D-NURBS for Physics-Based Shape Modeling

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Geometric modeling is critical to computer graphics, computer-aided design, and virtual environment. Among various geometric formulations, non-uniform rational B-splines, or NURBS [4] have become a *de facto* standard in commercial modeling systems. They provide a unified mathematical representation for free-form curves and surfaces and for standard analytic shapes such as conics, quadrics, and surfaces of revolution. Experienced practitioners can design a large variety of NURBS objects by adjusting the positions of control points, setting the values of associated weights, and modifying the distribution of knots [1, 2]. Despite a great number of modern 3D interaction devices and techniques, this indirect geometric design can be clumsy and laborious when designing complex, real-world objects. In general, traditional design does not exploit the full potential of geometric NURBS.

This paper proposes a physics-based framework for shape modeling with NURBS that addresses these problems through a new class of models known as *Dynamic* NURBS, or D-NURBS [3]. D-NURBS are physics-based models that incorporate mass distributions, internal deformation energies, and other physical quantities into the NURBS geometric formulation. The models are governed by dynamic differential equations that, when integrated numerically through time, continuously evolve the control points and weights in response to applied forces. Therefore, shapes can be sculpted interactively using a variety of force-based "tools." For example, Fig. 1 illustrates the result of interactively sculpting a D-NURBS wine glass shape using simple spring forces. In addition, the dynamic model can be built upon the standard geometric NURBS foundation. While shape design may proceed interactively or automatically at the physical level, existing geometric toolkits are concurrently applicable at the geometric level. More importantly, the two types of toolkits are compatible with each other. Designers are free to choose either one or both to achieve design requirements.

Physics-based sculpting is intuitively appealing for shape design and control. It can free designers from having to make nonintuitive decisions, such as assigning weights to NURBS. In addition, with physics-based direct manipulation, non-expert users are able to concentrate on visual shape variation without necessarily comprehending the underlying mathematical formulation. Within the physical dynamics framework, standard geometric NURBS objects inherit some of the universally familiar behaviors of physical, real-world objects. Thus, physics-based design augments (rather than supersedes) standard geometry and geometric design, offering attractive new advantages. In particular, the elastic energy functionals associated with D-NURBS allow the imposition of global qualitative "fairness" criteria through quantitative means. Linear or nonlinear shape constraints may be imposed either as hard constraints that must not be violated, or as soft constraints to be satisfied approximately. Constraints may be interpreted intuitively as forces and optimal shape design results when D-NURBS are allowed to achieve static equilibrium subject to these forces.

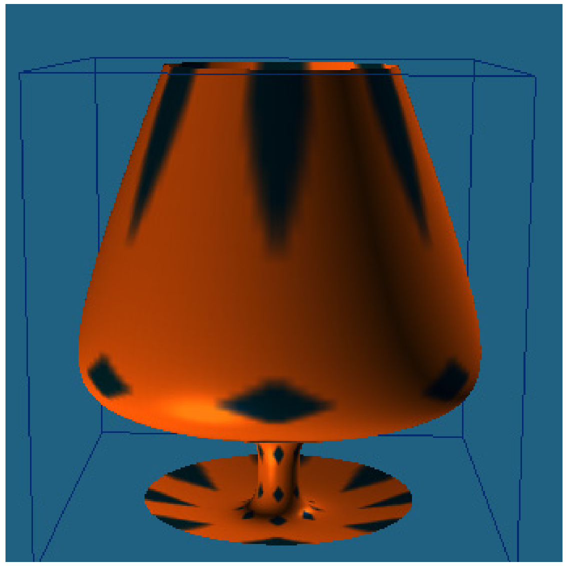

Figure 1: Interactive sculpting of a D-NURBS wine glass from a prototype cylinder.

We formulate four different varieties of D-NURBS: D-NURBS curves, tensor-product D-NURBS surfaces, swung D-NURBS surfaces, and triangular D-NURBS surfaces. We employ Lagrangian dynamics to arrive at the system of nonlinear ordinary differential equations that govern the shape and motion of D-NURBS. Next, we apply finite element analysis to reduce these equations to efficient numerical algorithms computable at interactive rates on common graphics workstations. Our D-NURBS formulation supports interactive direct manipulation of NURBS objects, which results in physically meaningful, hence intuitively predictable, deformation and shape variation.

We implement a prototype modeling environment based on D-NURBS. Using our D-NURBS modeling system, a modeler can interactively sculpt complex shapes not merely by kinematic adjustment of control points and weights, but dynamically as well—by applying simulated forces. Additional control over dynamic sculpting stems from the modification of physical parameters such as mass, damping, and elastic properties. We demonstrate that D-NURBS can be effective tools in a wide range of graphics applications such as shape blending, scattered data fitting, and interactive sculpting. Because D-NURBS can unify the features of the industry-standard geometry with the many demonstrated conveniences of interaction through physical dynamics, we anticipate that the physics-based design framework can be readily incorporated into commercial design systems for real-time shape modeling and design in virtual environments.

References

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