

Background Knowledge

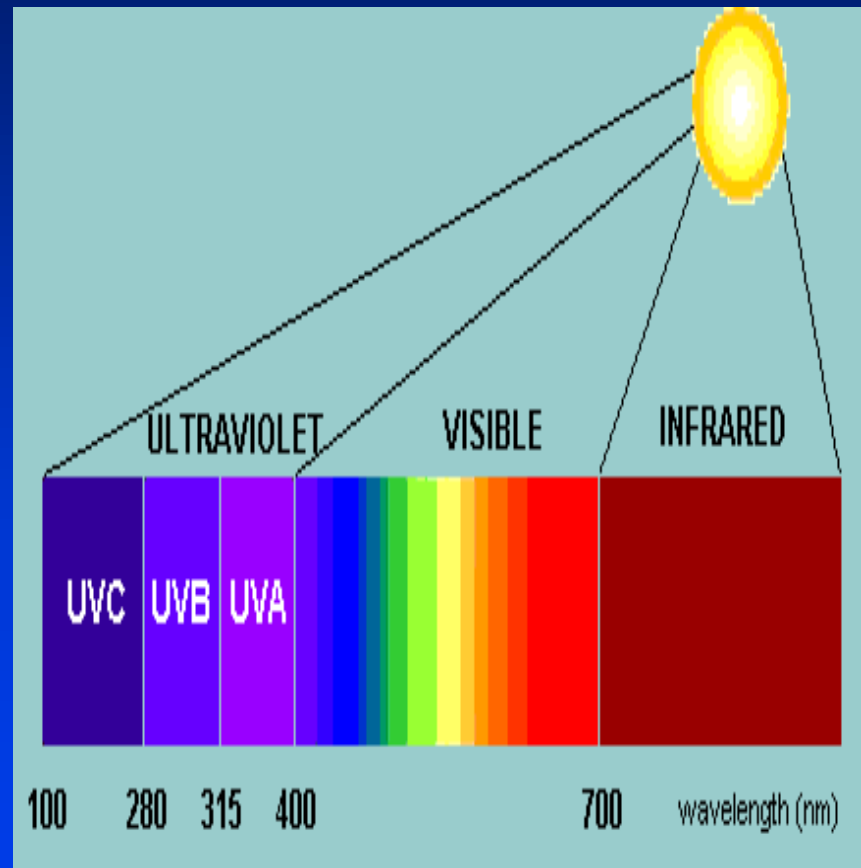
- Light
- Light Transport
- Radiometry
- Reflection Functions

Optics

- **Geometric optics**
 - Shadow, optical laws
- **Physical optics**
 - Interference
- **Quantum optics**
 - Photons
- **To study radiosity, geometric optics is needed**

Light

- The visible light can be polarized
- Optics is the area that studies about these radiations



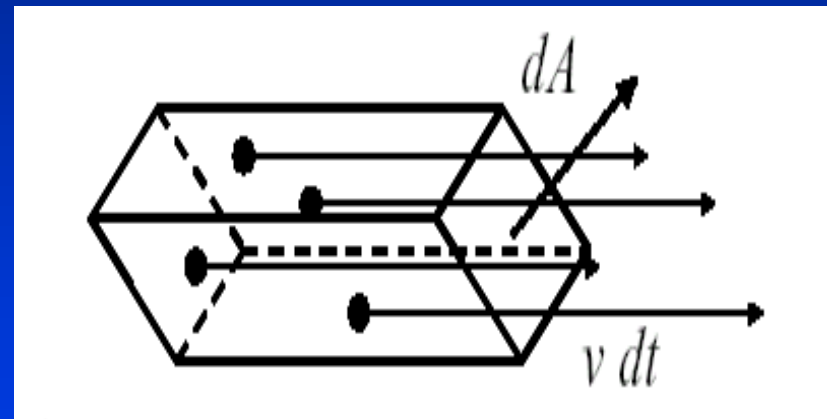
Light Transport

- Light travels in the form of particles (photons)
- Total number of particles in a small differential volume dV is

$$P(\mathbf{x}) = p(\mathbf{x}) dV$$



particle density



$$P(\mathbf{x}) = p(\mathbf{x}) (v dt \cos(\theta)) dA$$

Light Transport

Not all particles flow with the same speed and same direction

The particle density is now a function of two independent variables x , ω .

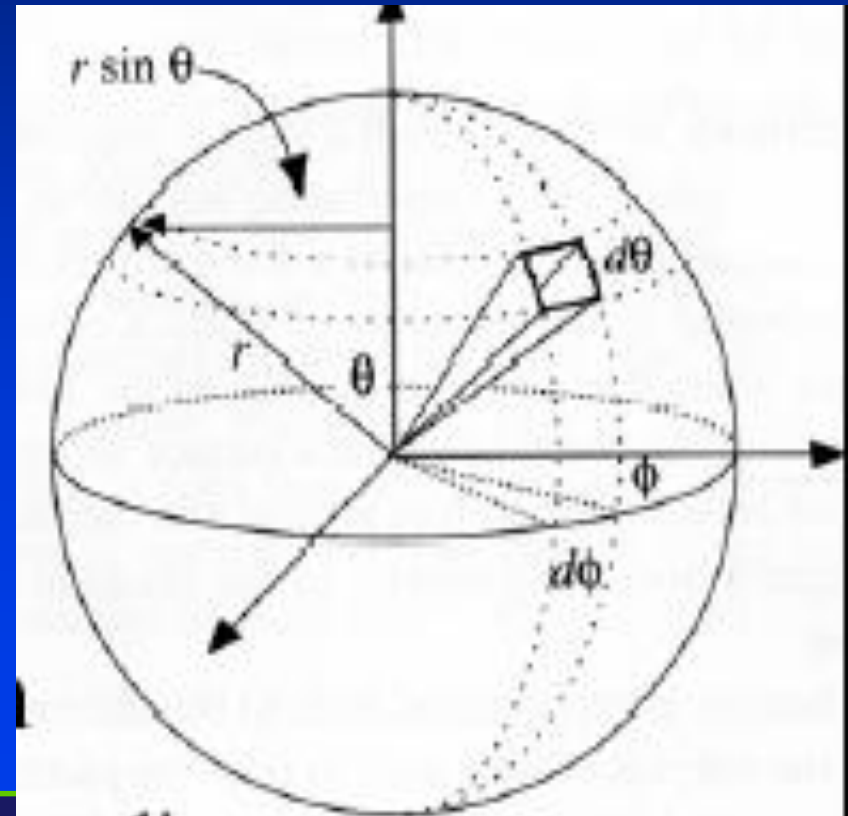
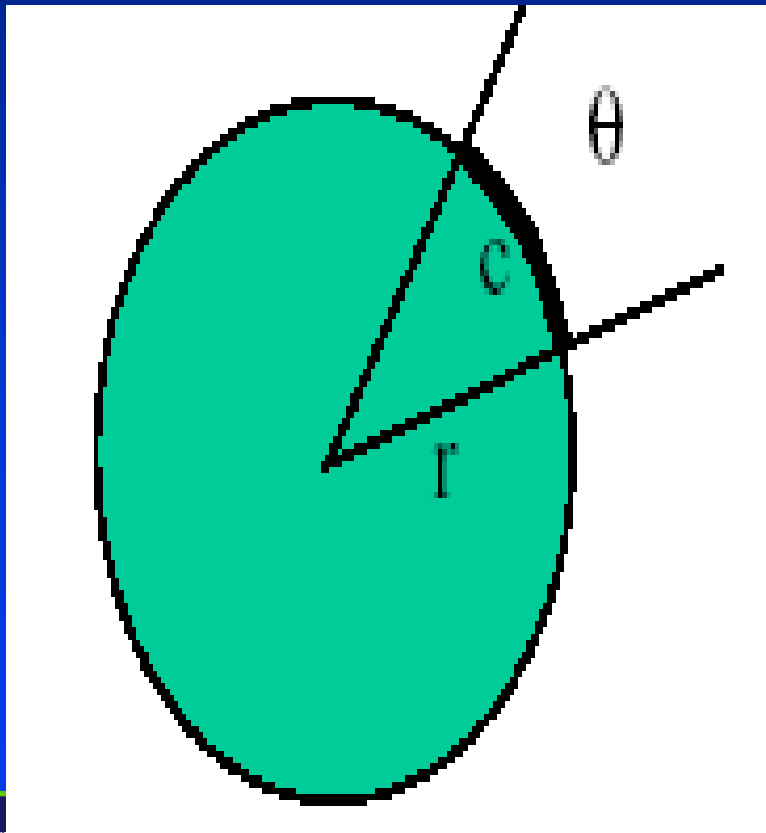
Then we have

$$P(x, \omega) = p(x, \omega) \cos\theta \, d\omega \, dA$$

Here $d\omega$ is called the differential solid angle

Angles

- 2D Angle vs. 3D Solid Angle



Radiometry

- Science of measuring light
- Analogous science called Photometry is based on human perception

Radiometry for Surface Rendering

- Investigate formally some methods for physically-based realistic rendering
- Present a practical method for producing highly realistic (and also physically correct) images (renderings) of 3D worlds

Radiometry for Surface Rendering



Radiometry

The radiometric quantities that characterize the distribution of light in the environment are:

- Radiant Energy
- Radiance
- Radiant Power
- Irradiance
- Radiosity
- Radiant Intensity

Radiometric Quantities

- Functions of wavelength, time, position, direction, polarization

$$g(\lambda, t, X, \vec{\omega}, \gamma)$$

- Add polarization to Plenoptic function
- We will have to simplify this formulation

Wavelength

- Assume wavelengths are independent
 - Phosphorescence: material traps energy and re-emits it for an extended period of time
 - No phosphorescence
 - R, G, B components behave identically

$$g(t, X, \vec{\omega}, \gamma)$$

Time

- **Equilibrium states considered only**
 - Light is traveling fast...
 - No luminescence
 - Fluorescence

$$g(\mathbf{x}, \vec{\omega}, \gamma)$$

Polarization

- Ignore it
 - Would likely need wave optics to simulate

$$g(\mathbf{x}, \vec{\omega})$$

A Function with Five Dimensions

- With little loss in usefulness
- Two quantities

\mathbf{x} Position (3 components)

ω Direction (2 components)

$$g(\mathbf{x}, \vec{\omega})$$

Radiant Energy – (Q)

- Fundamental quantity we start with
- Consider photon as carrying quantum of energy (hc/λ , where c is speed of light, and h is Planck's constant)
- Radiant energy per unit volume is the photon volume density times the energy of a single photon (hc/λ)
- Total energy, Q , is energy of the total number of photons

Radiant Energy – (Q)

Rendering systems consider the stuff that flows as radiant energy or radiant power (Φ)

$$L(\mathbf{x}, \omega) = \int p(\mathbf{x}, \omega, \lambda) (hc/\lambda) d\lambda$$

L is called radiance

Radiant Power – (Φ)

- Flow of energy (important for transport)
- Power is the energy per unit time (joules / s)
- Also called as *radiant flux*.
- Unit: *Watt*
- $\Phi = dQ/dt$

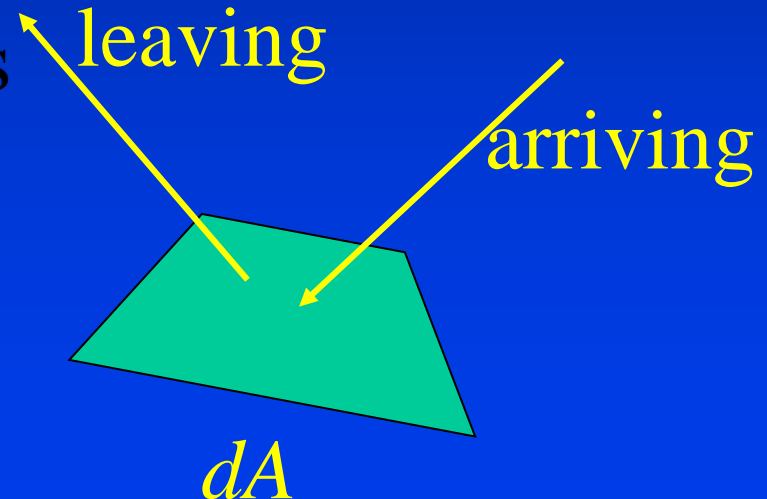
The differential flux is the radiance in small beam with cross sectional area dA and solid angle $d\omega$

$$d\Phi = L(x, \omega) \cos\theta d\omega dA$$

Radiant Flux Area Density

- We render stuff on surfaces
- So we need a measure for the energy arriving/leaving a surface
- Units: watts per meter squared
- Graphics does NOT use this term!

$$u = \frac{d\Phi}{dA}$$

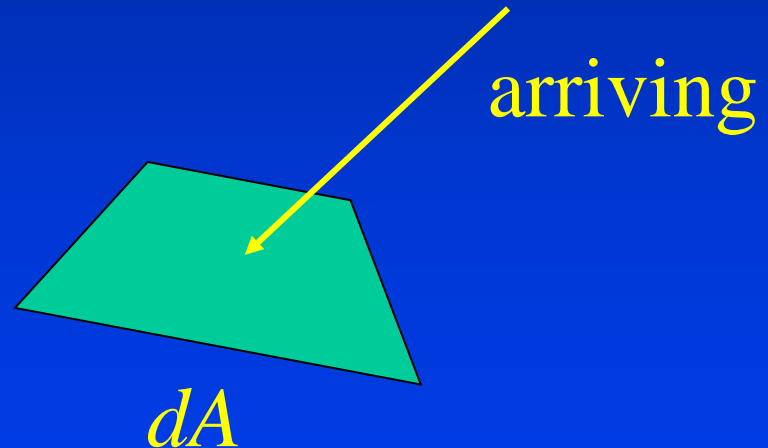


Irradiance

- Power per unit area incident on a surface

$$E = d\Phi/dA$$

- Unit: Watt / m²

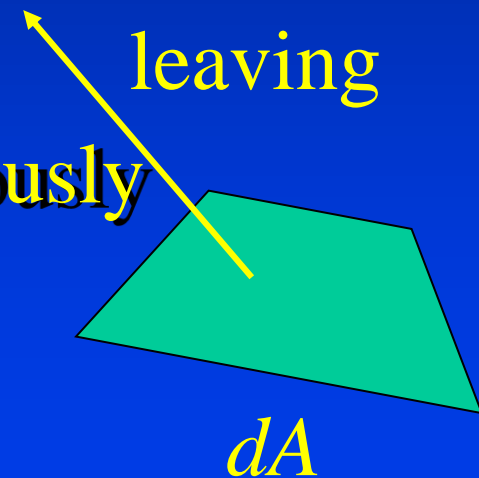


Radiant Exitance

- Power per unit area leaving surface
- Also known as *radiosity*

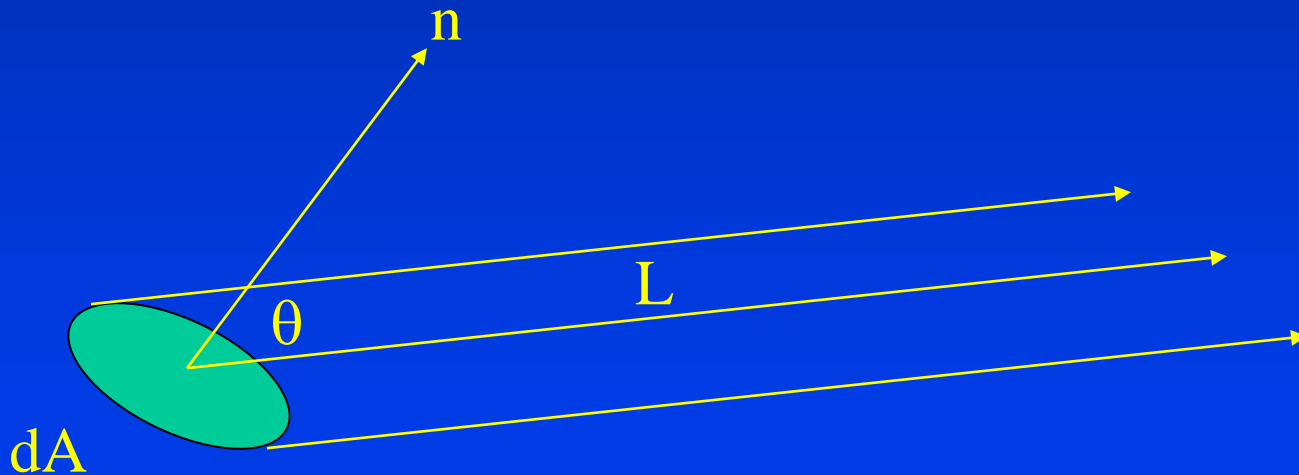
$$B = d\Phi/dA$$

- Same units as *irradiance*, obviously
- Just direction changes



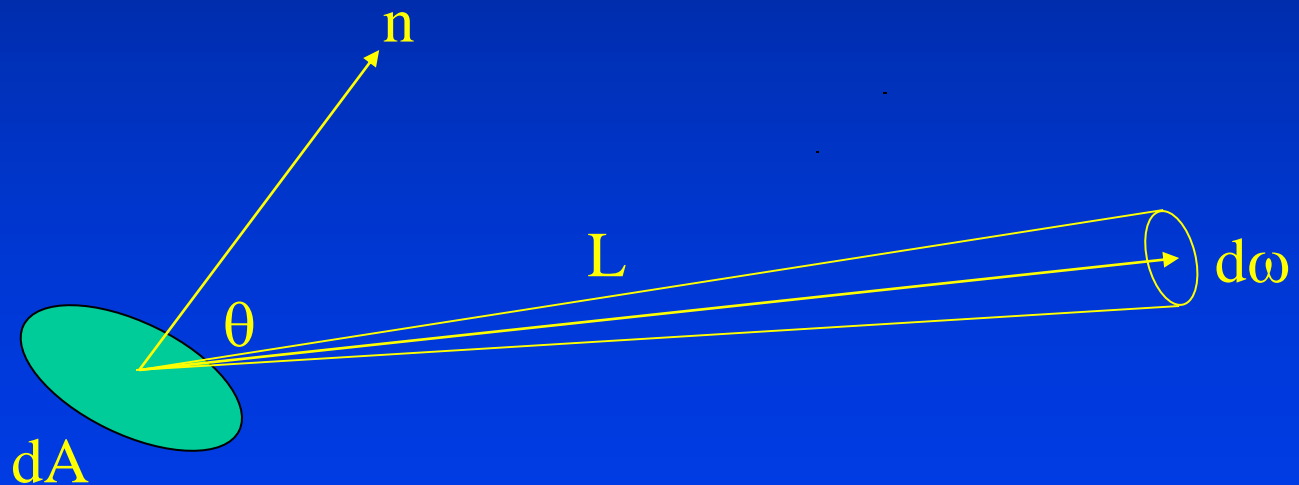
Radiance

- Radiance (L) is the flux that leaves a surface, per unit projected area of the surface, per unit solid angle of direction.



Radiance

- For computer graphics the basic particle is not the photon and the energy it carries but the ray and its associated radiance.



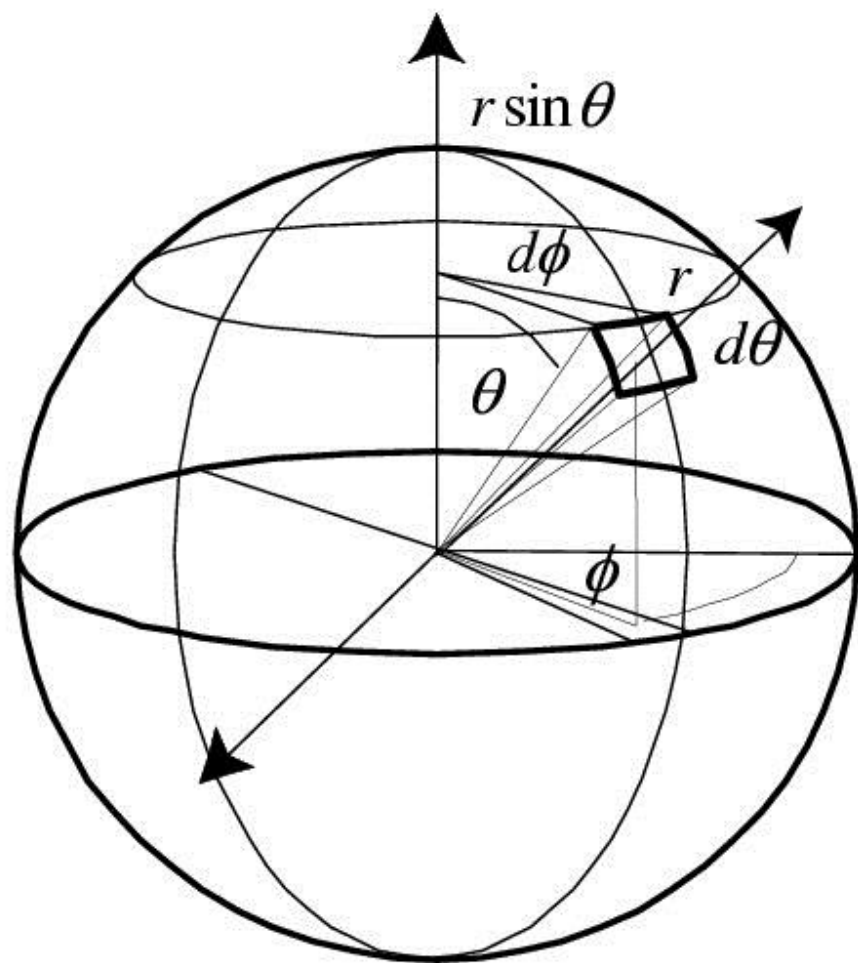
Radiance is constant along a ray.

Radiant Intensity

- **Radiant Intensity:** Radiant power per solid angle of a point source
- Units – watts per steradian
- Note: the term “Intensity” is heavily overloaded
- What is a solid angle?

$$I(\omega) = \frac{d\Phi}{d\vec{\omega}}$$

Differential Solid Angles



$$\begin{aligned}dA &= (r d\theta)(r \sin \theta d\phi) \\ &= r^2 \sin \theta d\theta d\phi\end{aligned}$$

$$d\omega = \frac{dA}{r^2} = \sin \theta d\theta d\phi$$

$$S = \int_0^{\pi} \int_0^{2\pi} \sin \theta d\theta d\phi = 4\pi$$

Solid Angle

Definition: The solid angle (SA) subtended by an object from a point P is the area of projection of the object onto the unit sphere centered at P, the size of a differential patch, dA ,

$$dA = (rd\theta)(r \sin \theta d\phi) = r^2 \sin \theta d\theta d\phi$$

The differential solid angle:

$$d\omega = dA / r^2 = \sin \theta d\theta d\phi$$

Solid Angle

- Size of a patch, dA , is

$$dA = (r \sin \theta d\varphi)(r d\theta)$$

- Solid angle is

$$d\vec{\omega} = \frac{dA}{r^2}$$

- Measured in steradians (sr)

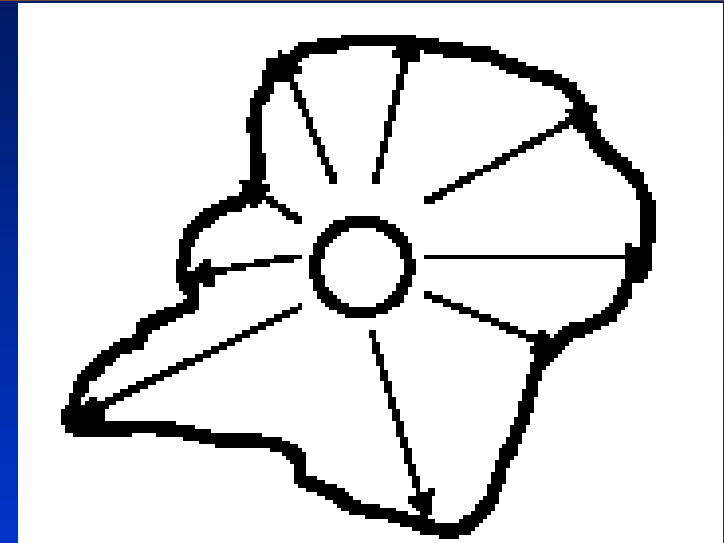
What about a Point Source?

- Not a lot of area.....

Radiant Intensity

$$I(\omega) = \frac{d\Phi}{d\vec{\omega}}$$

$$\Phi = \int_{\Omega} I(\omega) d(\vec{\omega})$$



For an isotropic point source: $I(\omega) = \Phi/4\pi$

Isotropic Point Source

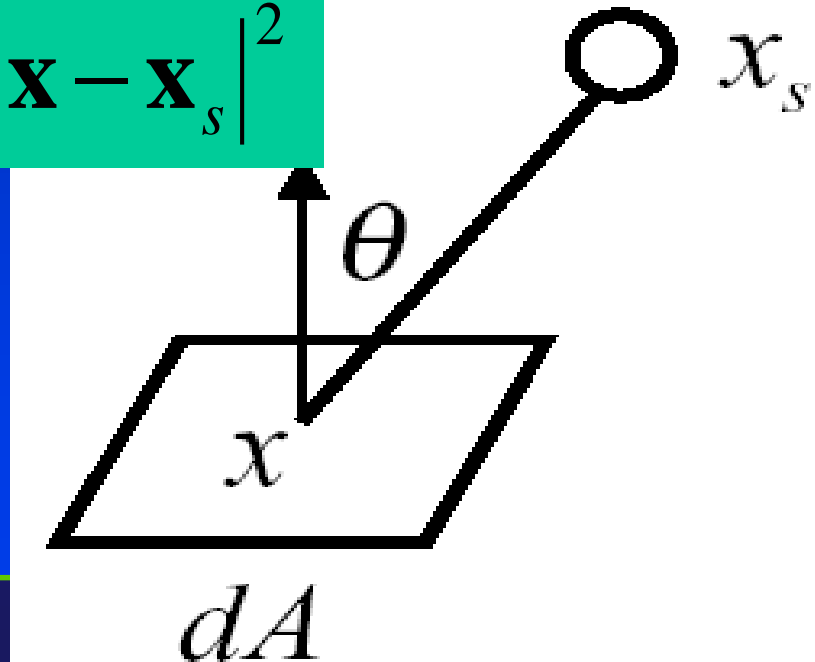
- Irradiates equally in all directions
- Even distribution of power over sphere
- Intensity is power over whole sphere

$$I = \frac{d\Phi}{d\vec{\omega}} = \frac{\Phi}{4\pi}$$

Irradiance due to a Point Light

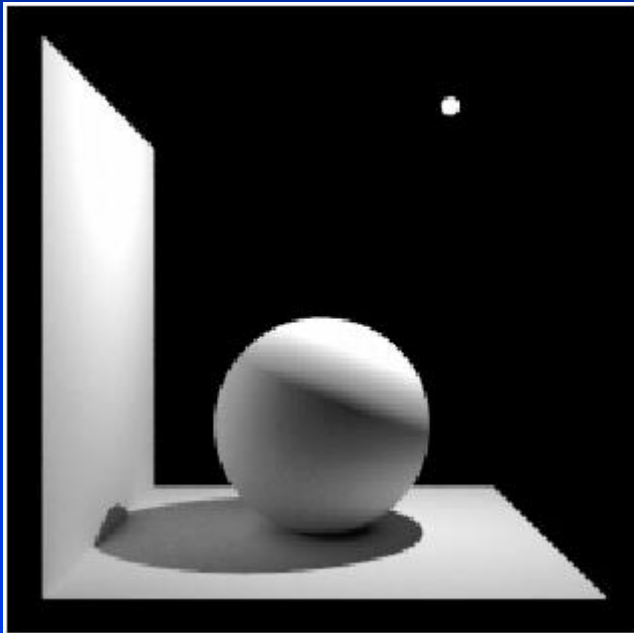
Irradiance on a differential surface due to an isotropic point light source is

$$E = \frac{d\Phi}{dA} = I(\omega) \frac{d\omega}{dA} = \frac{\Phi}{4\pi} \frac{\cos\theta}{|\mathbf{x} - \mathbf{x}_s|^2}$$

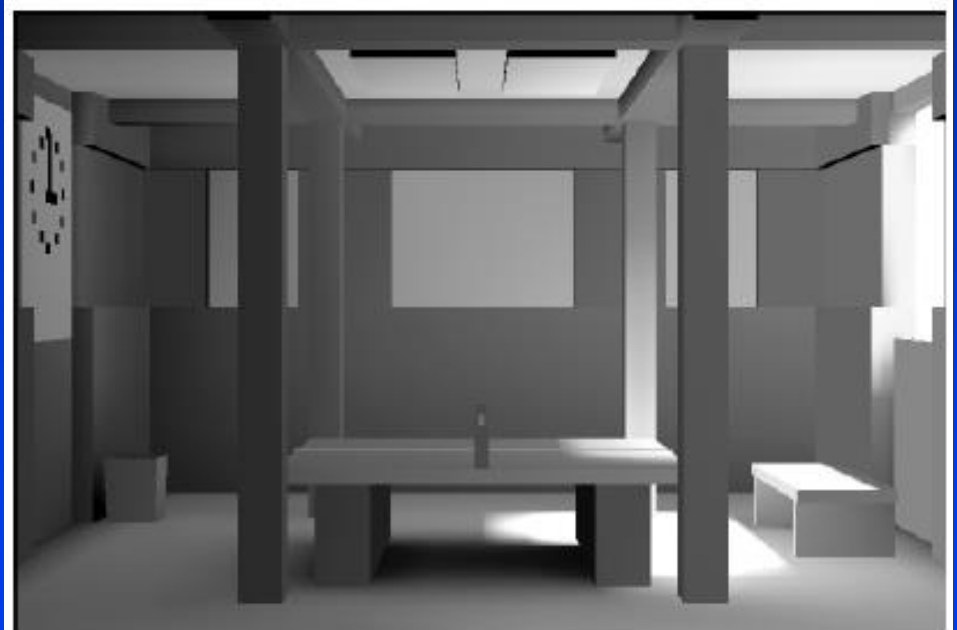


Point Light Source

point source



area source(s)

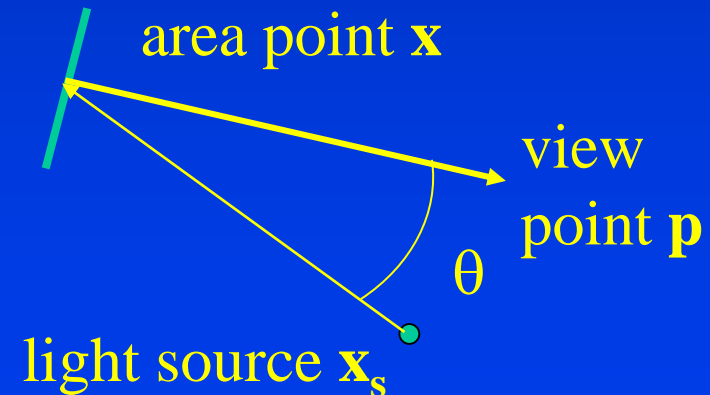


Irradiance on Differential Patch

- What is the irradiance of a differential area, illuminated by a point source at \mathbf{x}_s , seen from a light point \mathbf{p} ?
- This is the “inverse square law”

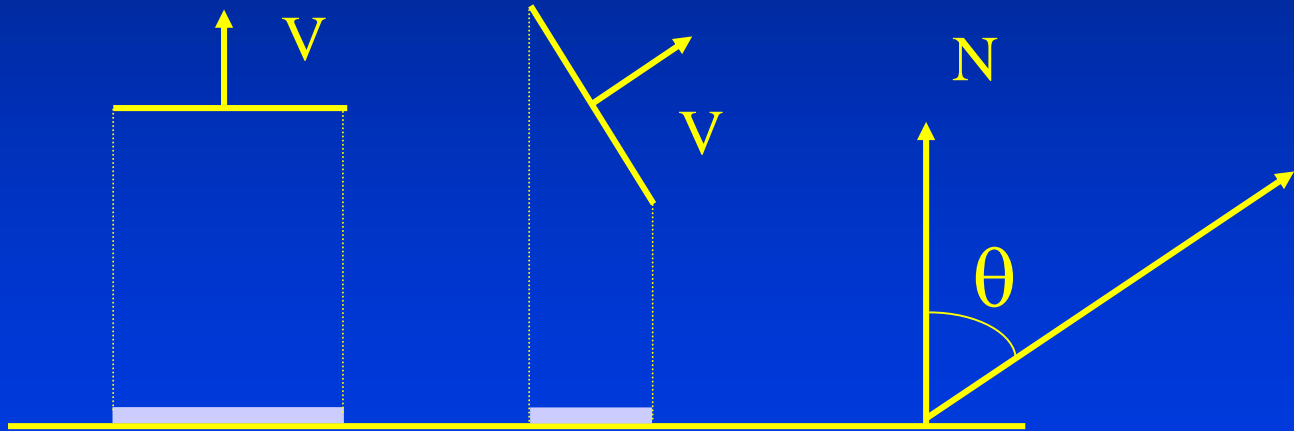
$$E = I \frac{d\omega}{dA} = \frac{\Phi}{4\pi} \frac{\cos \theta}{|\mathbf{x} - \mathbf{x}_s|^2}$$

accounts for
projected area



Projected Area

- $A_p = A (\mathbf{N} \cdot \mathbf{V}) = A \cos \theta$



Radiance

- Power per unit projected area per unit solid angle.

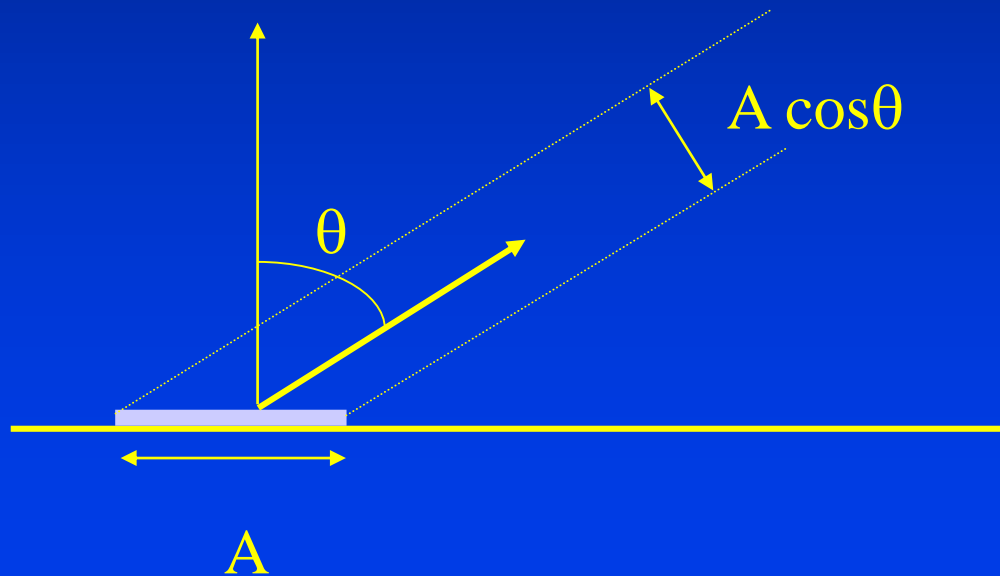
$$L = \frac{d\Phi}{dA_p d\vec{\omega}}$$

- Units: watts per steradian m^2
- We have now introduced *projected area*, a cosine term.

$$L = \frac{d\Phi}{dA \cos \theta d\vec{\omega}}$$

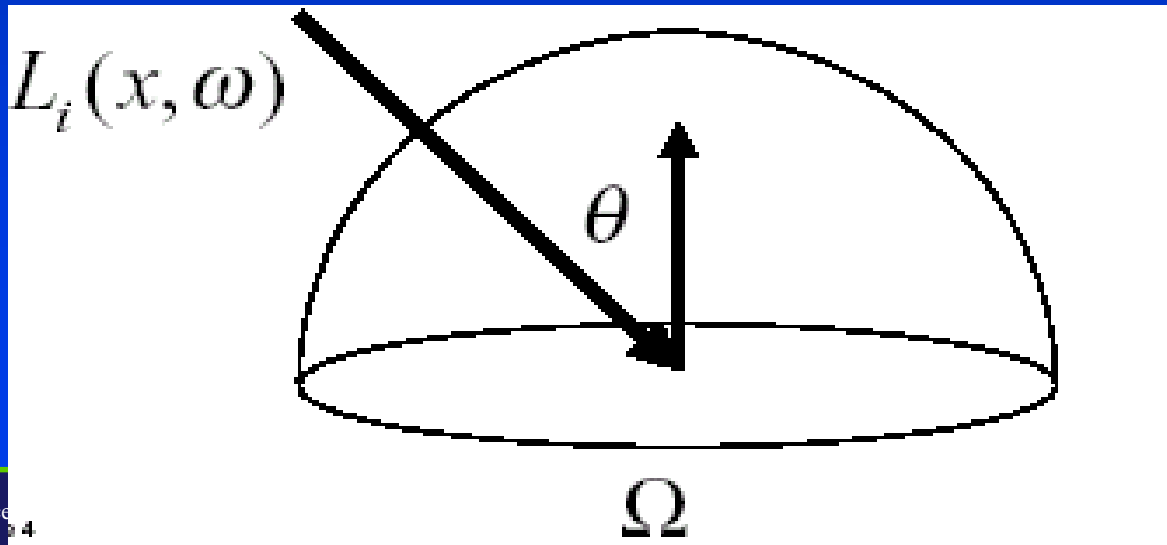
Why the Cosine Term?

- Foreshortening is by cosine of angle.
- Radiance gives energy by *effective* surface area, as seen from the view direction



Irradiance from Radiance

- **Irradiance:** Radiant power per unit area incident on a surface
- Just look at definitions of E and L...
- $\cos\theta d\omega$ is projection of a differential patch



Irradiance from Radiance

$$E = \int_{\Omega} L_i(\mathbf{x}, \omega) \cos \theta \, d\omega$$

Radiosity

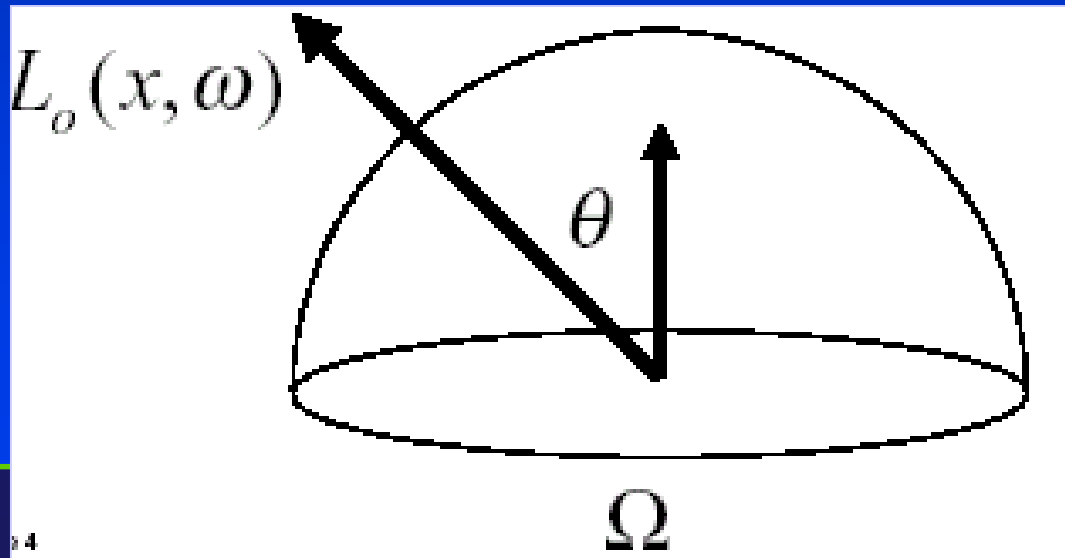
- Surfaces in a scene reflect & emit light
- Some of this light reaches the viewer; this makes the surface visible
- But much of this reflected/emitted light will illuminate other surfaces
- This light will then reflect off these other surfaces; in fact, *every* surface in a scene will illuminate other surfaces in the scene

Radiosity

Official term : Radiant Exitance

Radiosity: Radiant power per unit area exiting a surface

$$B = \int_{\Omega} L_o(x, \omega) \cos \theta d\omega$$



Properties of Radiance

(1) Fundamental quantity

-all other quantities derived from it

(2) Invariant along a ray

- quantity used by ray tracers

(3) Sensor response is proportional to radiance

-eye/camera response depends on radiance

Properties

- **What's Effect of Distance on Radiance?**
 - Let's look at thin pencil of light
- **What's radiance on a sensor?**

Radiance at a Sensor

- Sensor of a fixed patch sees more of a surface that is farther away.
- However, the solid angle is inversely proportional to distance.
- Response of a sensor is proportional to radiance.

$$d\vec{\omega} = dA / r^2$$

Radiance as a Unit of Measure

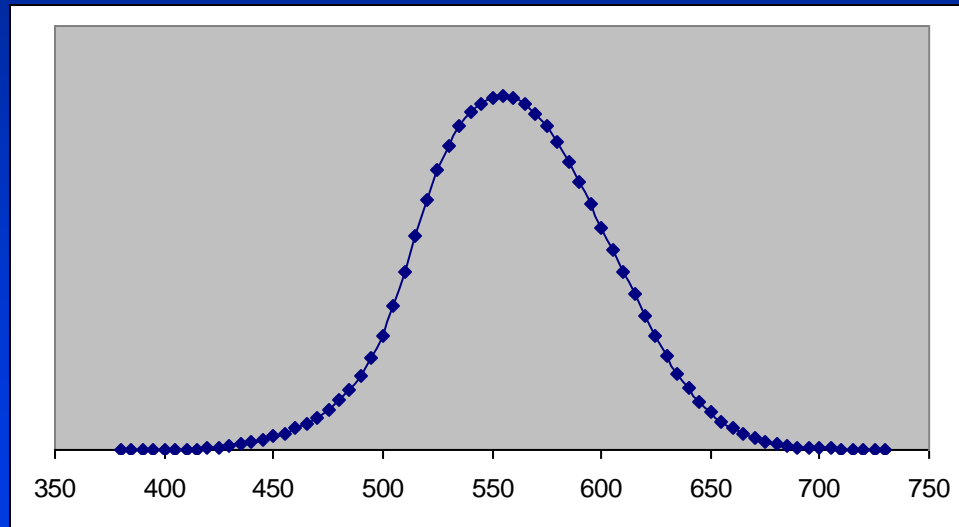
- Radiance doesn't change with distance
 - Therefore it's the quantity we want to measure in a ray tracer.
- Radiance proportional to what a sensor (camera, eye) measures.
 - Therefore it's what we want to output.

Radiometry and Photometry

- Photometry (begun 1700s by Bouguer) deals with how humans perceive light.
- All measurements relative to perception of illumination
- Units different from radiometric but conversion is scale factor -- weighted by spectral response of eye (over about 360 to 800 nm).

CIE Curve

- Response is the integral over all wavelengths



Radiometry Summary

- **Energy**
 - photons...
- **Power**
 - energy / time
- **Irradiance and Radiosity**
 - power / projected-area
- **Intensity**
 - power / solid-angle
- **Radiance** power / ((projected-area)*(solid-angle))

Radiometry

Radiometry describes light in itself

- What about interaction of light with objects?

Light-Surface Interaction

- **Surface Properties**
- Reflected radiance is proportional to incoming flux and to irradiance (incident power per unit area)

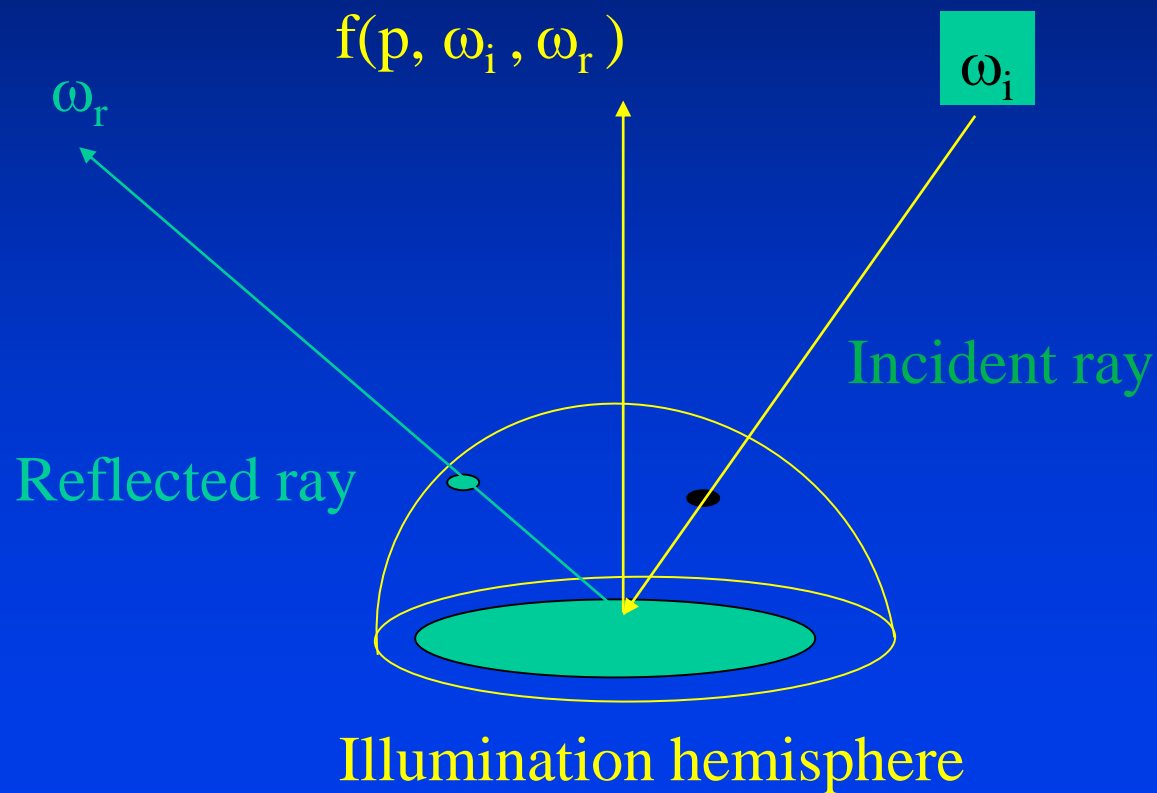
$$dL_r(\vec{\omega}_r) \propto dE(\vec{\omega}_i)$$

Reflection Functions

Reflection is defined as the the process by which the light incident on a surface leaves the surface from the same side.

The nomenclature and the general properties of reflection functions are discussed.

BRDF



Bidirectional Reflection Distribution Functions (BRDF)

Bidirectional Reflection Distribution Function

$$f(x, \omega_i, \omega_r) = L_r(x, \omega_r) / dE_i(x, \omega_i)$$

In short, this is the ratio of radiance in a reflected direction to the differential irradiance that created

Bidirectional Reflectance Distribution Function (BRDF)

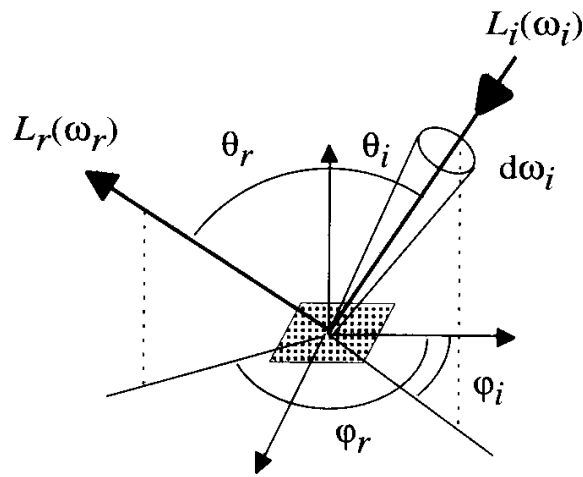


Figure 2.9: Bidirectional reflection distribution function.

Relates incoming and outcoming radiances at reflection

$$f_r(\vec{\omega}_i \rightarrow \vec{\omega}_r) \equiv \frac{L_r(\vec{\omega}_r)}{L_i(\vec{\omega}_i) \cos \theta_i d\omega_i}$$

BRDF Dimensionality

- **Function of**
 - position,
 - four angles (two incident, two reflected),
 - Wavelength and polarization (usually ignored!)
- **Material is usually considered uniform, so position is ignored!**
- **If isotropic, one angle goes away.**
- **Result: 3 or 4 dimensional.**

BRDF Properties

- Reciprocity (of incoming and outgoing directions)
- Natural condition: material is ‘symmetric’

$$f_r(\vec{\omega}_i \rightarrow \vec{\omega}_o) = f_r(\vec{\omega}_o \rightarrow \vec{\omega}_i)$$

Properties of the BRDF

- (1) Reciprocity

- $$f(\mathbf{x}, \omega_i, \omega_r) = f(\mathbf{x}, \omega_r, \omega_i)$$

- (2) Anisotropy

If the incident and the reflected light are fixed and the underlying surface is rotated about the surface normal, the percentage of light reflected may change.

Reflectance Equation

The BRDF allows us to calculate outgoing light, given incoming light:

$$\begin{aligned}L_r(\mathbf{x}, \omega_r) &= f(\mathbf{x}, \omega_i, \omega_r) * dE_i(\mathbf{x}, \omega_r) \\ &= f(\mathbf{x}, \omega_i, \omega_r) * L_i(\mathbf{x}_i, \omega) \cos \theta d\omega_i\end{aligned}$$

Integrating over the hemisphere gives the reflectance equation:

$$L_r(\mathbf{x}, \omega_r) = \int_{\Omega} f(\mathbf{x}, \omega_i, \omega_r) * L_i(\mathbf{x}_i, \omega) \cos \theta d\omega_i$$

Reflectance

- **Reflectance:** ratio of reflected flux to incident flux

$$\rho = \frac{d\Phi_r}{d\Phi_o} = \frac{\int_{\Omega_r} L_r(\mathbf{x}, \omega_r) \cos \theta_r d\omega_r}{\int_{\Omega_i} L_i(\mathbf{x}, \omega_i) \cos \theta_i d\omega_i}$$

- Reflectance is always between 0 and 1
but depends on incident radiance distribution

Lambertian Diffuse Reflection

$$\rho = \frac{d\Phi_r}{d\Phi_o} = \frac{\int_{\Omega_r} L_r(\mathbf{x}, \omega_r) \cos \theta_r d\omega_r}{\int_{\Omega_i} L_i(\mathbf{x}, \omega_i) \cos \theta_i d\omega_i}$$
$$= L_{r,diffuse} \frac{\int_{\Omega_r} \cos \theta_r d\omega_r}{E} = \pi f_{r,diffuse}$$

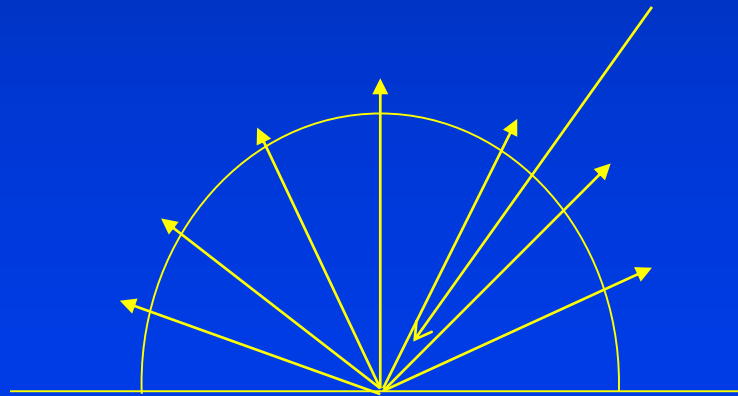
Lambertian (Diffuse) Surfaces

Diffuse BRDF

- BRDF is a constant.

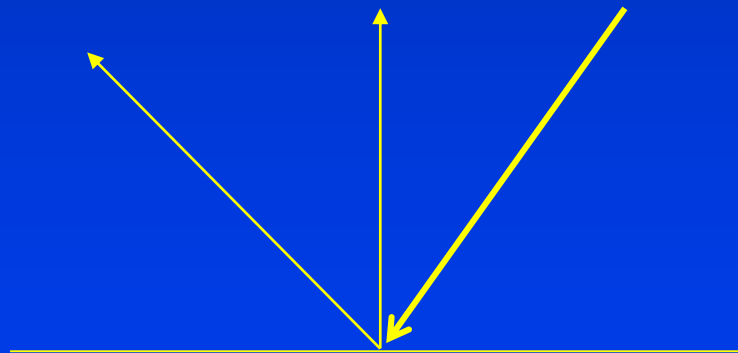
$$L_r(\vec{\omega}_r) = k E$$

- Independent of direction of incoming light.
- Radiosity over irradiance is constant.



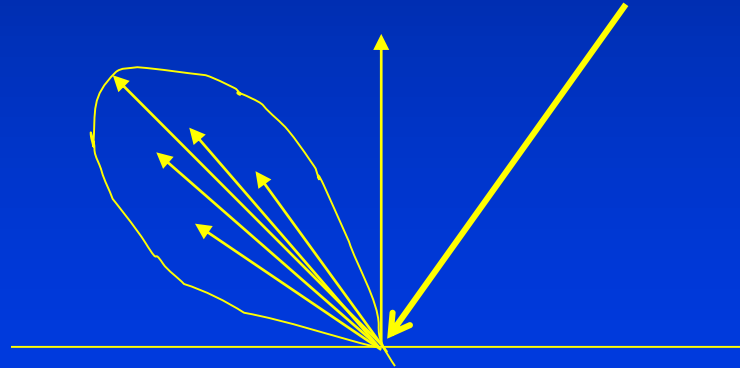
Mirror (Ideally Specular) Surfaces

- Reflection takes place on a plane perpendicular to surface
- Angle of reflectance = angle of incidence
- BRDF modeled by delta functions



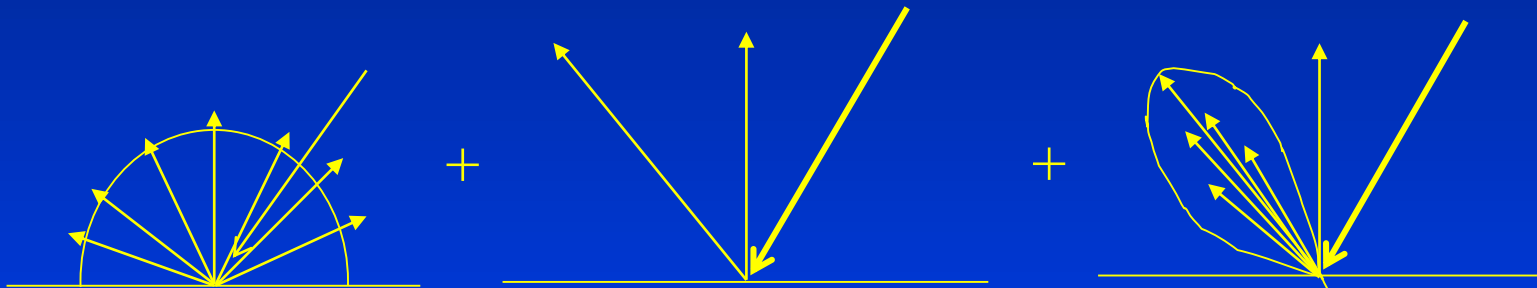
Glossy (Shiny) Surfaces

- Between lambertian and specular.



Complex BRDFs

- Combinations of the three



diffuse

mirror

glossy

- An interesting BRDF is a retroreflector
- What's a range of values of BRDF?

Representations

- 4D function, so awkward to represent directly.
- Most often, it is represented as parametric equation (Phong, Cook-Torrance, etc.).
- Sometimes with basis functions (such as spherical harmonics, sum of cosines, etc.).

Reflectance

- Ratio of reflected to incident flux
- Always 0 to 1; convenient

$$\rho(\vec{\omega}_i \rightarrow \vec{\omega}_r) \equiv \frac{\int_{\Omega_r} \int_{\Omega_i} f_r(\vec{\omega}_i \rightarrow \vec{\omega}_r) \cos \theta_i d\vec{\omega}_i \cos \theta_r d\vec{\omega}_r}{\int_{\Omega_i} \cos \theta_i d\vec{\omega}_i}$$

- Can be over part or all of incident and exitant hemispheres

The Rendering Equation

$$L_o(\mathbf{x}, \vec{\omega}_o) = L_e(\mathbf{x}, \vec{\omega}_o) + \int_{\Omega} f_r(\mathbf{x}, \vec{\omega}_o, \vec{\omega}_i) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

The equation is annotated with red text: "emitted" above L_e , "outgoing" below L_o , and "reflected" above the integral term.

- “Essence” of physically-based rendering
- Basically, it is an energy balance equation
- Oftentimes, approximated by splitting diffuse, specular, and glossy (shiny) components

The Rendering Equation

$$L_o(\mathbf{x}, \vec{\omega}_o) = L_e(\mathbf{x}, \vec{\omega}_o) + \int_{\Omega} f_r(\mathbf{x}, \vec{\omega}_o, \vec{\omega}_i) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

- Not exactly like Kajiya '86 (more like Radiosity equation).
- Often approximated by splitting diffuse, specular, and glossy.

Transport of Energy

- Now we have a model of the light-surface interaction (i.e., the ‘rendering equation’ for reflection modeling)
- How do we transfer energy from light sources to all surfaces in a 3D scene?
- Approximations are used to make computational feasible
 - Only certain paths accounted for

Transport Approximations

- **Classical ray tracing**
 - Direct lambertian
 - Global specular
 - View dependent
- **Radiosity**
 - Global illumination between diffuse surfaces
 - View independent

Rendering Equation

- Recall

$$L_o(\mathbf{x}, \vec{\omega}_o) = L_e(\mathbf{x}, \vec{\omega}_o) + \int_{\Omega} f_r(\mathbf{x}, \vec{\omega}_o, \vec{\omega}_i) L_i(\mathbf{x}, \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

- We want to simplify enough to solve

Radiosity Assumptions

1. Opaque surfaces
 2. Vacuum
 3. Purely diffuse surfaces
- Solve in object space
 - Solution represented in object space
 - View independent; render as triangles w/ vertex color (or a radiosity texture)

Other Surfaces

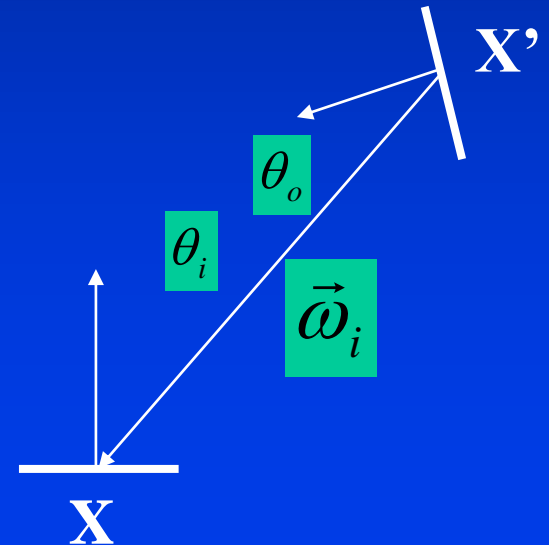
Let's relate incoming radiance to other surfaces

$$L_i(\mathbf{x}, \vec{\omega}_i) = L_o(\mathbf{x}', \vec{\omega}_o') V(\mathbf{x}', \mathbf{x})$$

where

$$\vec{\omega}_i = -\vec{\omega}_o'$$

and $V(\mathbf{x}', \mathbf{x})$ is 0 or 1.



Radiance at x from x'

So now rendering equation is (without emitter)

$$L_o(\mathbf{x}, \vec{\omega}_o) = \int_{\Omega} f_r(\mathbf{x}, \vec{\omega}_o, \vec{\omega}_i) L_o(\mathbf{x}', \vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

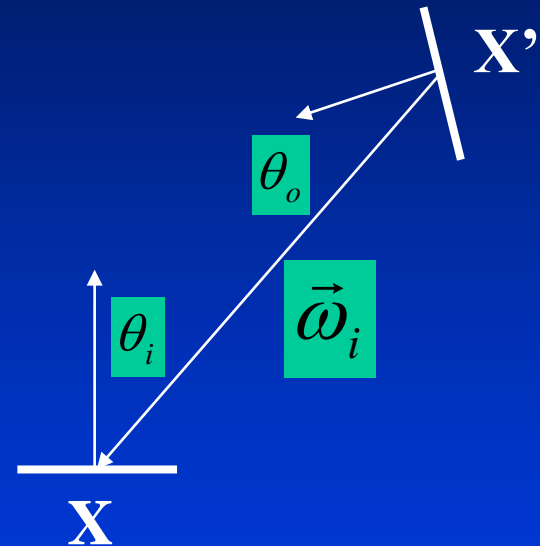
Next, let's make our integral over surfaces instead of solid angles

From Solid Angle to Area

Recall that

$$d\vec{\omega} = \frac{dA}{r^2}$$

$$d\vec{\omega}_i = \frac{\cos \theta_o dA'}{|\mathbf{x}' - \mathbf{x}|^2}$$



So

$$L_o(\mathbf{x}, \vec{\omega}_o) = \int_{\Omega} f_r(\mathbf{x}, \vec{\omega}_o, \vec{\omega}_i) L_o(\mathbf{x}', \vec{\omega}_o') V(\mathbf{x}', \mathbf{x}) \frac{\cos \theta_o \cos \theta_i}{|\mathbf{x}' - \mathbf{x}|^2} dA'$$

Geometry Term

For simplicity, define

$$G(\mathbf{x}', \mathbf{x}) = G(\mathbf{x}, \mathbf{x}') = \frac{\cos \theta_o \cos \theta_i}{|\mathbf{x}' - \mathbf{x}|^2} V(\mathbf{x}', \mathbf{x})$$

Therefore

$$L_o(\mathbf{x}, \vec{\omega}_o) = \int_S f_r(\mathbf{x}, \vec{\omega}_o, \vec{\omega}_i) L_o(\mathbf{x}', \vec{\omega}_o') G(\mathbf{x}, \mathbf{x}') dA'$$

Diffuse Assumption

All surfaces diffuse, so replace BRDF with a constant

$$\rho(\mathbf{x}) = f_r(\mathbf{x}, \vec{\omega}_o, \vec{\omega}_i)$$

Also angles are now irrelevant, so

$$L_o(\mathbf{x}) = \rho(\mathbf{x}) \int_S L_o(\mathbf{x}') G(\mathbf{x}, \mathbf{x}') dA'$$

Convert to Radiosities

$$B = \int_{\Omega} L_o \cos \theta d\omega$$

- So $L = B / \pi$, and

$$B(\mathbf{x}) = \rho(\mathbf{x}) \int_S \frac{B(\mathbf{x}')G(\mathbf{x}, \mathbf{x}')}{\pi} dA'$$

Radiosity Equation

For convenience subsume the π into $G()$. Also, add the emissive term back to get

$$B(\mathbf{x}) = E(\mathbf{x}) + \rho(\mathbf{x}) \int_S B(\mathbf{x}') G(\mathbf{x}, \mathbf{x}') dA'$$

where

$$G(\mathbf{x}, \mathbf{x}') = \frac{\cos \theta_o \cos \theta_i}{\pi |\mathbf{x}' - \mathbf{x}|^2} V(\mathbf{x}', \mathbf{x})$$

Radiosity Equation

More importantly the outgoing radiance is the same in all directions and in fact equals B/π .

$$B(x) = E(x) + \rho(x) \int_s B(x') \frac{G(x, x') V(x, x')}{\pi} dA$$

Where Are We?

- We have an expression relating radiosity at a point to radiosity at ALL other points
- But no method to solve for the values yet!

Next

- Formulation of the radiosity method
- We need to address practical aspects for computing a solution (i.e., ‘render’ a 3D scene)
- Later
 - Monte Carlo methods
 - Bi-directional ray tracing

More Readings on This Topic

- Chapter 2 (by Hanrahan) in Cohen and Wallace, *Radiosity and Realistic Image Synthesis*.
- Glassner, *Principles of Digital Image Synthesis*, pp. 648 – 659 and Chapter 13.

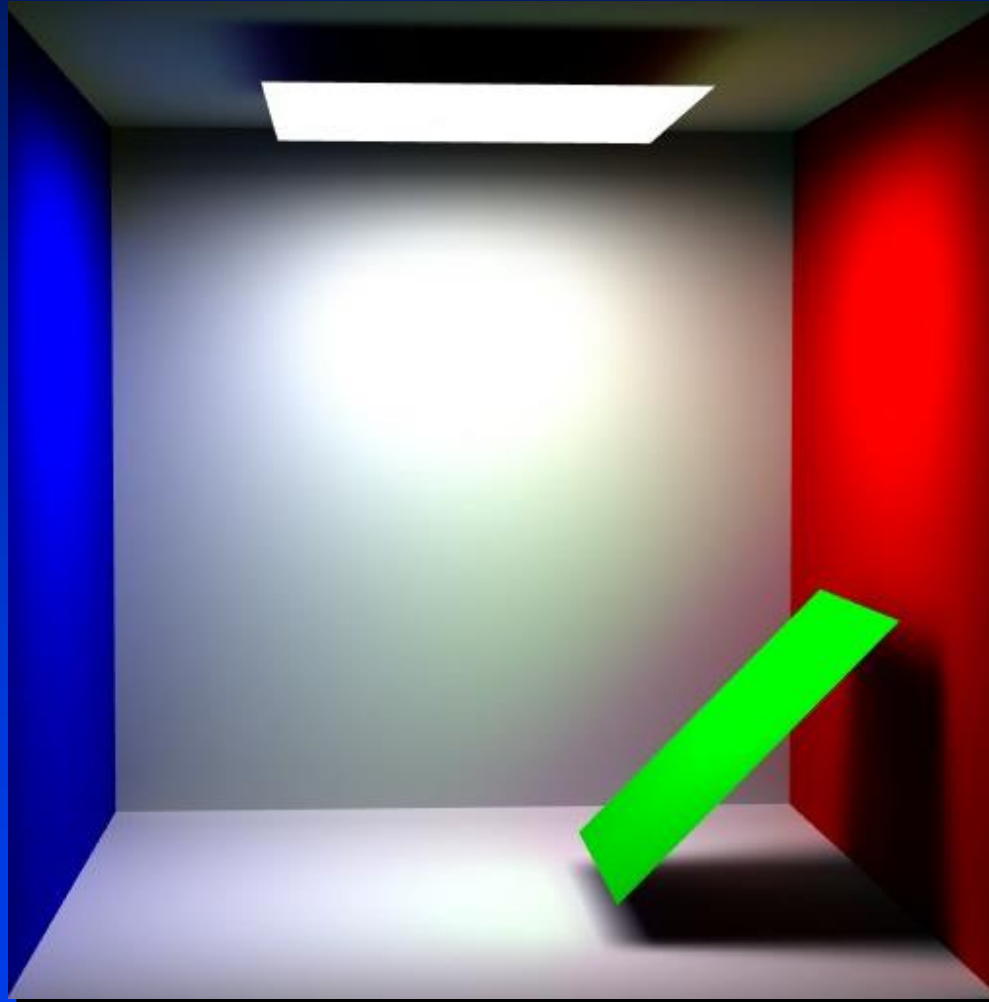
References

- Geometrical Considerations and Nomenclature for Reflectance, F.E. Nicodemus, J.C. Richmond, J.J. Hsia, I.W. Ginsberg, and T. Limperis, Nat. Bureau Stand. (1977)
- Link to PDF is
<http://physics.nist.gov/Divisions/Div844/facilities/specphoto/pdf/geoConsid.pdf>

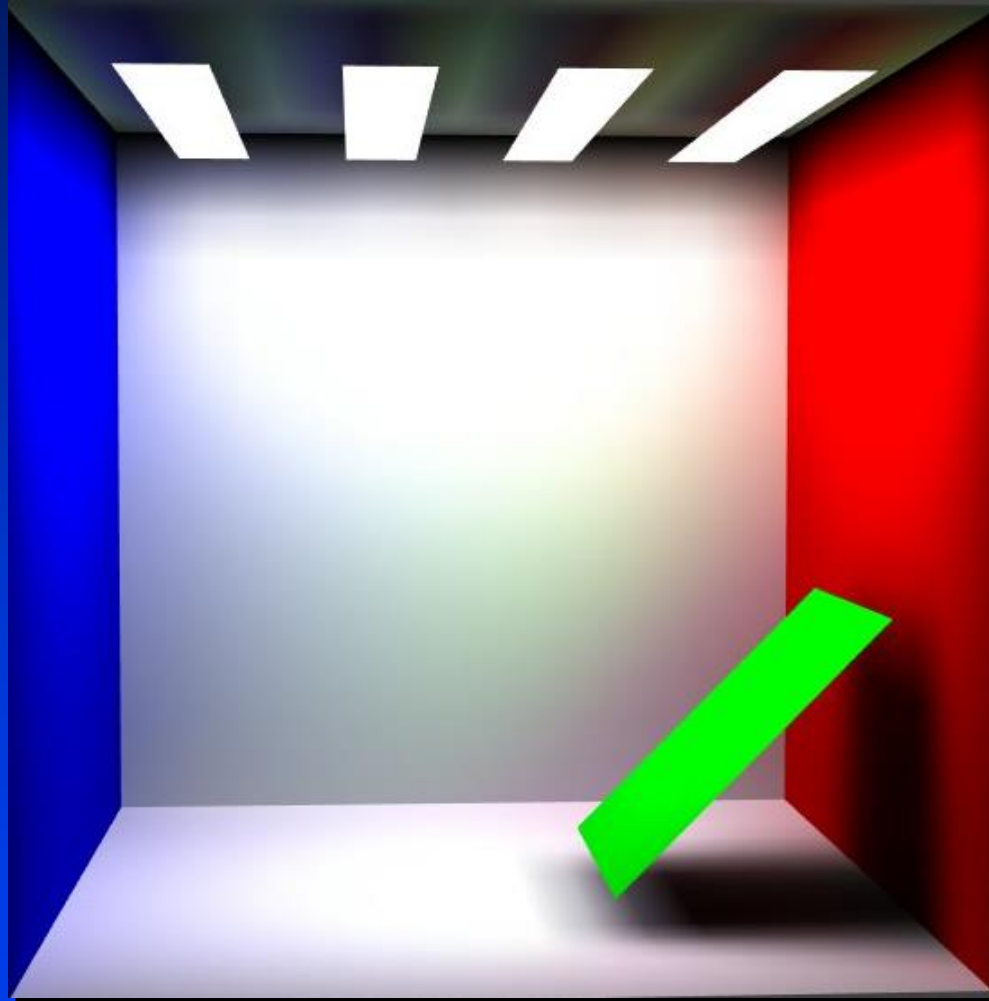
References

- **Bastos dissertation, Chapter 3 in**
<http://www.cs.unc.edu/~bastos/PhD/2and3.pdf>
- **Heckbert, Adaptive radiosity textures for bidirectional ray tracing**
– <http://doi.acm.org/10.1145/97879.97895>

One Light Source



Four Light Sources



Examples

- Realistic rendering: color bleeding



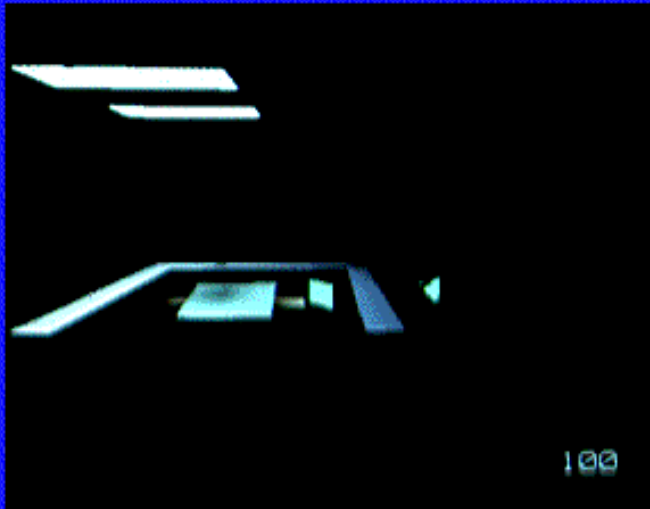
Progressive Rendering



PROGRESSIVE SOLUTION

The above images show increasing levels of global diffuse illumination. From left to right: 0 bounces, 1 bounce, 3 bounces.

Progressive Rendering

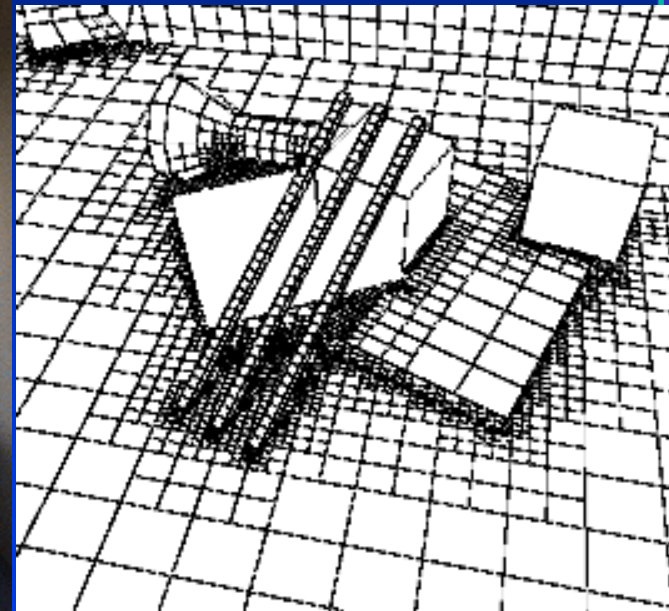
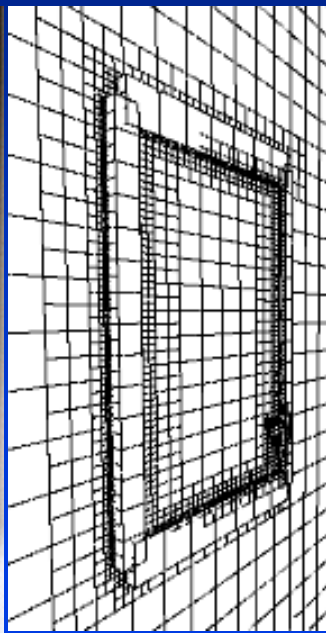


Progressive Rendering



Mesh Refinement

solution (rendering)



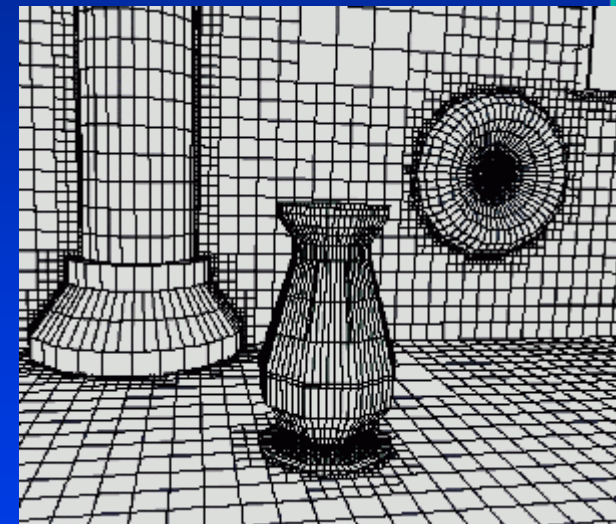
refined mesh

Mesh Refinement

whole scene



mesh detail



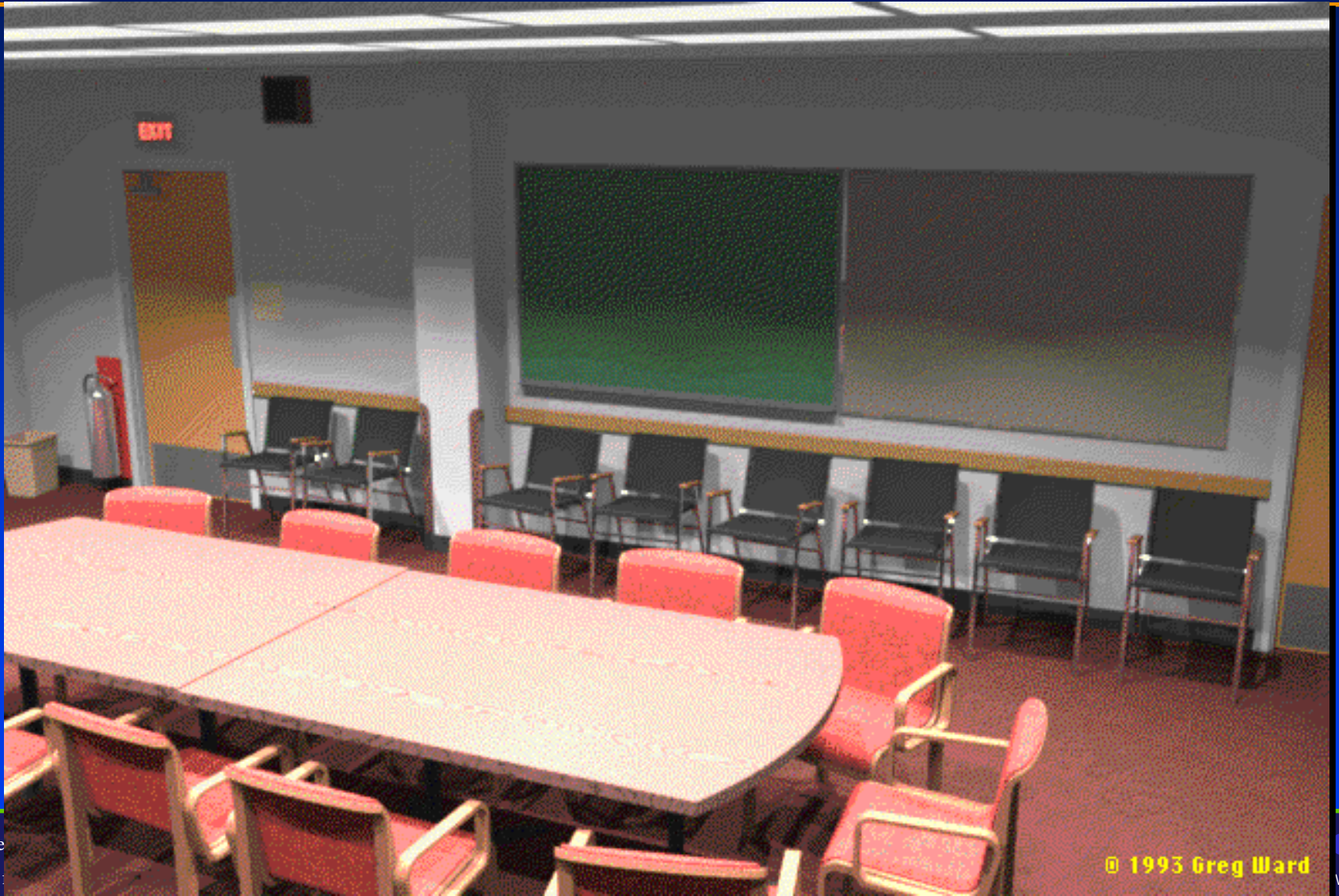
Discrete Meshing



Rendering + Textures



Real Photograph



... and The Rendered Scene



A Complex Scene



Another Complex Scene

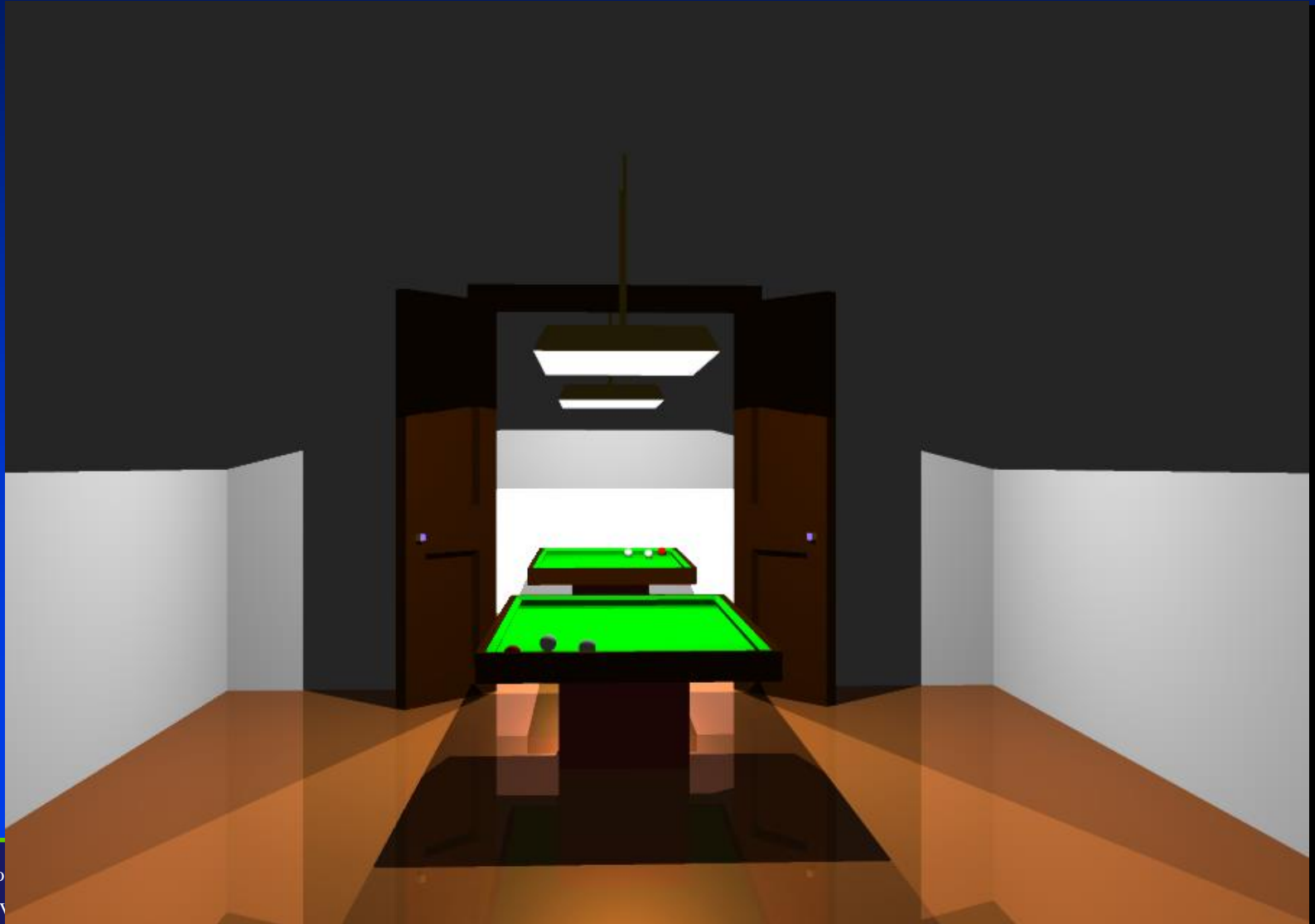
- 100,000 polygons



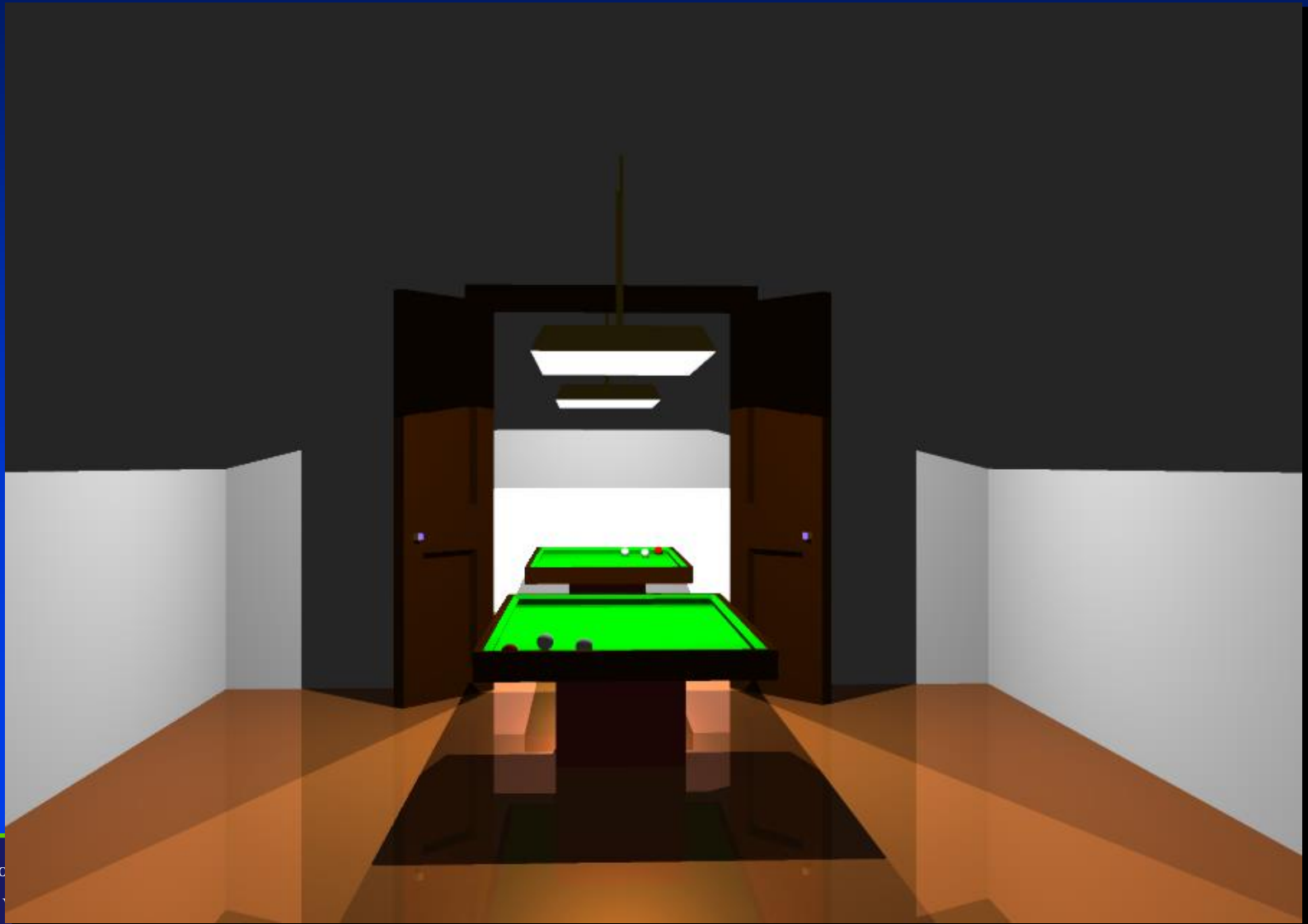
Rendering with Volumetric Effects



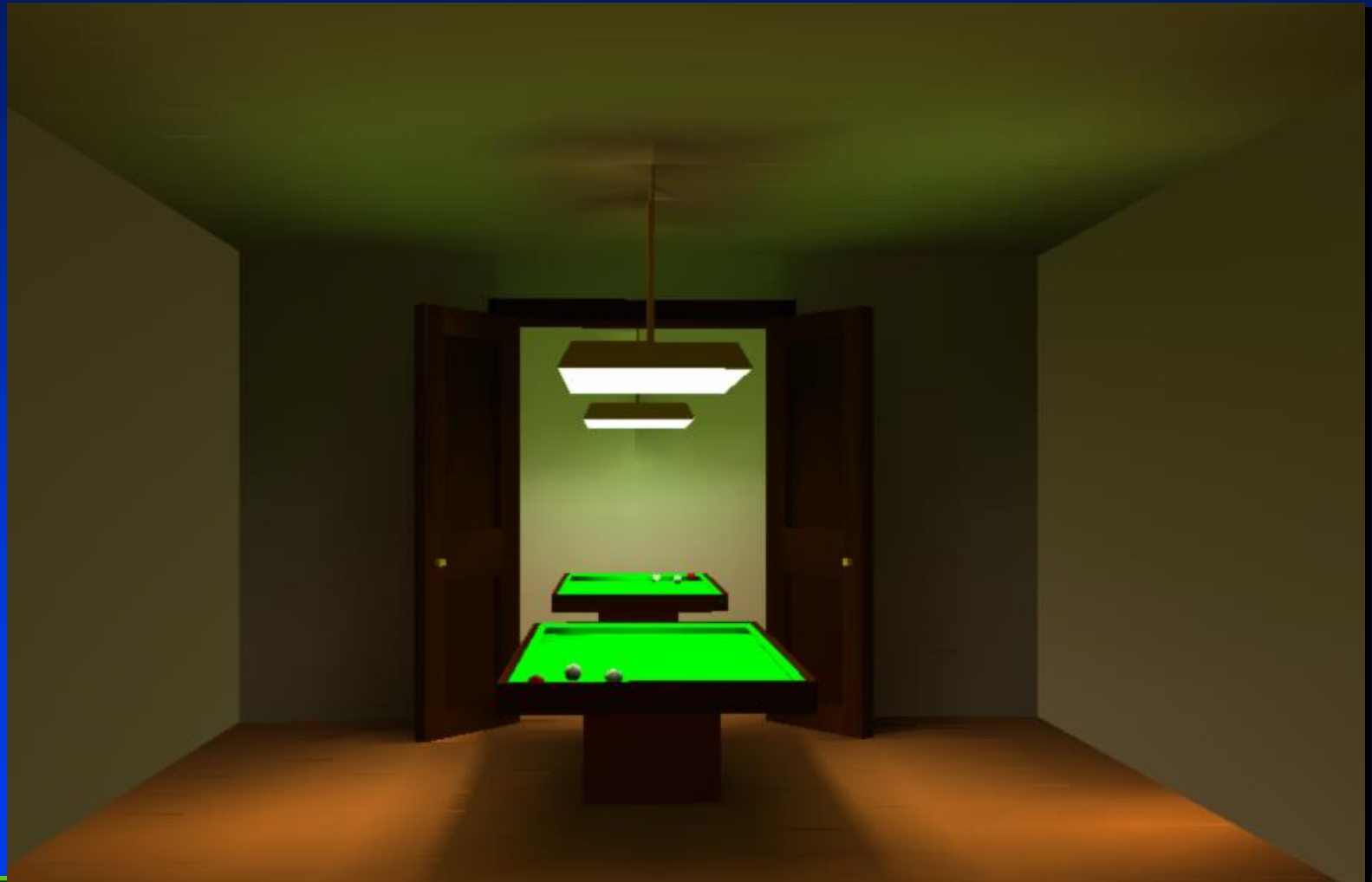
Ray Tracing



Ray Tracing



Radiosity

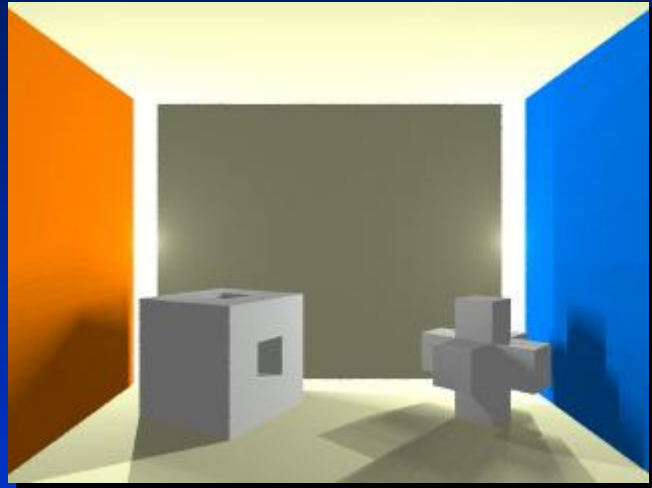


Radiosity



Combined Method

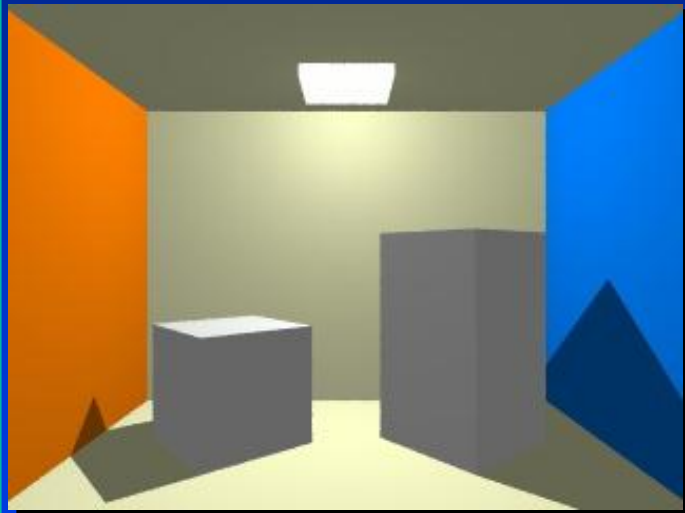




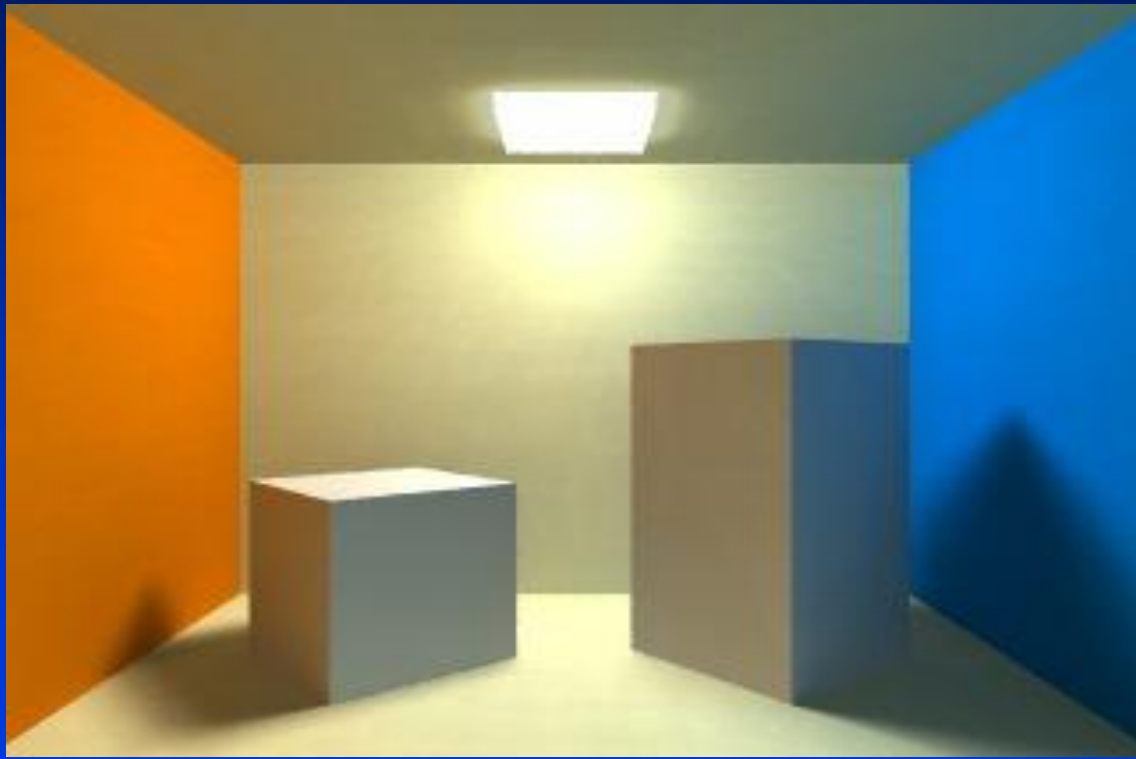
Radiosity off

Radiosity on



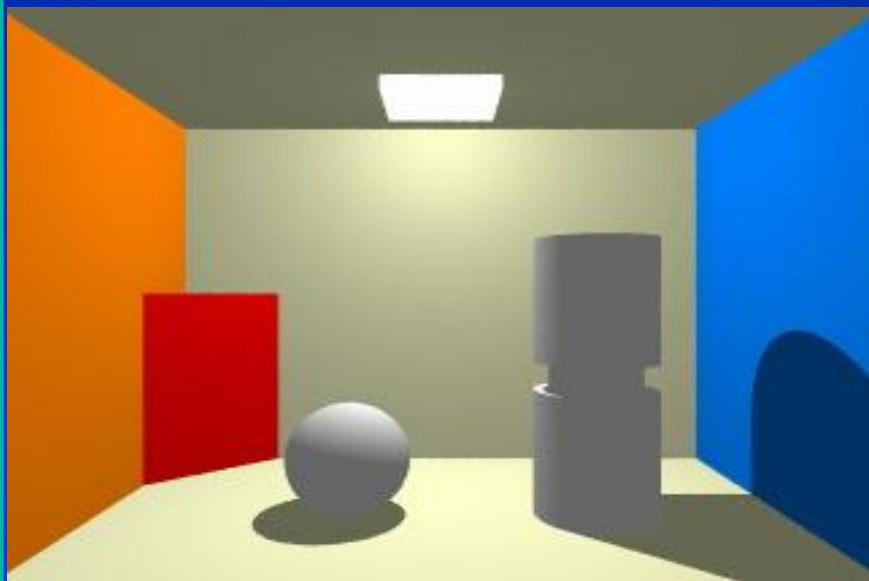


Radiosity off

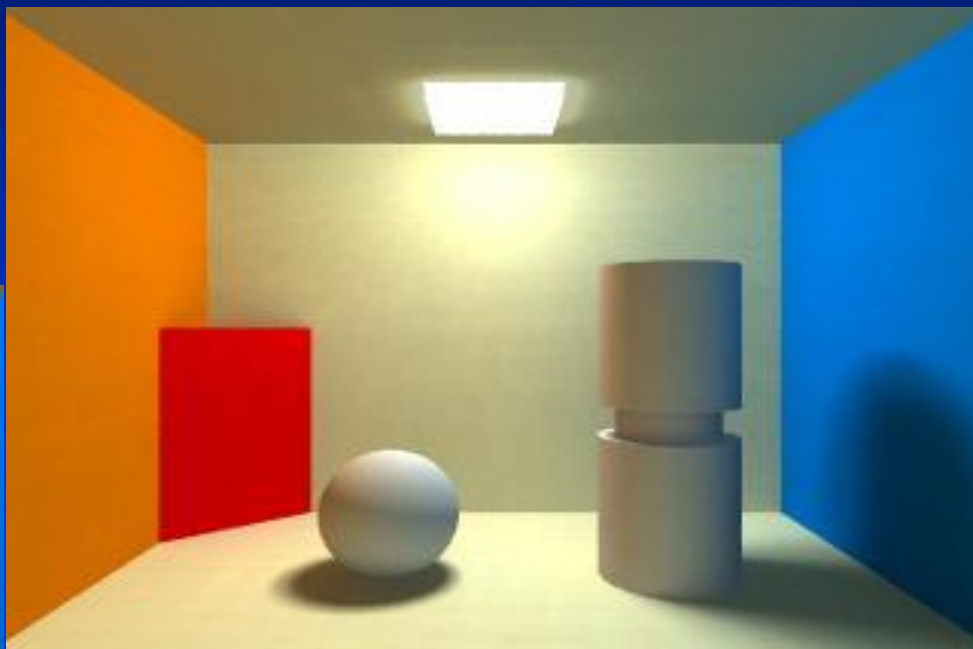


Radiosity on

Radiosity off



Radiosity on



Radiosity



Standard Radiosity



Smoke



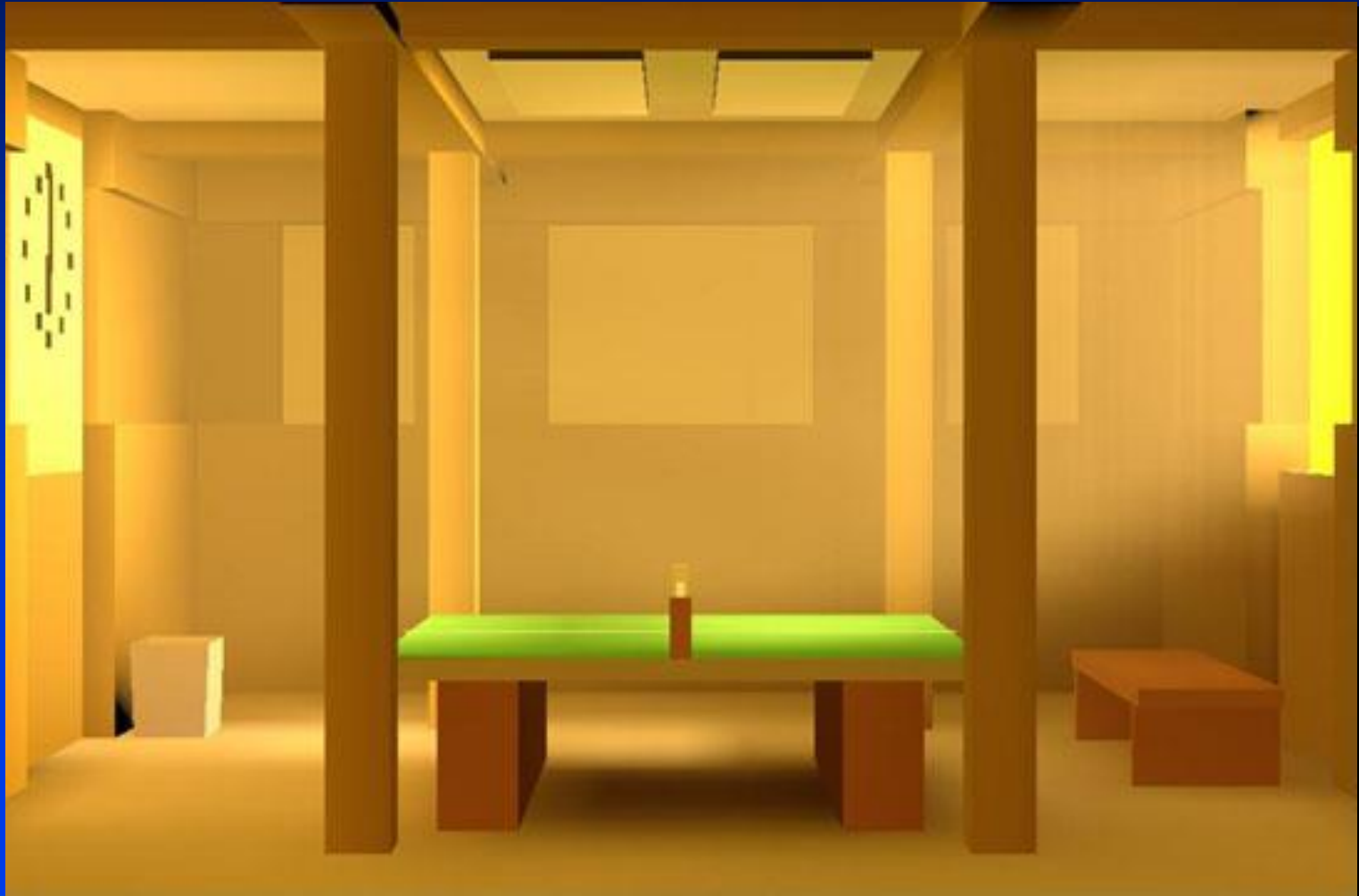
Ceiling Light (3am)



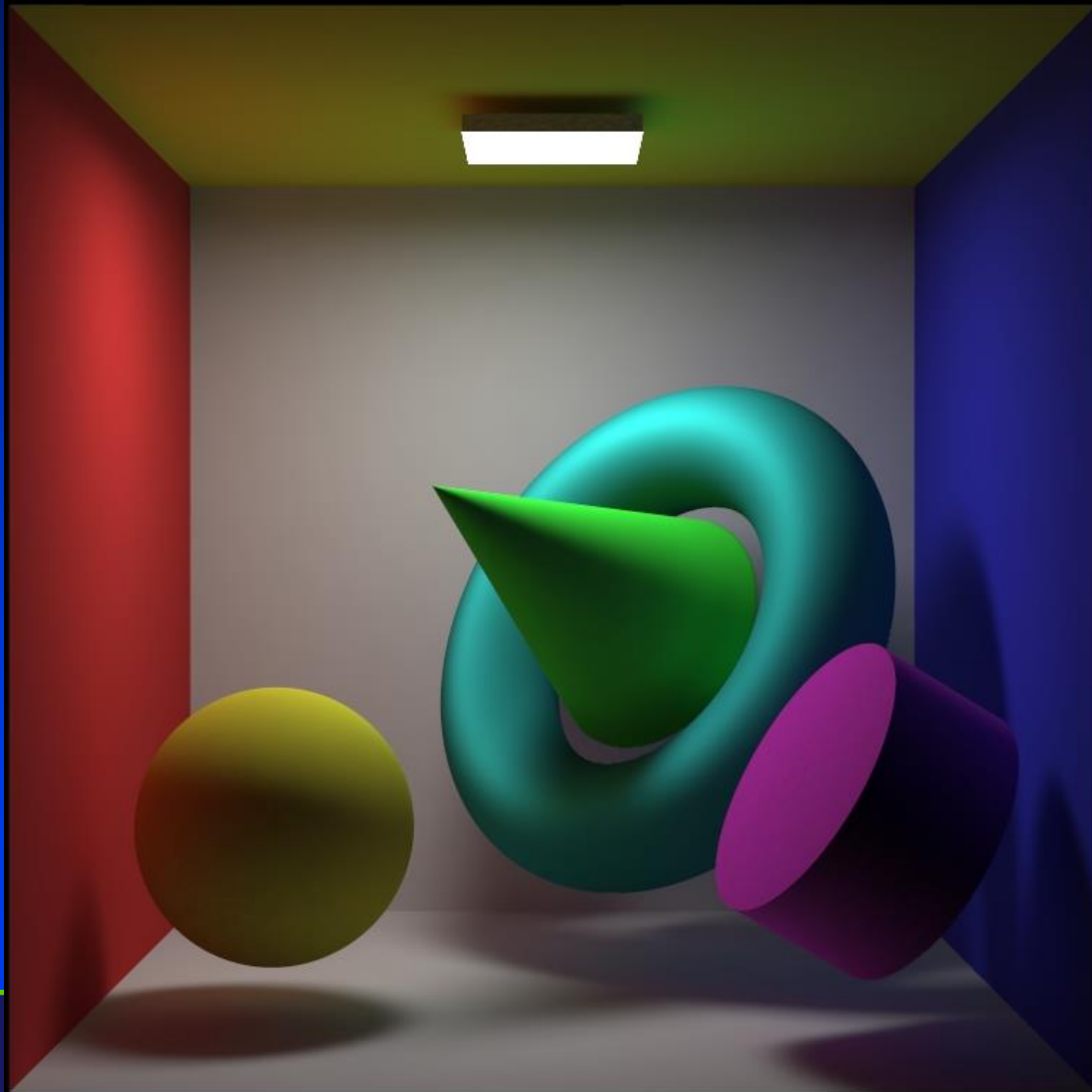
Afternoon (3pm)



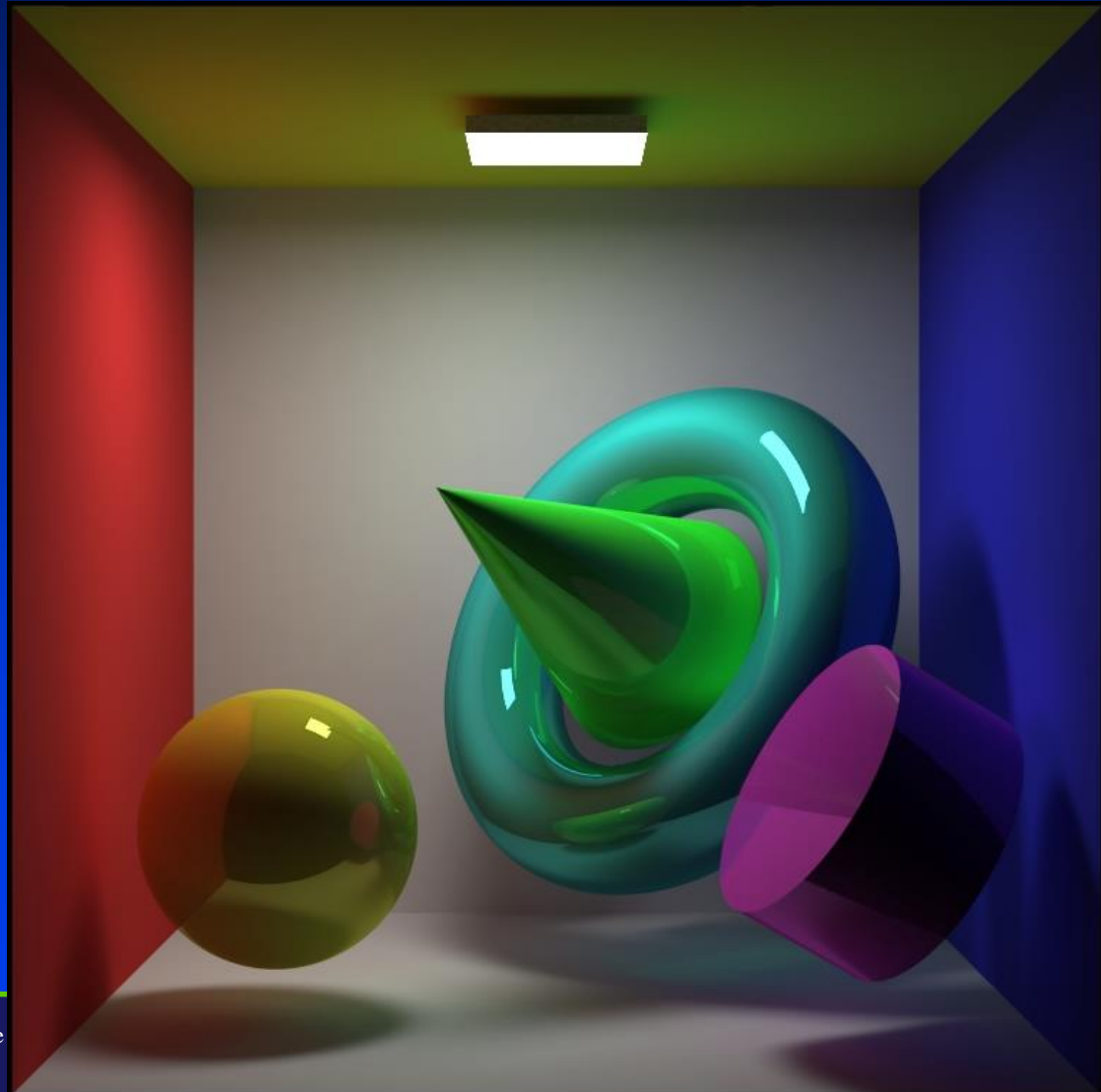
Evening (Sunset, 6pm)



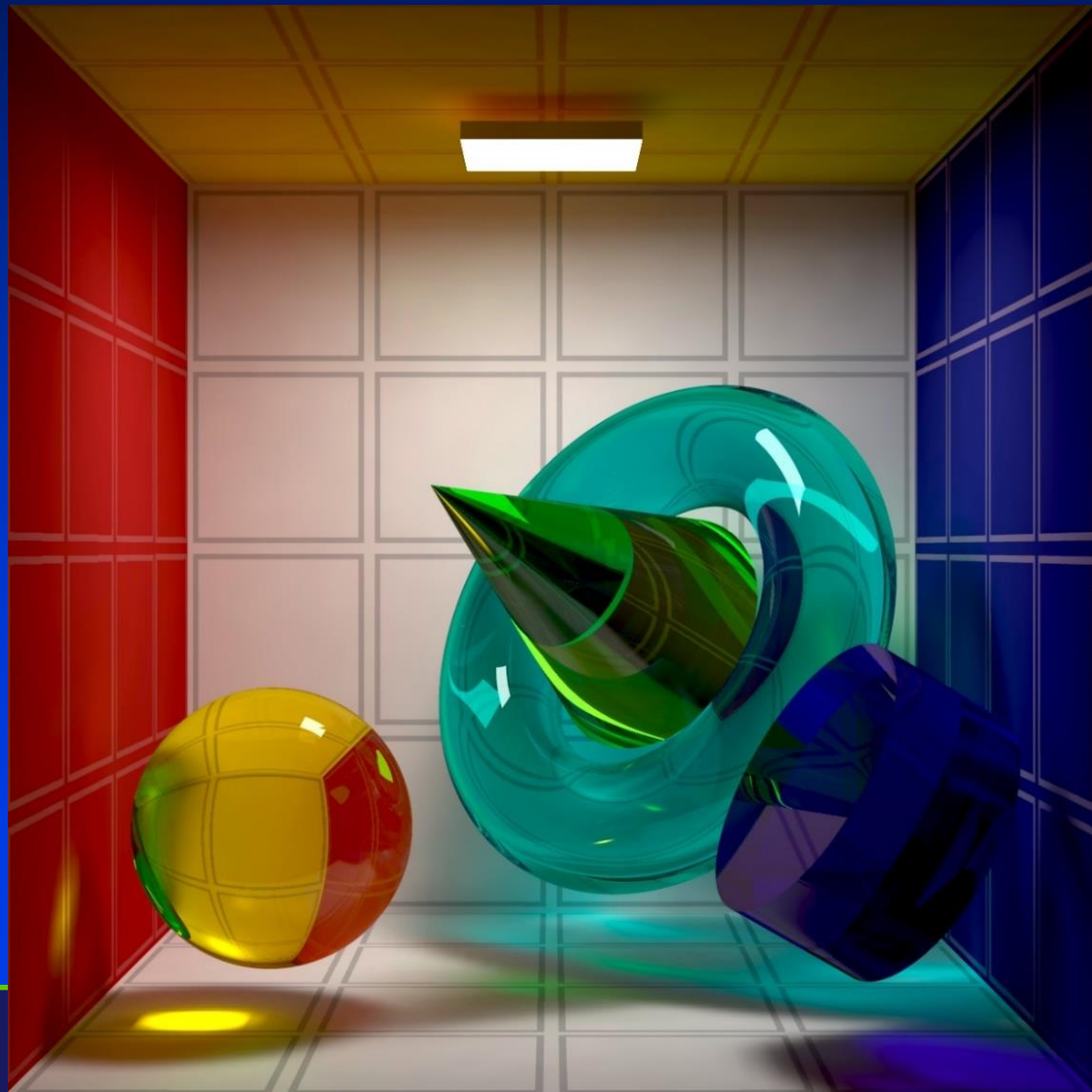
Diffuse



Specular



Transparent



Is This rendered or Real?



Advantages

- (1) Highly realistic quality of the resulting images by calculating the diffuse inter-reflection of light energy in an environment.
- (2) Accurate simulation of energy transfer.
- (3) The viewpoint independence of the basic radiosity algorithm provides the opportunity for interactive "walkthroughs" of environments.
- (4) Soft shadows and diffuse inter-reflection.

Disadvantages

- (1) Large computational and storage costs for form factors.
- (2) Must preprocess polygonal environments.
- (3) Non-diffuse components of light not represented.
- (4) Will be very expensive if object(s) is moving in the scene.

Heckbert's Notation

- For transport paths
- From Heckbert, SIGGRAPH 90
 - L – light
 - E – the eye
 - S – specular reflection
 - D – diffuse reflection
 - Sometimes also G for glossy
- Example: Path from light, to specular, to eye is **LSE**

Regular Expressions

- $(k)^+$ -- one or more
- $(k)^*$ -- zero or more
- $(k)?$ -- zero or one
- $(k|k')$ -- either one

Possible Paths

- From Heckbert 90

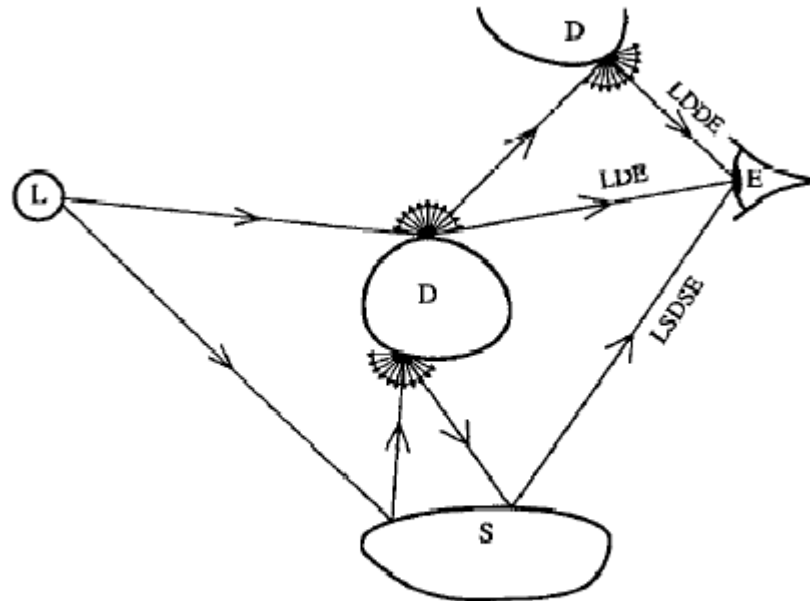


Figure 2: Selected photon paths from light (L) to eye (E) by way of diffuse (D) and specular (S) surfaces. For simplicity, the surfaces shown are entirely diffuse or entirely specular; normally each surface would be a mixture.

Transport Approximations

- **Classical ray tracing**
 - LD^*S^*E
 - Direct lambertian
 - Global specular
- **Radiosity**
 - LD^*E
 - Diffuse to diffuse global illumination
 - View independent
- **Bi-directional ray tracing**
 - Can be $L(S|D)^*E$