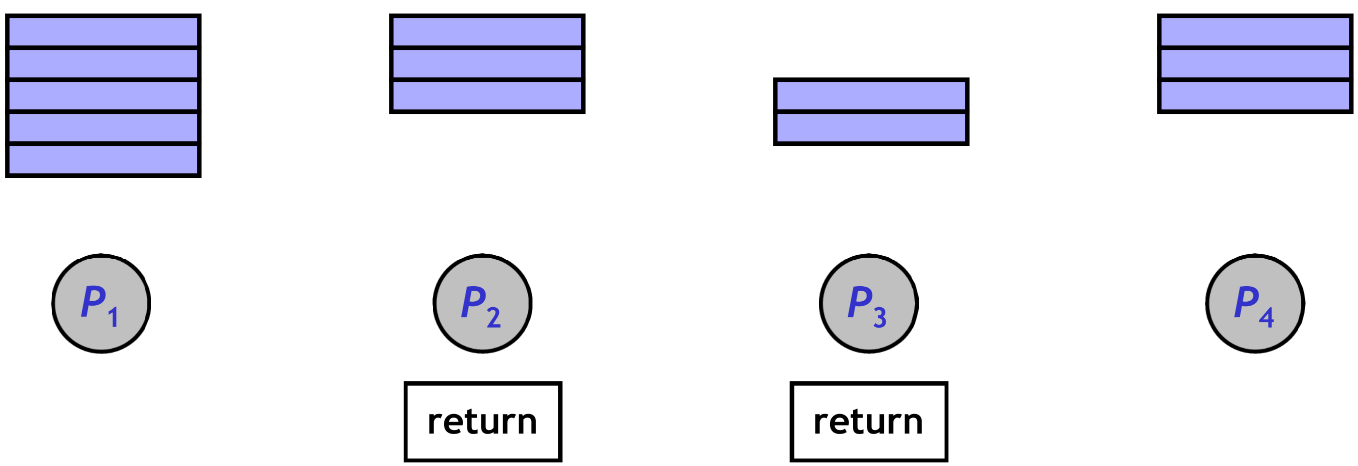

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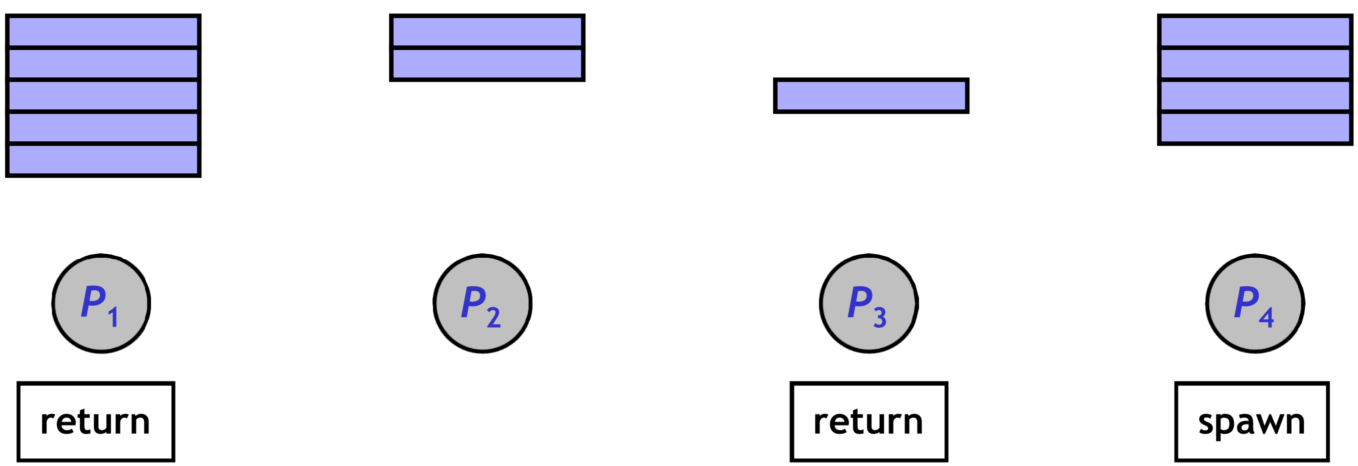

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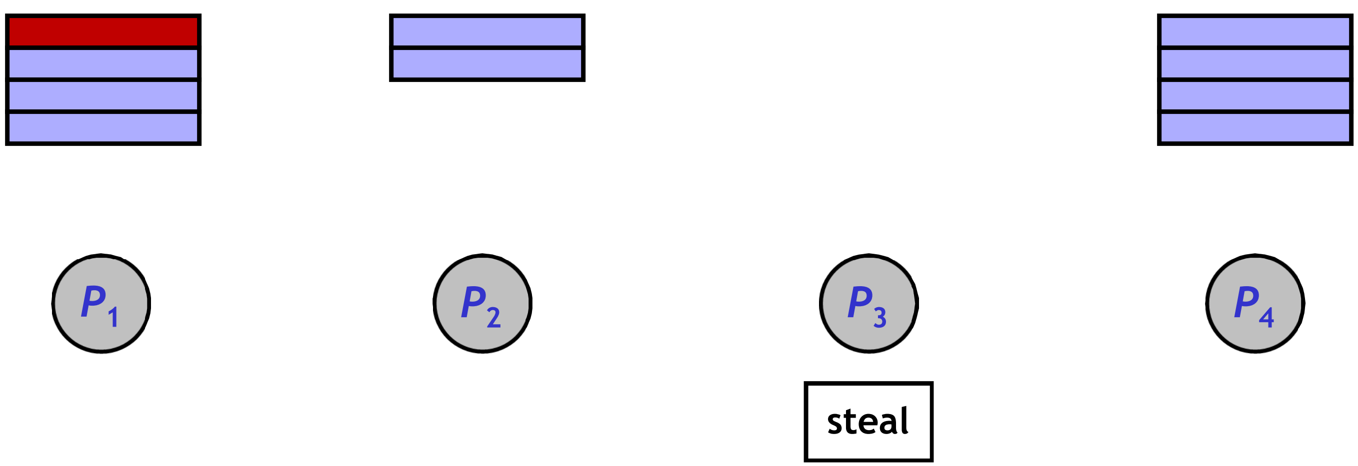

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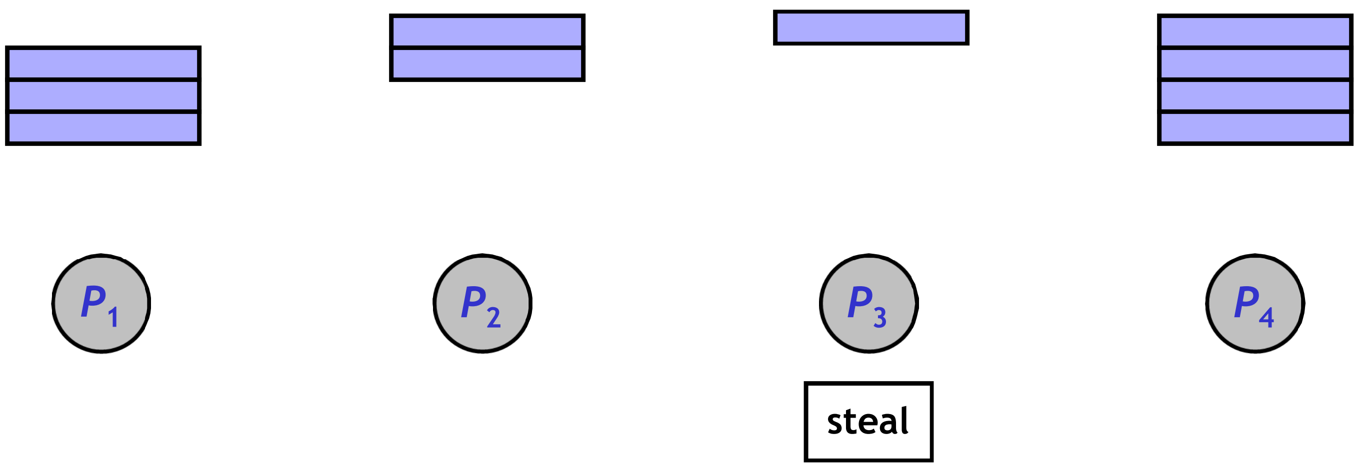

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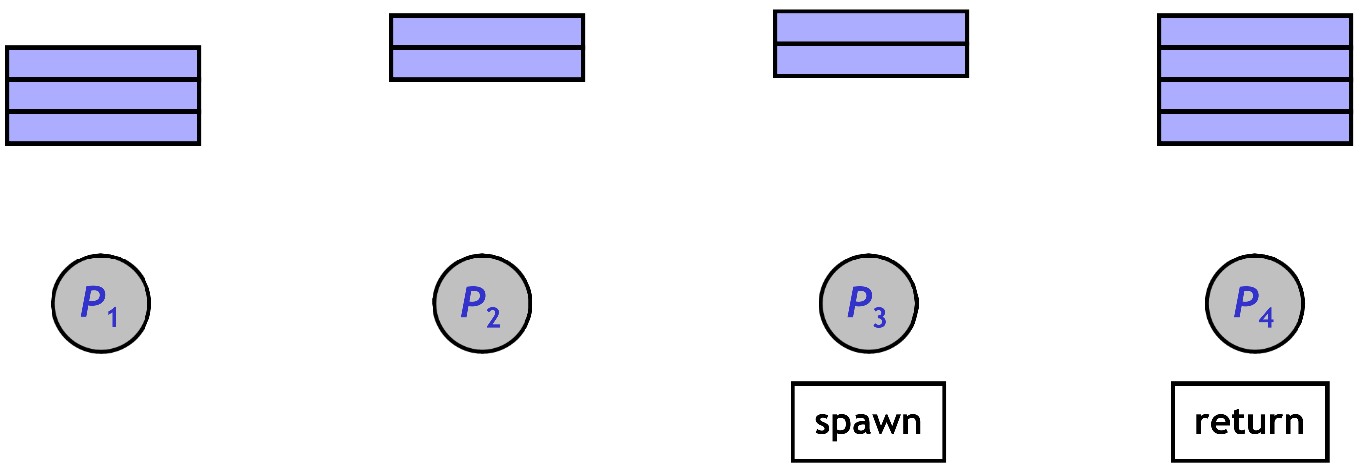
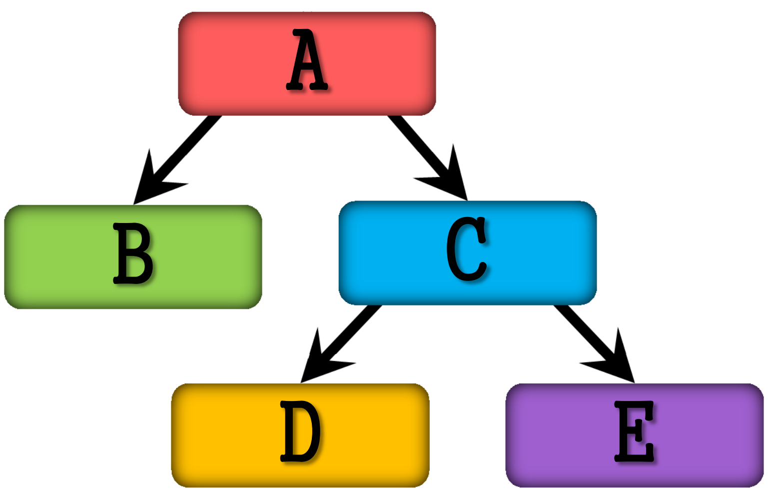


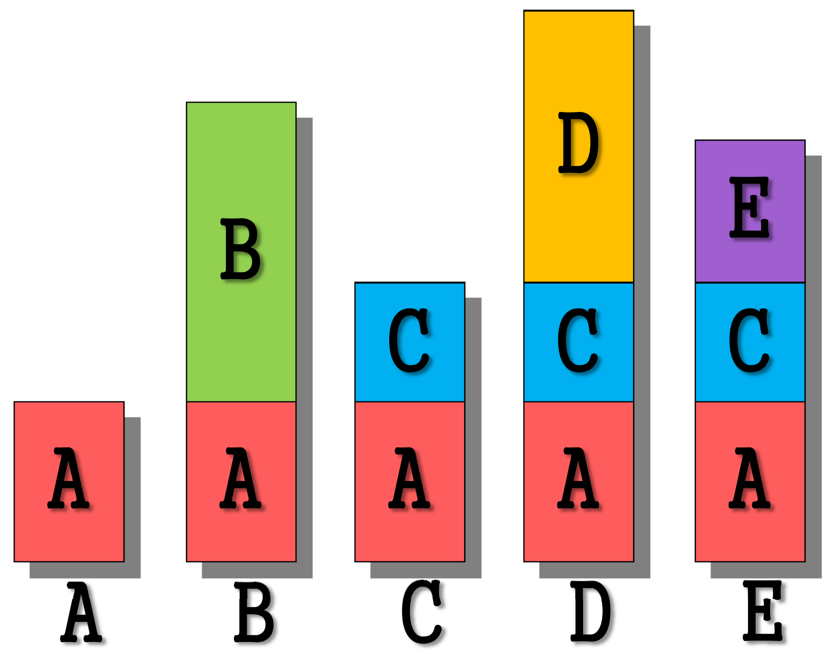
Space Usage of Cilk++'s Scheduler (Problem with Linear Stacks)

- C/C++ uses a *linear* (contiguous) *stack* to store function activation records (i.e., stack frames)
- When a function is called
  - The caller pushes the return address onto the stack
  - The callee allocates its local variables in the stack space
- The callee's stack frame lies directly above the caller's one
- But linear stacks do not work well for parallel programs (why?)

Space Usage of Cilk++'s Scheduler (Cactus Stack)

– Cilk++ uses a *cactus stack*
  - A heap allocated tree of stack frames
  - Not necessarily contiguous
– A cactus stack supports several views of the stack in parallel





Space Usage of Cilk++'s Scheduler

Theorem: Let S_1 be the stack space required by a serial execution of a Cilk++ program. Then the stack space used when run on *p* processing elements is, $S_p \leq pS_1$.

Proof:

– At any given time step, the spawn subtree can have at most *p* leaves

– For each such leaf, the stack space used by it and all its ancestors is at most S_1