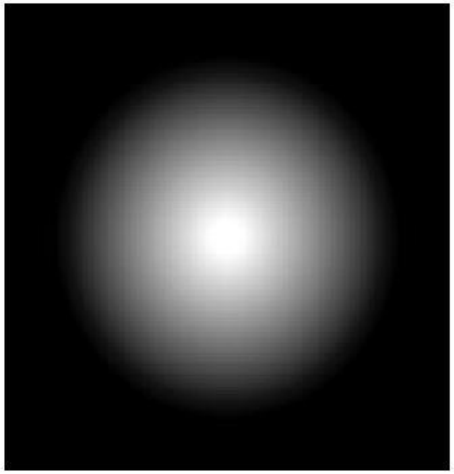
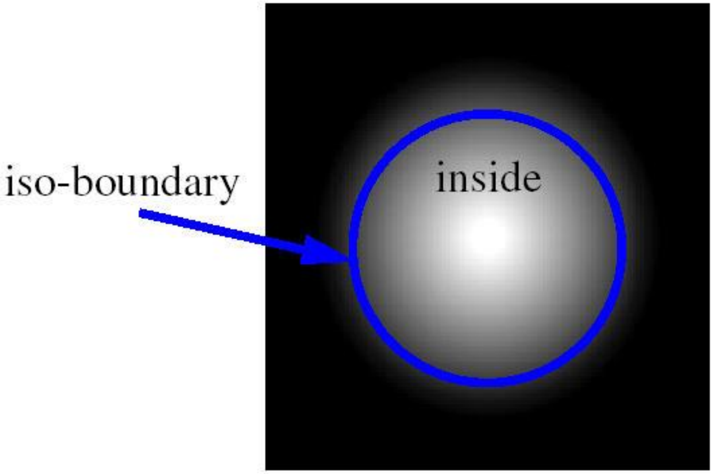
Iso-Surface Rendering

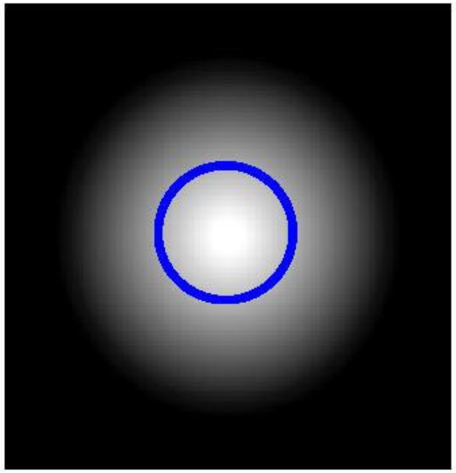
- A closed surface separates ‘outside’ from ‘inside’ (Jordan theorem)
- In iso-surface rendering we say that all voxels with values > some threshold are ‘inside’, and the others are ‘outside’
- The boundary between ‘outside’ and ‘inside’ is the *iso-surface*
- All voxels near the iso-surface have a value close to the *iso-threshold* or *iso-value*
- Example:



cross-section of a smooth sphere



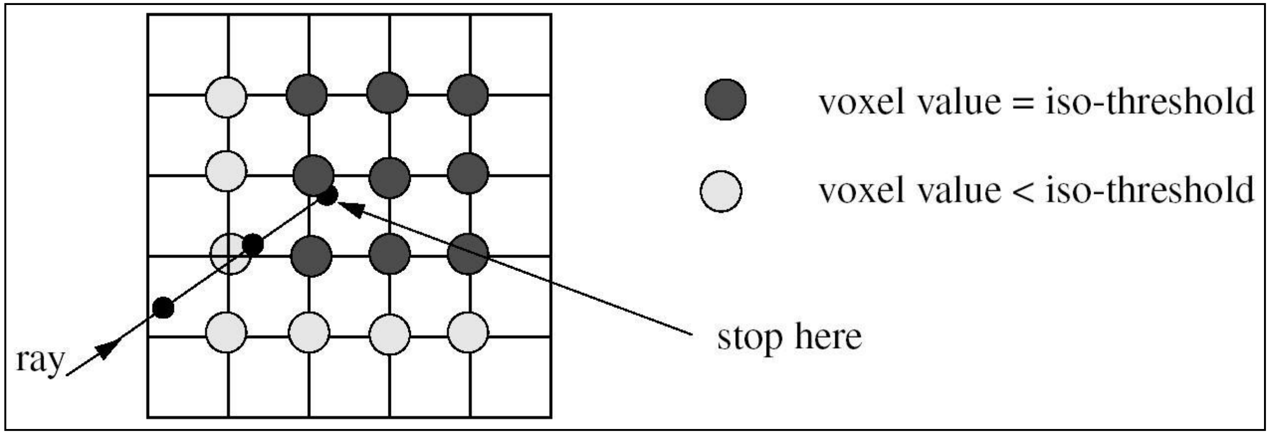
iso-value = 50
will render a large sphere

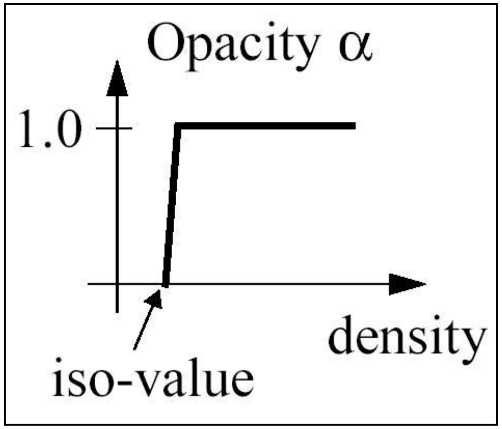


iso-value = 200
will render a small sphere

Iso-Surface Rendering

- To render an iso-surface we cast the rays as usual...
- But we stop once we have interpolated a value iso-threshold





- The easiest way to select the iso-surface is with the transfer function for α
- We would like to illuminate (shade) the iso-surface based on its orientation to the light source
- Recall that we need a normal vector for shading
- The normal vector N is the local gradient, normalized

Iso-Surfacing Example

Foot of the Visible Woman

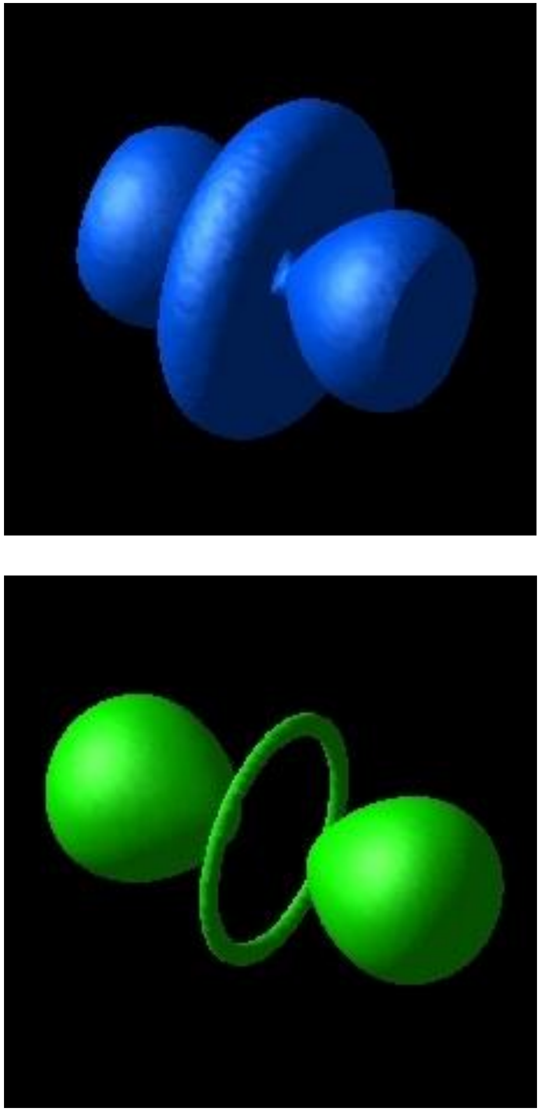
iso-value = 30

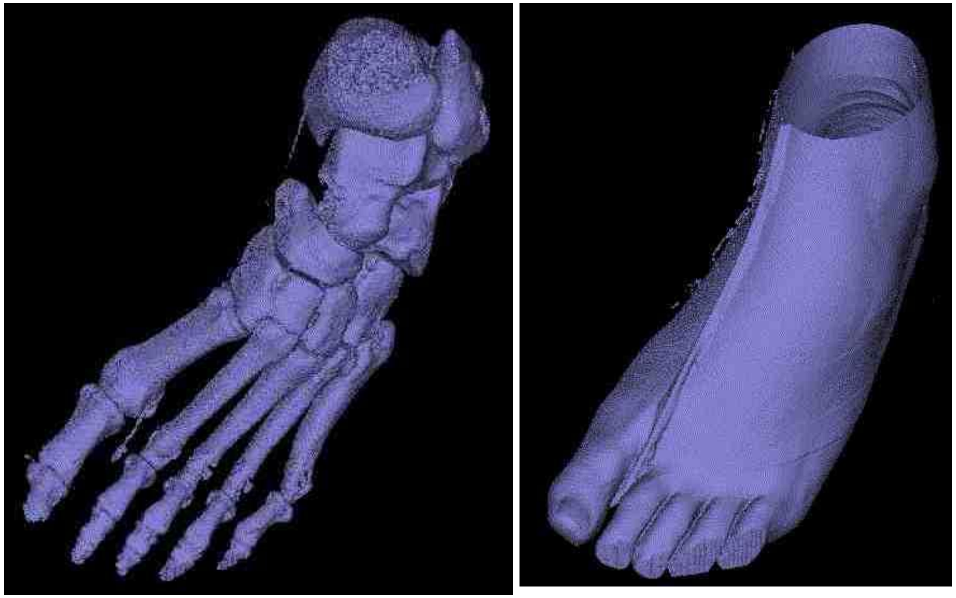
iso-value = 80

iso-value = 200

Different Iso-Levels

- Same data-sets, different extracted iso-surfaces
- Note that like all *surfaces*, the interior of the foot is “empty”





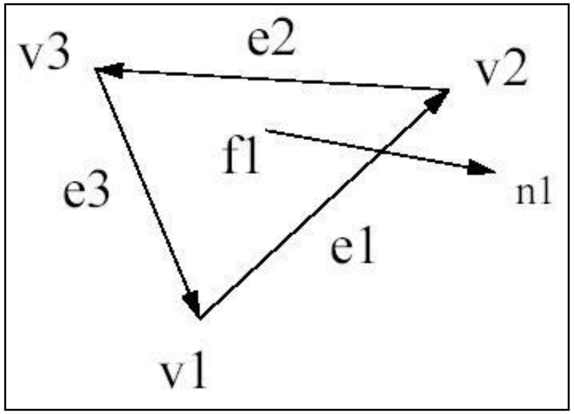
Surface Rendering with Polygons

- We have looked at several *direct rendering* algorithms for volume visualization
- Process volume itself with no conversion to other formats
- Speed and efficiency issues for software-based ray-casting
- Much of splatting can be implemented with commodity hardware
- Modern graphics hardware is all triangle-based since much of computer graphics is still surface-only
- Most applications require only surface rendering
- Today we will see algorithms for exploiting triangle-rendering hardware for volume visualization

Motivations for Iso-surface Polygonization

- Take advantage of surface graphics techniques
- Exploit inexpensive, yet powerful graphics hardware
- Use OpenGL (DirectX, etc.) to specify shading parameters
- Incorporate polygonized surfaces into other polygon-based software systems easily
- Familiar object representation format used widely across graphics and visualization
- Use object-order polygon mesh projection algorithms for rendering (described next)

Polygon Mesh Definitions



v1, v2, v3: vertices (3D coordinates)

e1, e2, e3: edges

$e1 = v2 - v1$ and $e2 = v3 - v2$

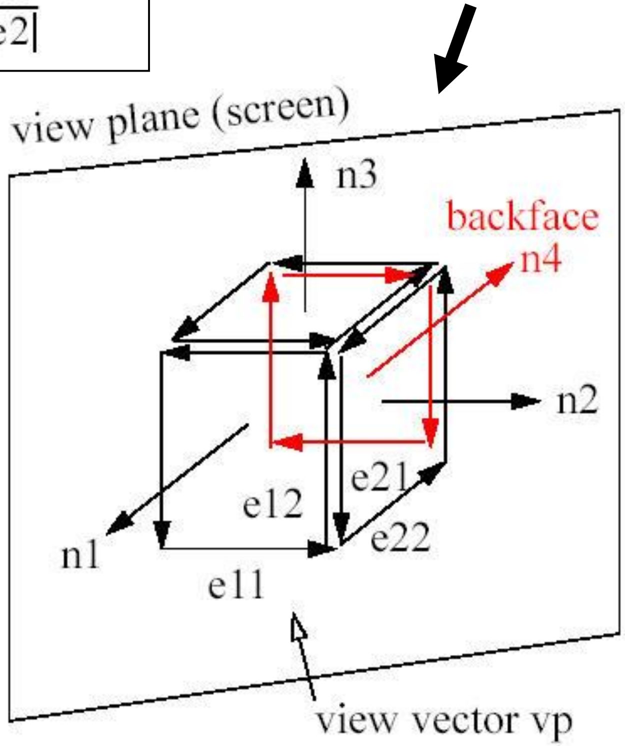
f1: polygon or *face*

n1: face normal $n1 = \frac{e1 \times e2}{|e1 \times e2|}$

$$n1 = \frac{e11 \times e12}{|e11 \times e12|}$$

$$n2 = \frac{e21 \times e22}{|e21 \times e22|}, \ e21 = -e12$$

- Rule: if all edge vectors in a face are ordered counterclockwise, then the face normal vectors will always point towards the outside of the object.
- This enables quick removal of *back-faces* (back-faces are the faces hidden from the viewer):
  back-face condition: $vp \bullet n > 0$



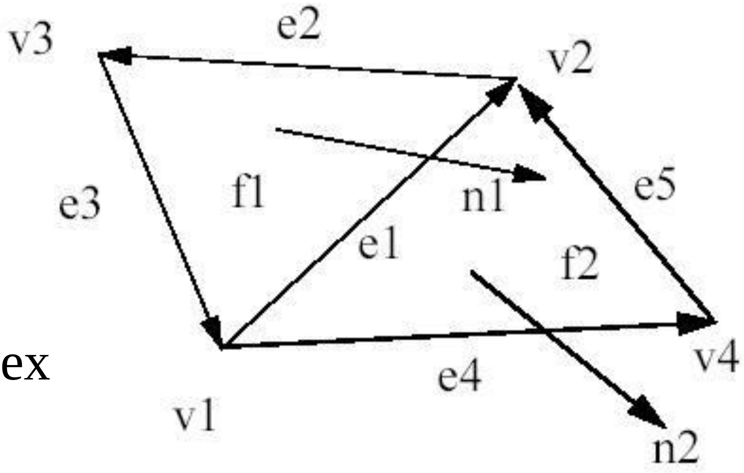
Polygon Mesh Data Structure

- Vertex list (v1, v2, v3, v4, ...):
  - (x1, y1, z1), (x2, y2, z2), (x3, y3, z3), (x4, y4, z4), ...

- Edge list (e1, e2, e3, e4, e5, ...):
  - (v1, v2), (v2, v3), (v3, v1), (v1, v4), (v4, v2), ...

- Face list (f1, f2, ...):
  - (e1, e2, e3), (e4, e5, -e1), ... or
  - (v1, v2, v3), (v1, v4, v2), ...

- Normal list (n1, n2, ...), one per face or per vertex
  - (n1x, n1y, n1z), (n2x, n2y, n2z), ...



- Use pointers or indices into vertex and edge list arrays, when appropriate
- Winged-edge / quad-edge / half-edge data structures

Hypothetical Polygonal Data Structure

- Your application determines which data you should store in order to maximize the cost/benefit of memory usage & computation time.

Vertex

    List of adjacent edges
    List of adjacent triangles

Edge

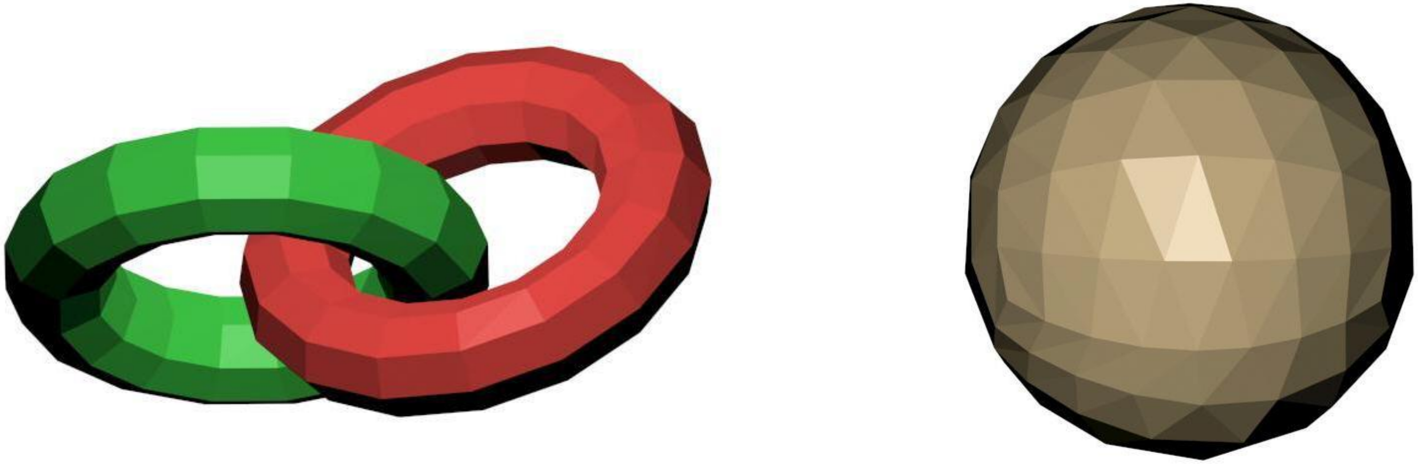
    Pair of end-points
    Pair of adjacent triangles (or one triangle if on the boundary)

Triangle

    Ordered list of edges

Polygon Shading Methods – Flat Shading

- How are the pixel colors determined?
- The simplest method is *flat or faceted shading:*
- Each polygon has a constant color
- Compute color at one point on the polygon (e.g., at center) and use everywhere
- Assumption: light source and eye are far away, i.e., N·L = const.
- Problem: discontinuities are likely to appear at face boundaries



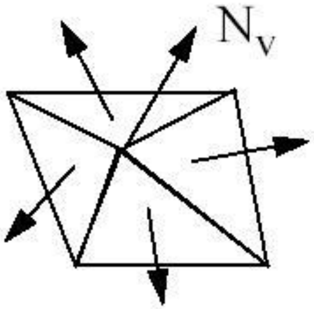
Polygon Shading Methods – Gouraud Shading

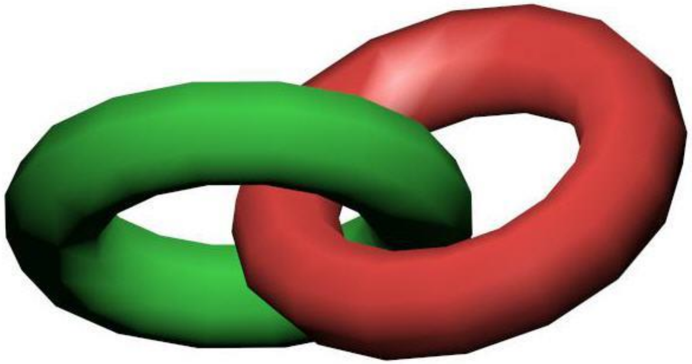
- Colors are averaged across polygons along common edges → no more discontinuities
- Steps:

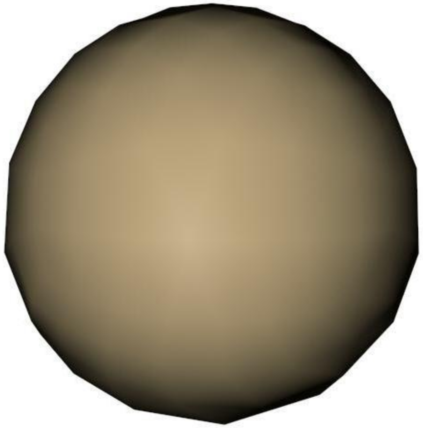
1. Determine average unit normal at each poly vertex:
2. n: number of faces that have vertex v in common
3. Apply illumination model at each poly vertex → C_v
4. Linearly interpolate vertex colors across edges
5. Linearly interpolate edge colors across scan lines

- Downside: may miss specular highlights at off-vertex positions or distort specular highlights

$$N_v = \sum_{k=1}^{n} N_k \Big/ \left| \sum_{k=1}^{n} N_k \right|$$





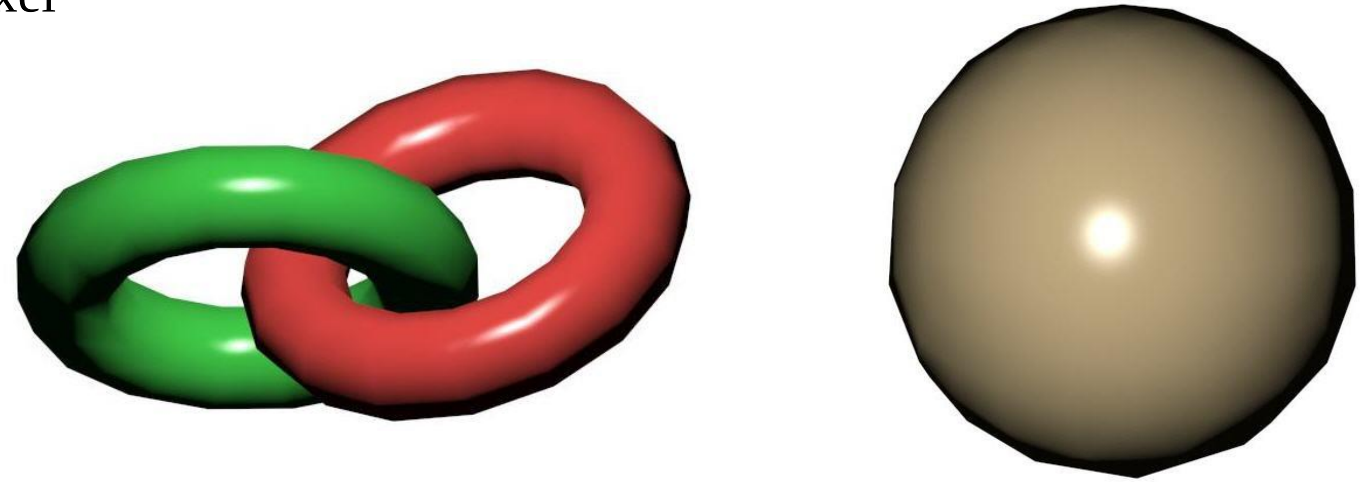


Polygon Shading Methods – Phong Shading

- Phong shading linearly interpolates normal vectors, not colors → more realistic specular highlights
- Steps:

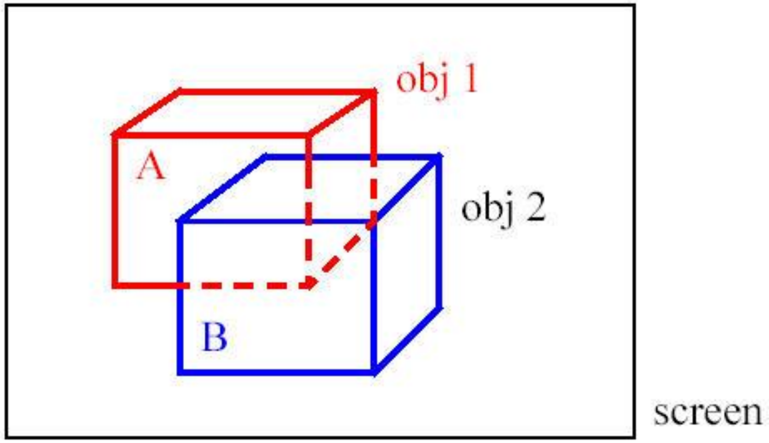
1. Determine average normal at each vertex
2. Linearly interpolate normals across edges
3. Linearly interpolate normals across scanlines
4. Apply illumination model at each pixel to calculate pixel color

- Downside: need more calculations since need to do illumination model at each pixel



Rendering Polygonal Objects – Hidden Surface Removal

- We have removed all faces that are *definitely* hidden: the back-faces
- But even the surviving faces are only *potentially* visible
- They may be obscured by faces closer to the viewer
- Face A of object 1 is partially obscured by face B of object 2
- Problem of identifying those face portions that are visible is called the *hidden surface problem*
- Solutions:
  - Pre-ordering of the faces and subdivision into their visible parts before display (expensive)
  - The z-buffer algorithm (cheap, fast, implementable in hardware)

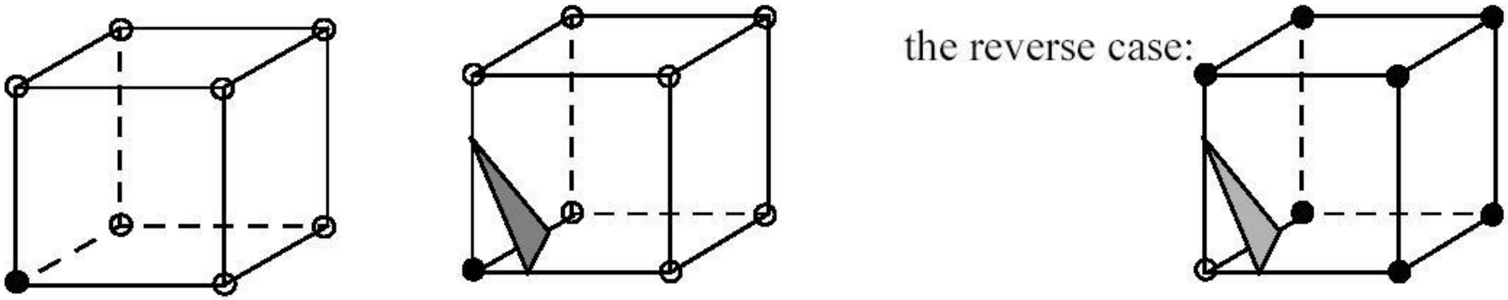


Overview and Motivation

- Algorithms extract surface of constant density (iso-surfaces) from 3D data and convert it into polygonal mesh
- Divide-and-conquer algorithm
- Process each row of voxels to build the triangulated surface in an incremental fashion
- Use table to decide on a case-by-case basis how each *cell* (group of 8 voxels) is used to generate triangles
- Normalized gradient will provide normal direction for the triangles so we can shade the surface
- *Marching Cubes* algorithm – developed in 1987, still very widely used
- Several enhancements since then, but fundamental algorithm remains the same

The Marching Cubes Polygonization Algorithm

- The *Marching Cubes (MC)* algorithm converts a volume into a polygonal model
- Allows us to render the iso-surfaces quickly and shade them using flat, Gouraud or Phong shading (or others)
- Steps:
- Imagine all voxels above the iso-value are set to 1, all others are set to 0
- The goal is to find a polygonal surface that includes all 1-voxels and excludes all 0-voxels
- Look at one volume cell (a cube) at a time → hence the term *Marching Cubes*
- Here are 2 of ___ possible configurations:



the reverse case:

only 1 voxel > iso-value

the polygon that separates inside from outside

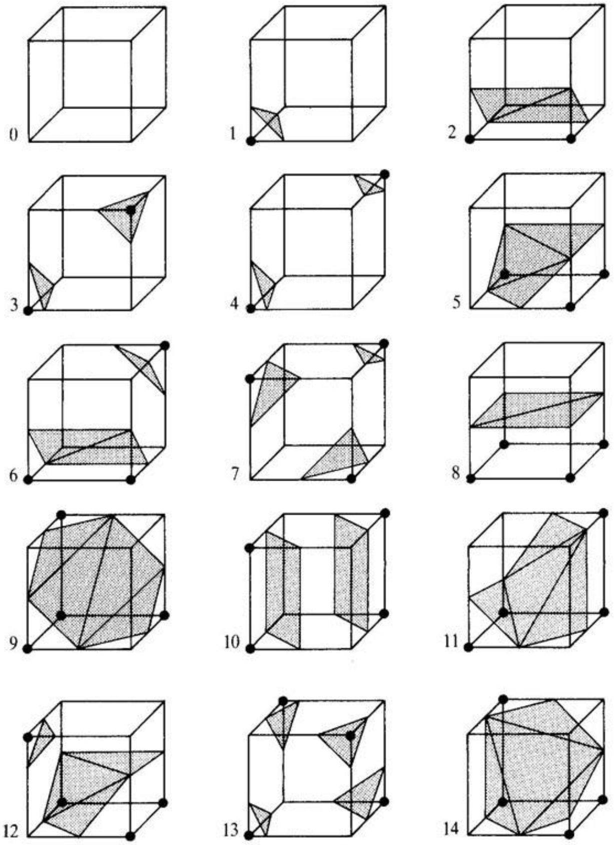
7 voxels > iso-value
the same polygon results

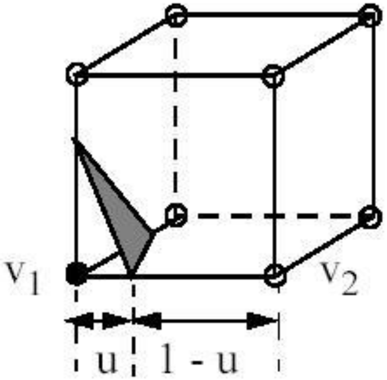
Marching Cubes

- One can identify 15 base cases and use symmetry and reverses to get the other 241 cases
- The exact position of the polygon vertex on a cube edge is found by linear interpolation:

$$iso = v_1 \cdot (1-u) + v_2 \cdot u \;\rightarrow\; u = \frac{v_1 - iso}{v_1 - v_2}$$

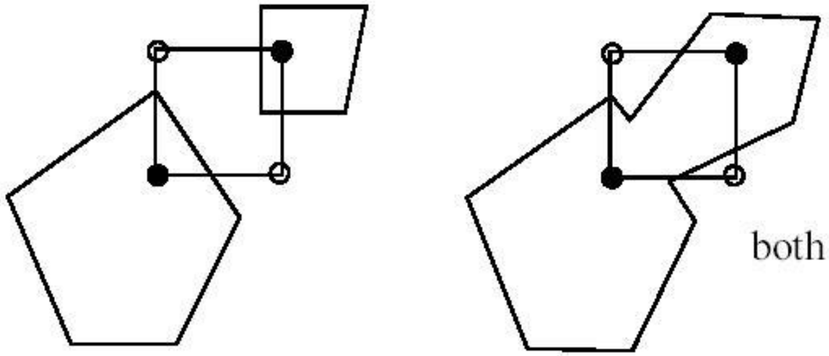
- Now interpolate the vertex color by:
  $c_1 = uc_2 + (1-u)c_1$
- Interpolate the vertex normal by:
  $n_1 = ug_2 + (1-u)g_1$
  g_1 and g_2 are the gradient vectors at v_1 and v_2 obtained by central differencing





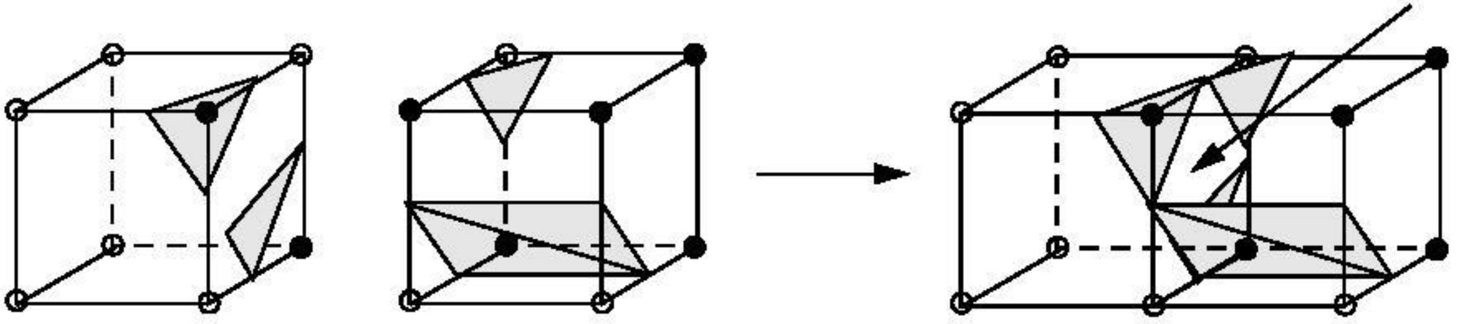
Marching Cubes – Ambiguous Cases

- 2D: ambiguous case:



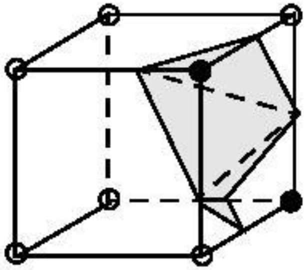
both versions are plausible

- 3D: what happens when cases are arbitrarily chosen:



case 3　case 6 (complementary)　connected

- Remedy: add 6 alternative cases for 3, 6, 7, 10, 12, 13 to prevent holes



Example: case 3c

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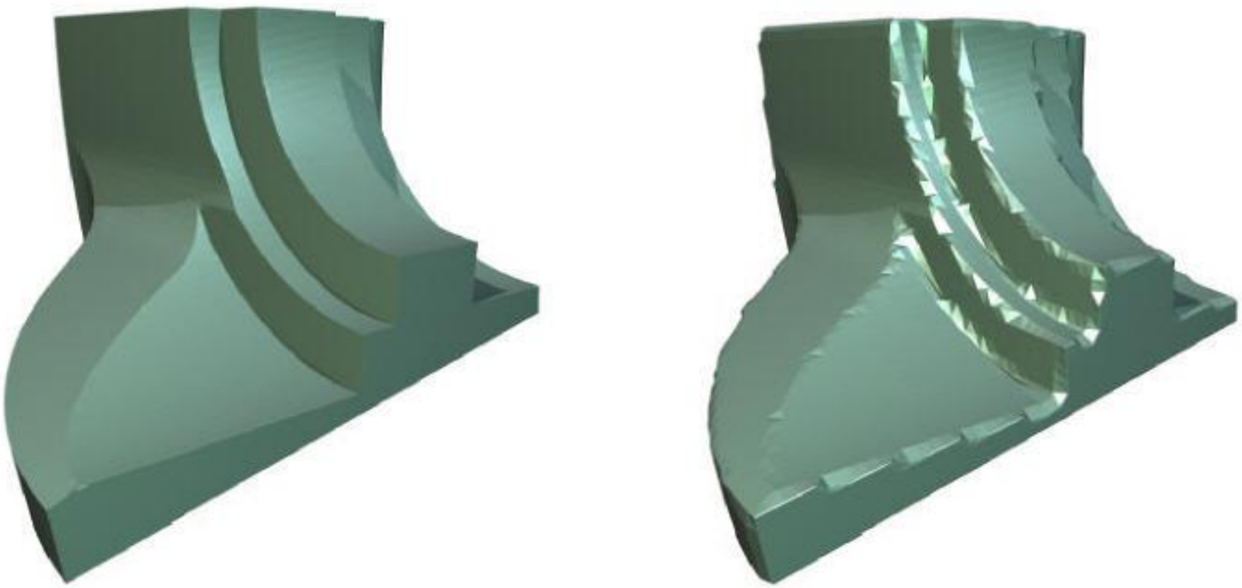
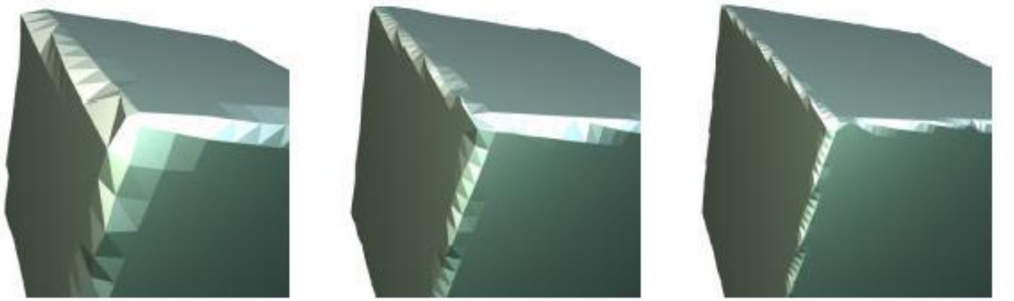
Problem with Marching Cubes

- Sharp features, like corners and hard edges, tend to be smoothed away by the Marching Cubes algorithm
- Finite grid → some details will be lost
- Continuous model discretized onto grid and Marching Cubes applied:



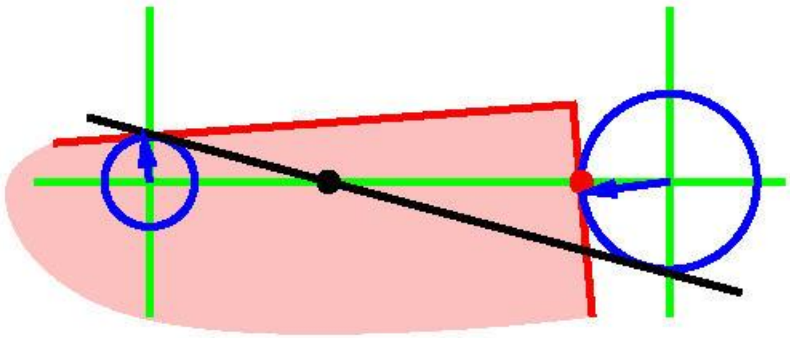
Model Conversion

- Suppose we wish to represent (convert) a surface model on a volumetric raster (grid)
- Possible motivation: sculpting operations to modify the object
- This means we need to discretize the 3D geometric shape
- After we have finished our work, we need to convert the 3D volume back to a surface model
- This can be done with Marching Cubes
- However, at what grid resolution do we store the shape?
- Certain features of the surface will *always* be lost by the regular MC algorithm



Cause of the Problem

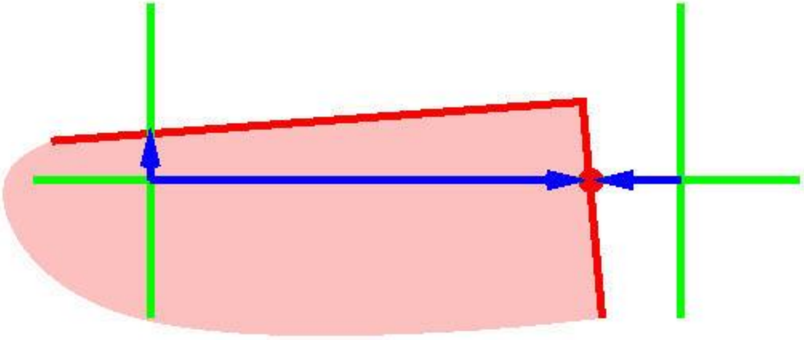
- When we discretize the object, at each voxel we store a distance of the voxel from the object surface
- Hence, the volume is what we call a *volumetric distance field* that approximates a smooth, continuou



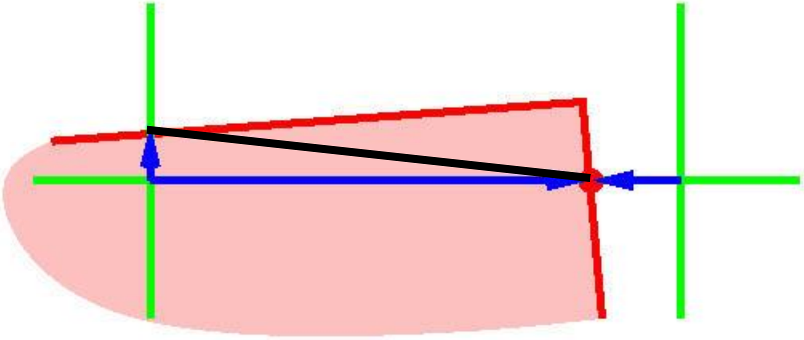
- Consider two neighboring grid points (green) in the vicinity of a sharp feature (corner) of the contour *S* (red)
- Sampling the scalar valued distance function *f* at both grid points (blue) and estimating the sample point by linear interpolation leads to a bad estimation (black) of the true intersection point between the red contour and the green cell edge

How About Storing a *Directed* Distance?

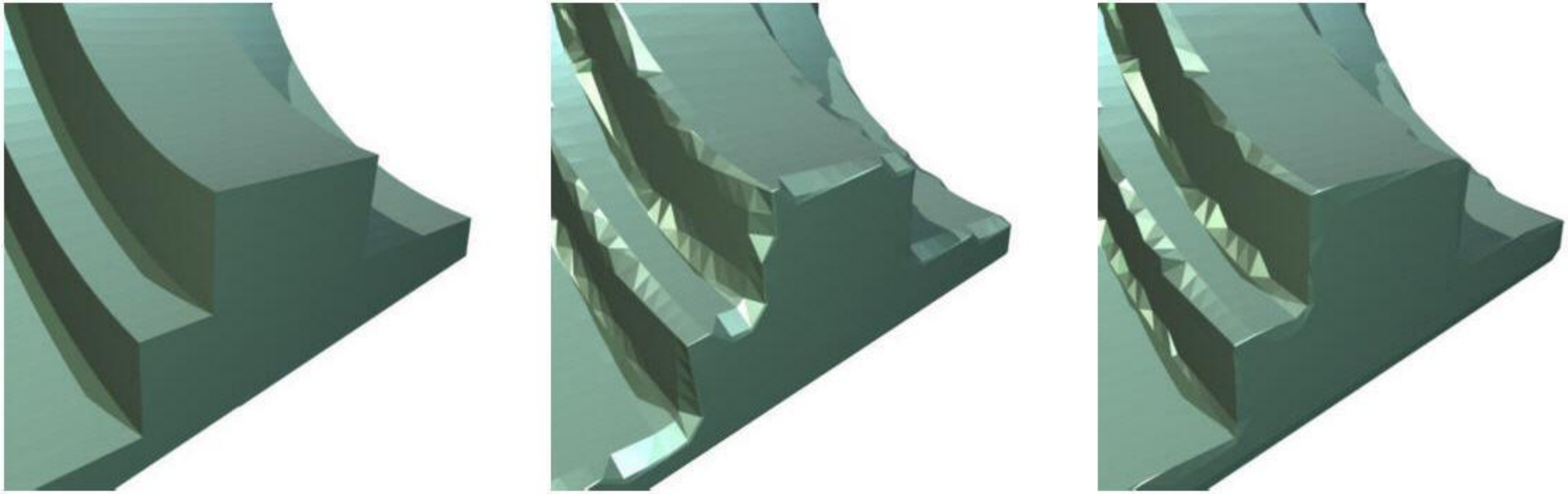
- Suppose instead of just storing a scalar value at each voxel, we store a vector that indicates the directed distance?



- This is still not enough and we replace sharp corners and other features with diagonal lines



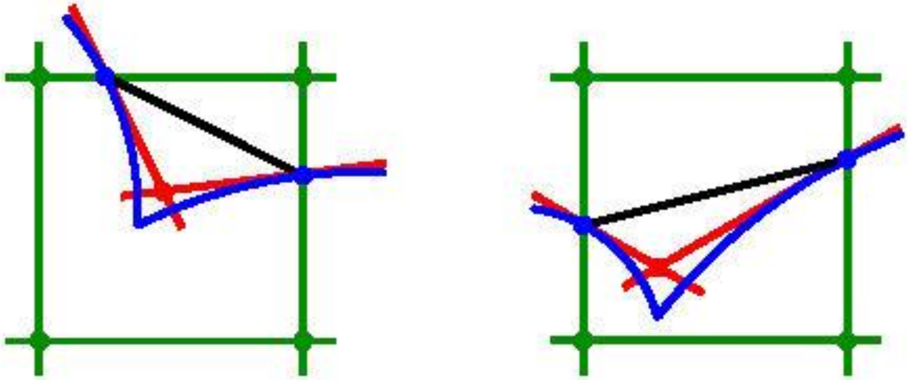
Scalar Distances vs. Directed Distances



- First image: original model
- Second image: discretized and MC applied to a scalar distance field
- Third image: discretized and MC applied to directed distance field

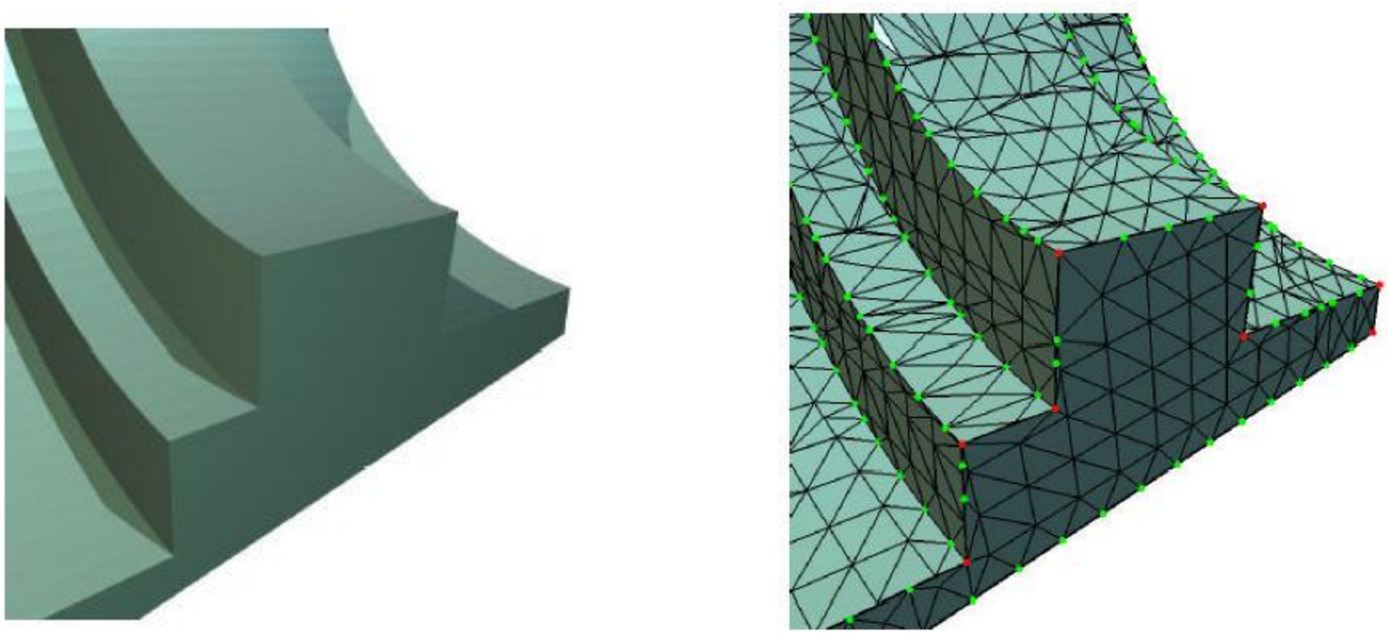
Solution to Loss of Features Problem

- The solution to this problem involves use more information inferred by the data
- During discretization, we compute and store *tangent* vectors that we compute using the surface normal
- These vectors basically tell you in what the direction(s) the surface is moving
- Then, when we are left with only the discrete grid, we *extend* these tangents into the center of the cells to approximate the character of the surface inside the cell
- Where these tangents intersect, we create a *feature point* we use to polygonize the surface
- Blue: original contour we discretized
- Red: extended tangents
- Vertex: feature point we will use to build polygons
- Black: what directed distances would have given us



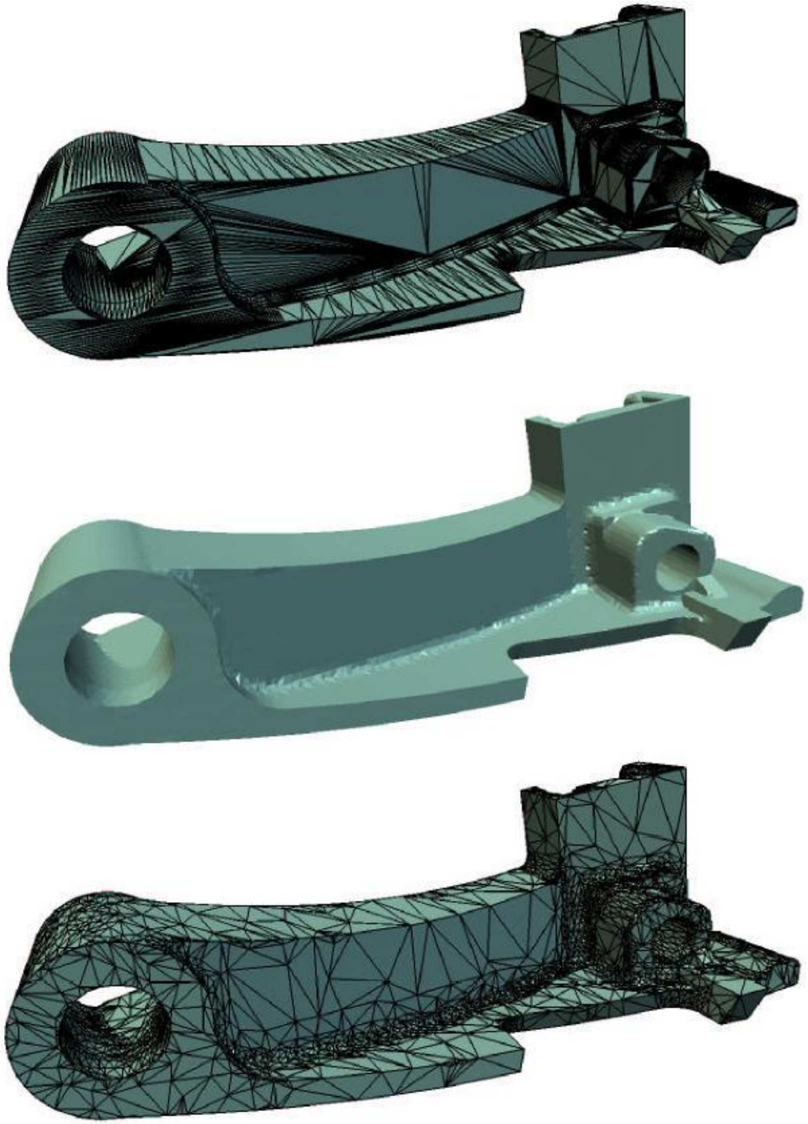
Extended Marching Cubes

- Algorithm:
- If cell contains a sharp feature, determine if an edge feature (green) or a corner feature (red) is present
- If yes, apply the new technique for selecting vertex positions
- Otherwise, apply the normal Marching Cubes algorithm



Application – Remeshing

- Remeshing of a polygonal mesh
- Generally speaking, skinny or *sliver triangles* are bad
- Poor rendering quality
- Interfere with mechanical simulation
- Often too many triangles present to represent the given object: wastes computation time, memory, storage space, etc.
- Extended MC algorithm takes discretized version of original mesh and extracts a new surface that has fewer triangles and also *higher quality* triangles

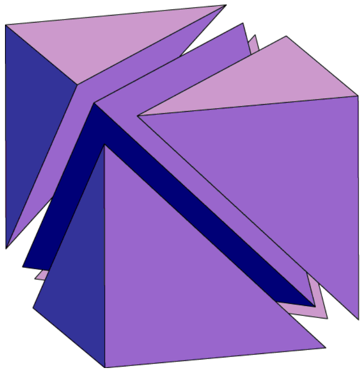


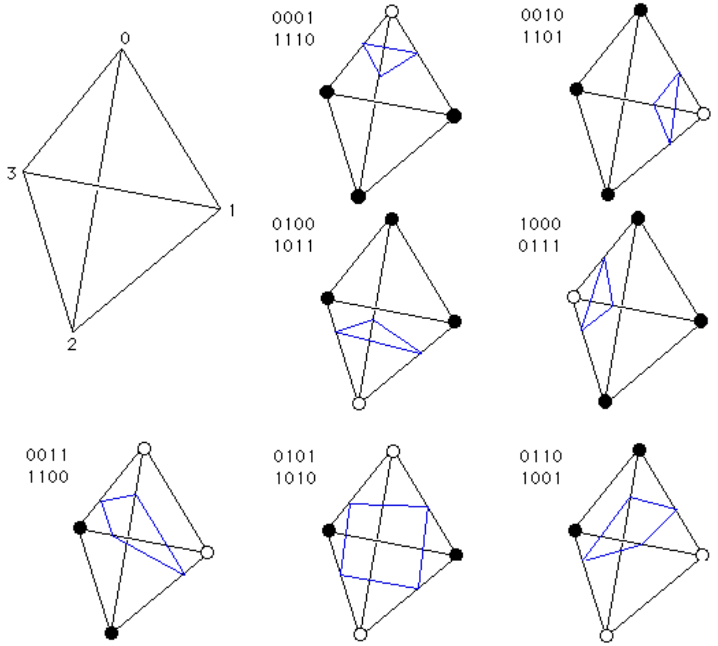
Application – CSG

- Constructive Solid Geometry (CSG) is a shape design technique
- Objects defined as the addition and subtraction of other objects
- Typically difficult to achieve <u>accurately</u> over a discrete grid
- Usually we have to compute intersections between design primitives exactly (spheres, cylinders, boxes, splines, etc.)
- Very expensive process that involves root-finding
- In discrete grid, problem much simpler by performing set inclusion/exclusion tests
- The extended Marching Cubes algorithms makes CSG feasible on a discrete grid because we can recover these intersected regions almost exactly

Marching Tetrahedra

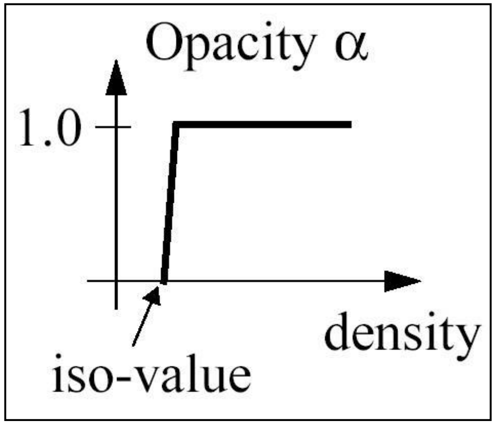
- Another iso-surface extraction algorithm is called Marching Tetrahedra
- Divide each cell into five tetrahedra
- Apply one of the three unique cases
- No ambiguity problem, as with Marching Cubes
- Easier to implement
- But surface quality is usually not as good since less information is taken into consideration (four values used for interpolation instead of eight)
- Also generates more triangles than MC, the latter of which might be able to generate a single large triangle instead of several small ones to cover the same surface area





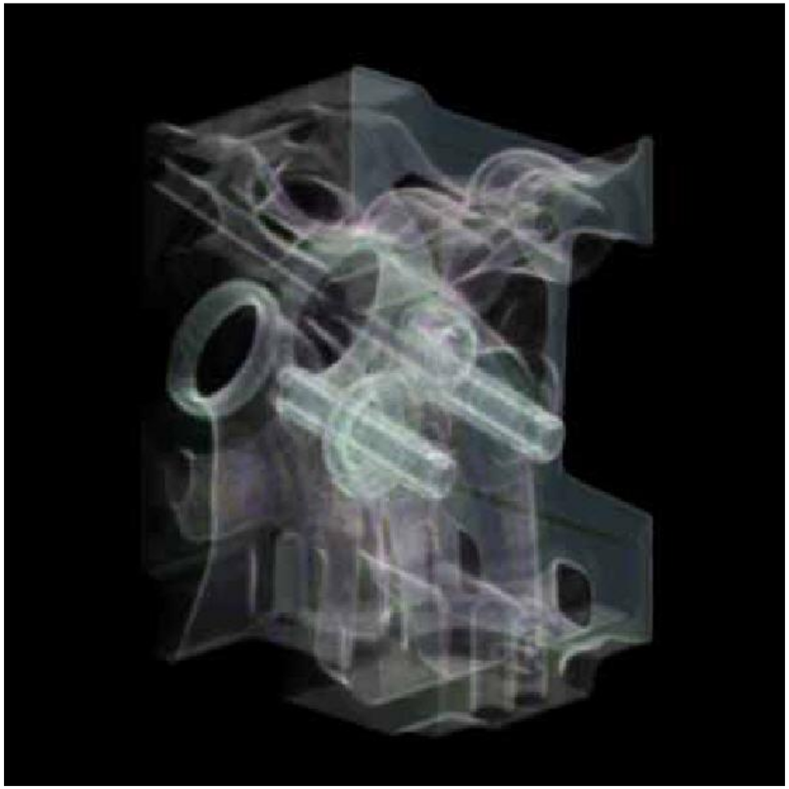
Use Volume Rendering to Handle Iso-surfaces

- We saw earlier how we can use ray-casting to render iso-surfaces by using an alpha transfer function with a sharp drop-off
- Suppose we don't have a ray-casting system available?
- We can instead use an *iso-surface extraction* algorithm to generate a polygonal approximation of the iso-surface implied by the volumetric data
- Pre-processing step, possibly slow
- User specifies the designed iso-level, and the algorithm produces the corresponding triangular iso-surface



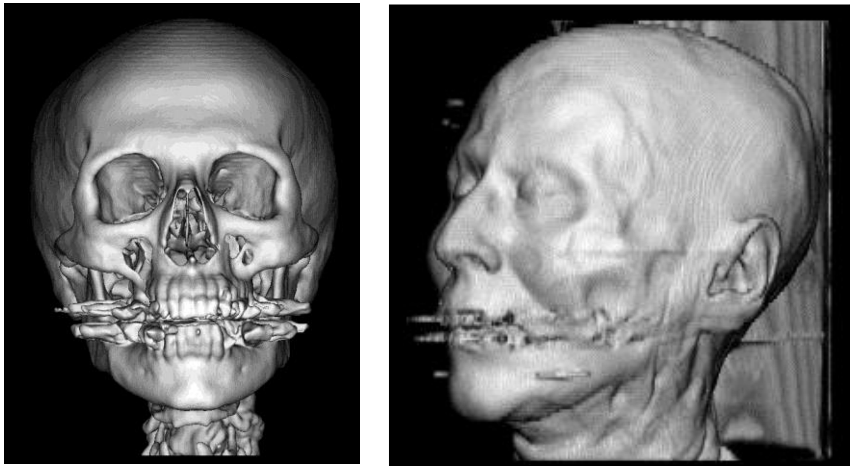
Gradient Modulation

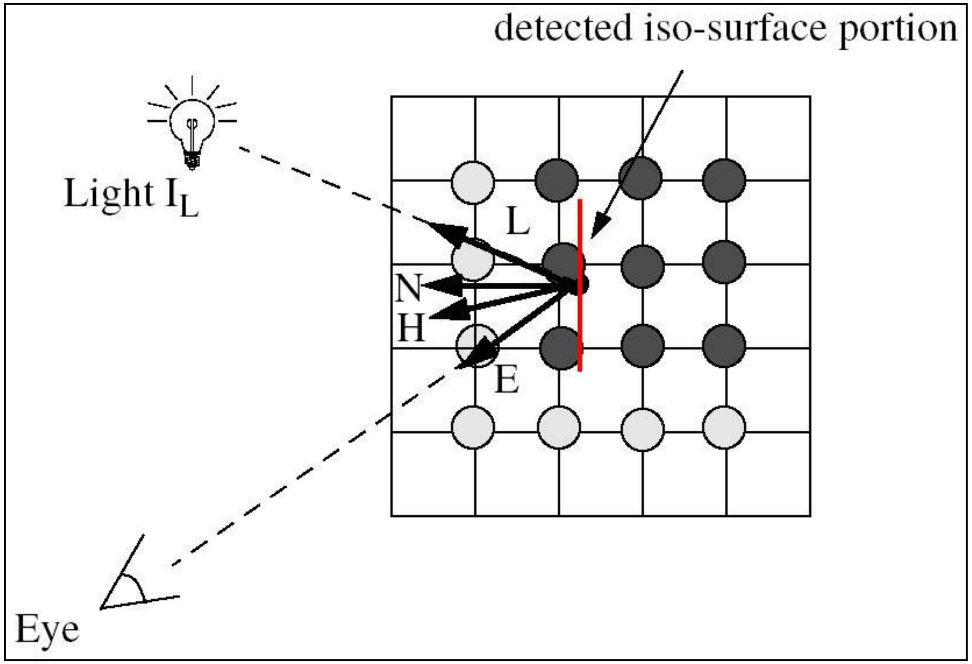
- One use of the gradient is in a process known as *gradient modulation* in which we modulate the opacity/color of a voxel by the gradient
- First we look up the voxel's opacity/color, given by the transfer function
- Then we multiply the opacity and color by some function of the gradient magnitude (also given by a transfer function, #5)
- Regions of high gradient magnitude cause an increase in opacity, whereas regions of low gradient magnitude cause the opacity to drop to near zero
- Remind us: what does a high magnitude signify?
- How does this explain the image on the right?



Iso-Surface Shading

- The normal vector is the *normalized* gradient vector g
- N = g / |g| (normal vector always has unit length)
- Once the normal vector has been calculated we shade the iso-surface at the sample point
- The color so obtained is then written to the pixel that is due to the ray
- Colors are computed using one of the standard illlumination models
- Let’s see a short movie to see this in practice (9)





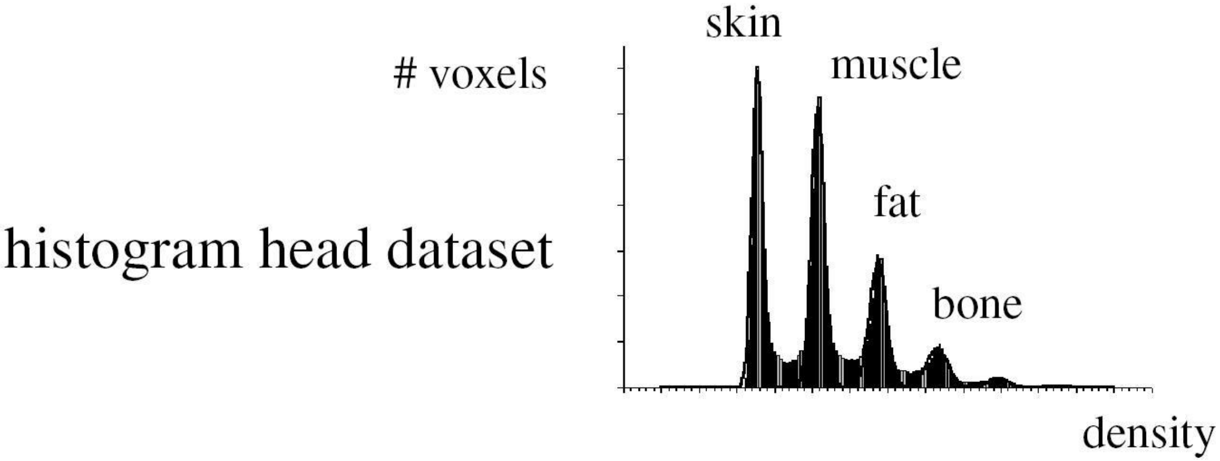
Iso-Surface Rendering – Algorithm (Perspective)

RenderIsoSurface(Volume V, int stepSize)

```
for each image pixel p(i, j)
    ray = (p(i, j) - eye) / | (p(i, j) - eye) |; // the ray direction vector, normalized
    t = 0; // start at the eye point
    do forever
        sampleLoc = eye + t · stepSize · ray // step along the ray
        intVal = Interpolate(V, sampleLoc)
        if opacityTransferFunction(intVal) > isoThreshold // found the iso-surface
            // interpolate 6 samples around sampleLoc and compute the gradient
            gradVec = ComputeGradientVector(V, sampleLoc);
            // shade the surface using standard illumination model and color transfer functions
            {r, g, b} = Shade(gradVec, lightSource, eye, sampleLoc, {R, G, B} TransFunc(intVal));
            value(p(i, j)) = {r, g, b}; // write color into image pixel p(i, j)
            break; // terminate this ray and go to next image pixel
        t = t + 1; // iso-surface not found yet, get ready to step to next sample point
```

Iso-Surface Rendering – Tips and Tricks (1)

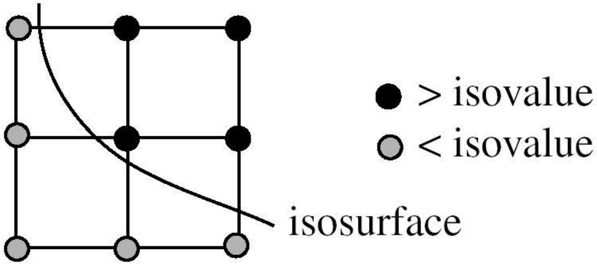
- Finding a good iso-value is not always easy
- Make a histogram of the volume densities and look for peaks (iso-value = onset of peak)



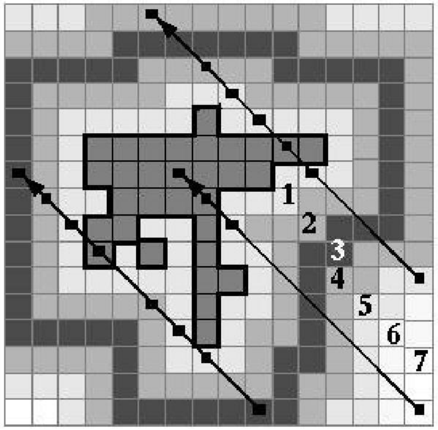
- Good shading requires good gradients around iso-surface
- Need smooth degradations at iso-surface for good gradient estimation
- Else get aliasing

Iso-Surface Rendering – Tips and Tricks (2)

- Ray stepsize must be chosen sufficiently small
- Choose stepsize of less than or equal to 1.0 voxel units (or we may get aliasing in the ray direction)
- But even for small stepsizes, we may never exactly hit the isosurface
- Iso-surface goes through a cell when at least one vertex, but not all, has a density > isoValue
- Compute exact location of the iso-surface within a cell by solving a cubic function in *t*. This is usually impractical.



- A variety of acceleration methods are possible:
- Enclose the object in a bounding box and start rays at the bounding box intersection (works also for general volume rendering)
- Store distance values in voxels outside the object → this enables quick *space leaping*
- Multi-resolution volume representation (octree) (works also for general volume rendering)



space leaping