# PDE-based Geometric Modeling and Interactive Sculpting for Graphics

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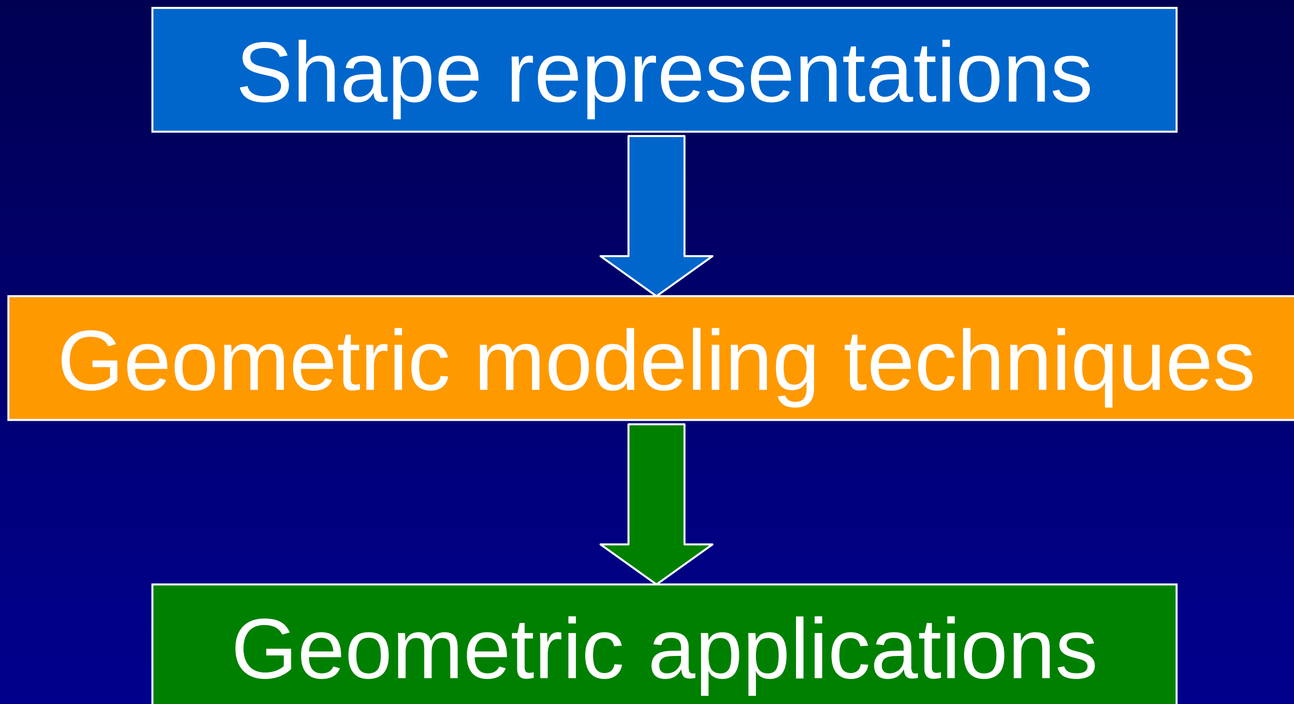
# Geometric Modeling



# Background Review

- Introduction to PDEs
- PDE techniques and applications

  Geometric modeling, visualization, simulation, animation, image processing, ……
- Other modeling techniques

  Free-form splines, implicit functions, physics-based techniques, medial axis extraction

# PDE Techniques and Applications

- Elliptic PDEs for geometric modeling
- Level set method
- Diffusion equations
- Other applications
  – Simulation and animation
  – Image processing

  ……

# PDEs for Geometric Modeling

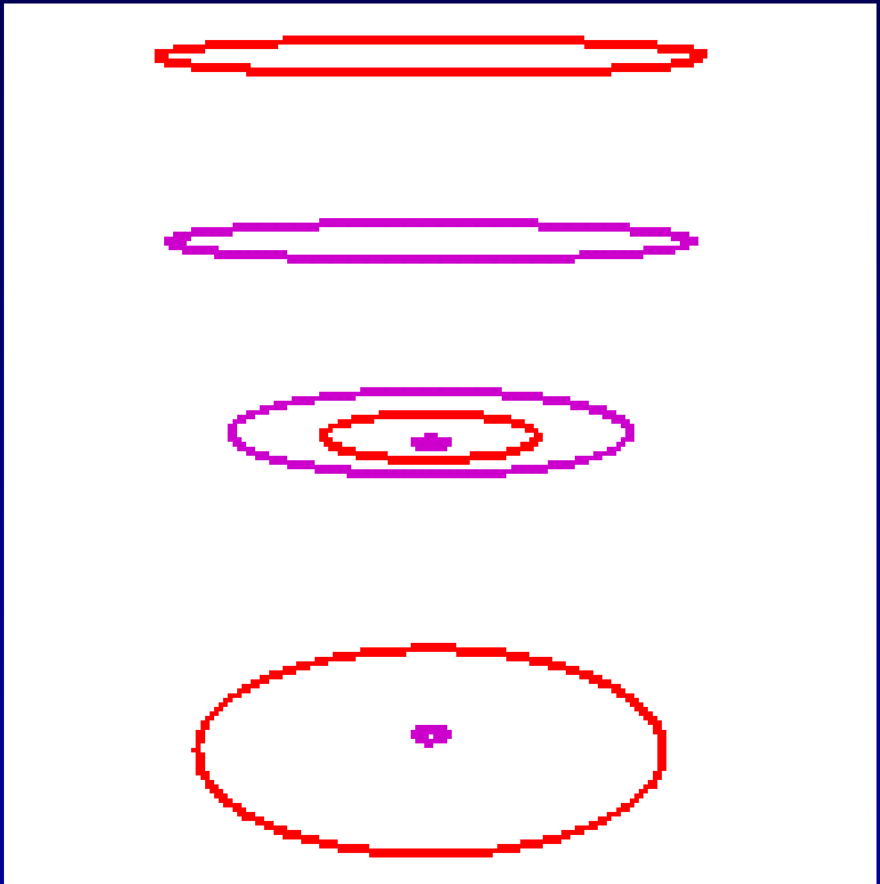
- Geometric objects are defined by a set of PDEs
- PDE objects are controlled by a few parameters
- Powerful numerical techniques to solve PDEs are available
- PDE is related to energy optimization
- PDE models can potentially unify geometric and physical aspects

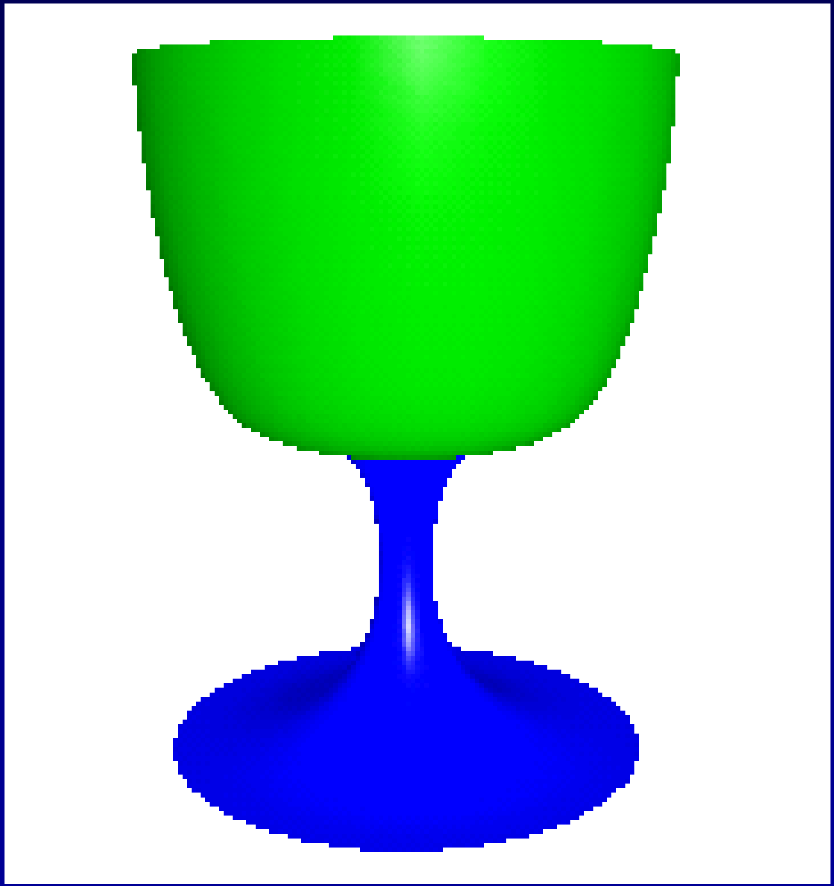
# Geometric Modeling

- Shape representations
  - Explicit model
    - Defines objects by positions
    - free-form splines, parametric PDE model, Subdivision model, ……
  - Implicit model
    - Defines objects by level set of scalar functions
    - CSG model, level-set model, splines,……

# Geometric Applications

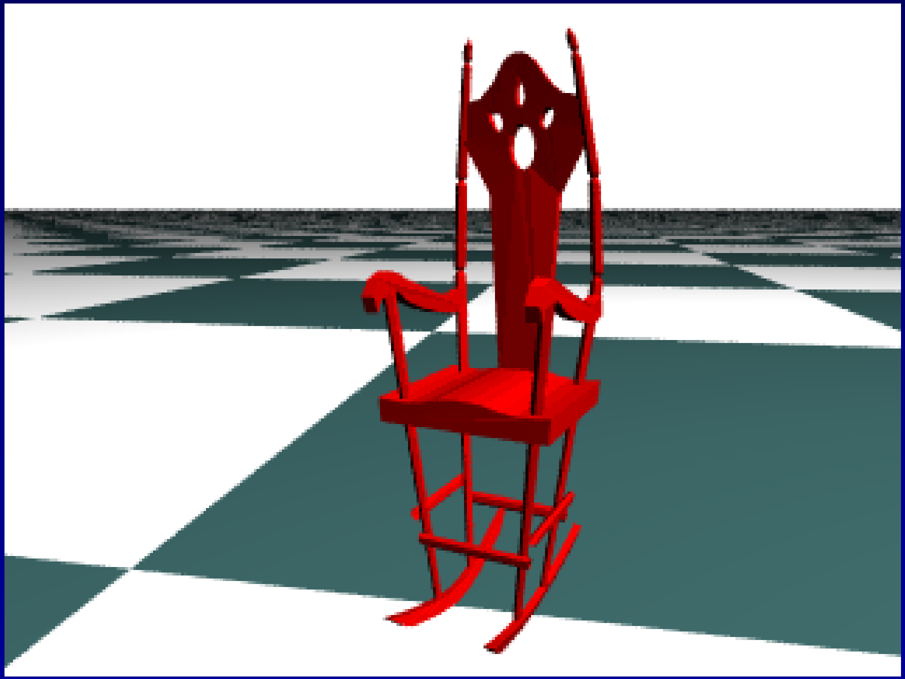
- Shape design

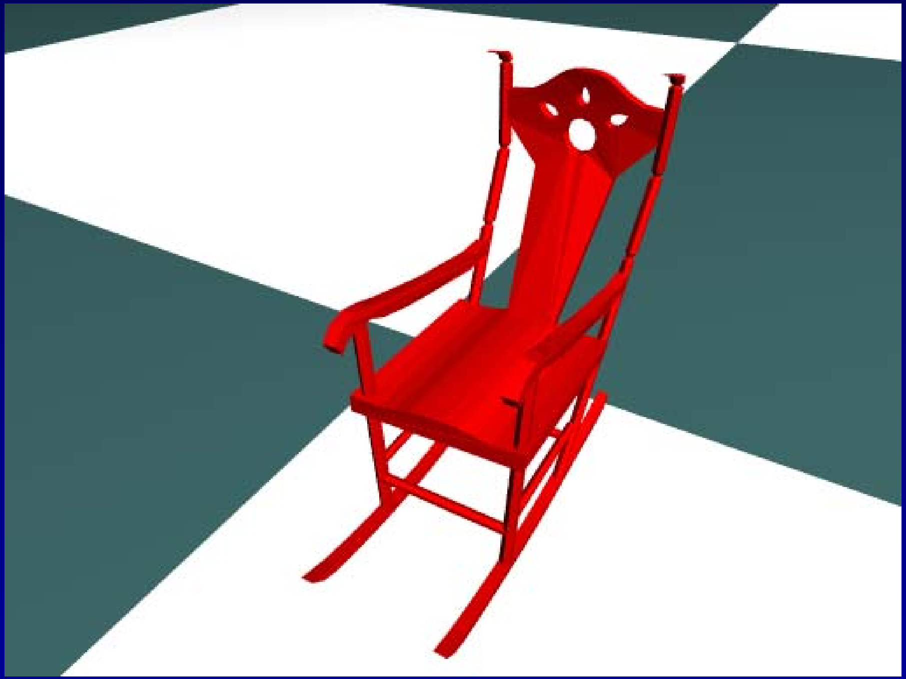




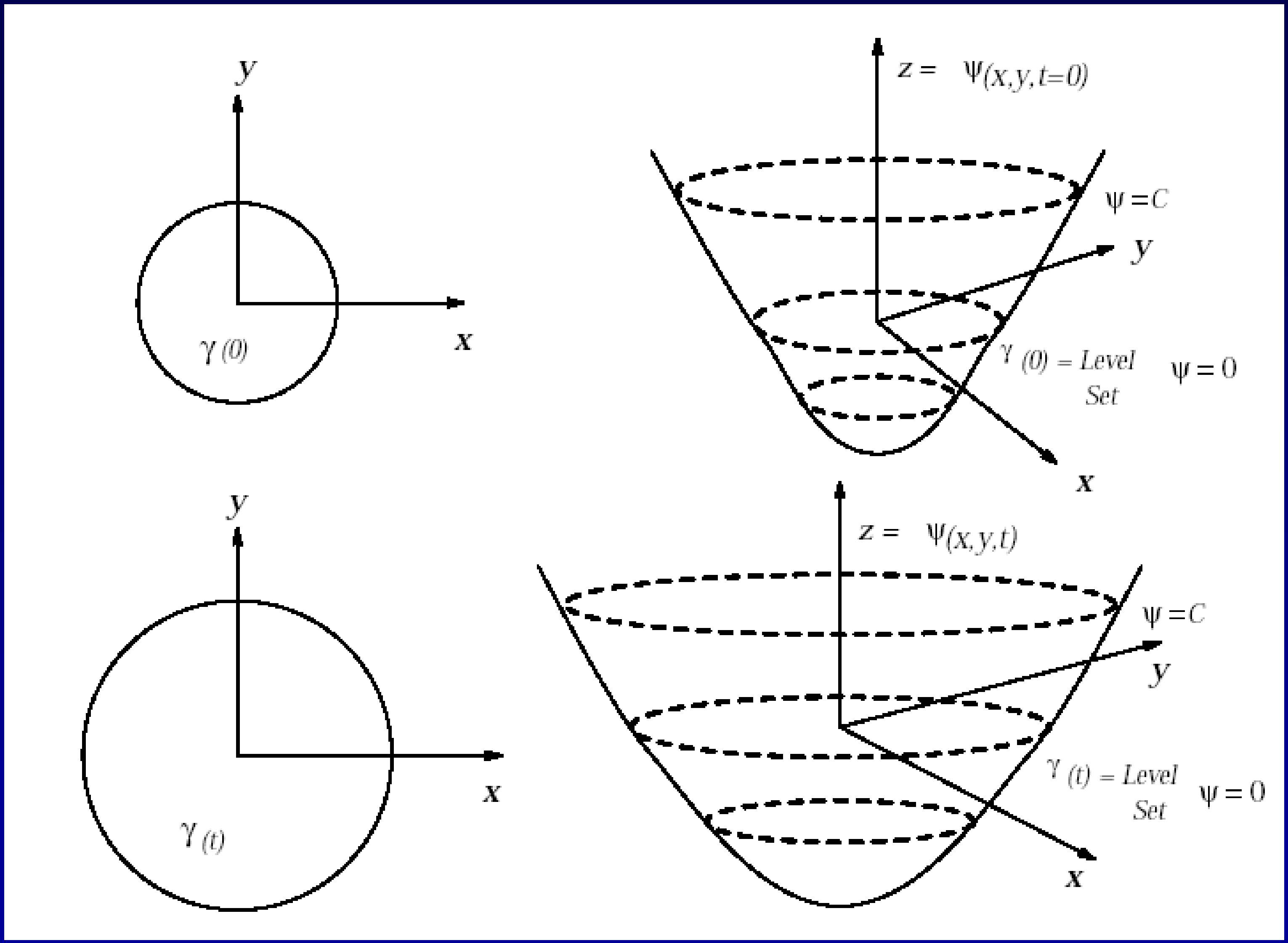
# Geometric Applications

- Shape design
- Object deformation



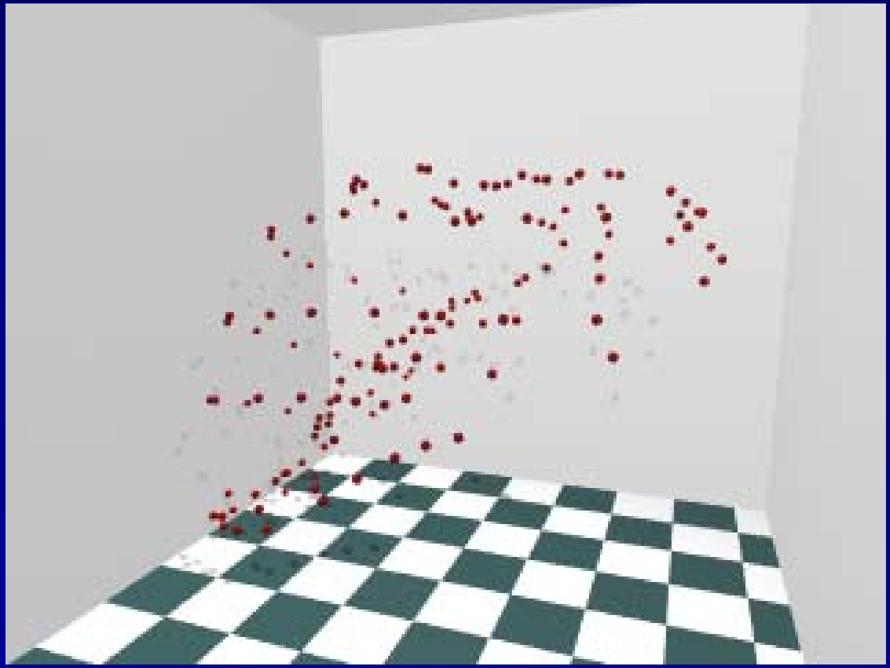


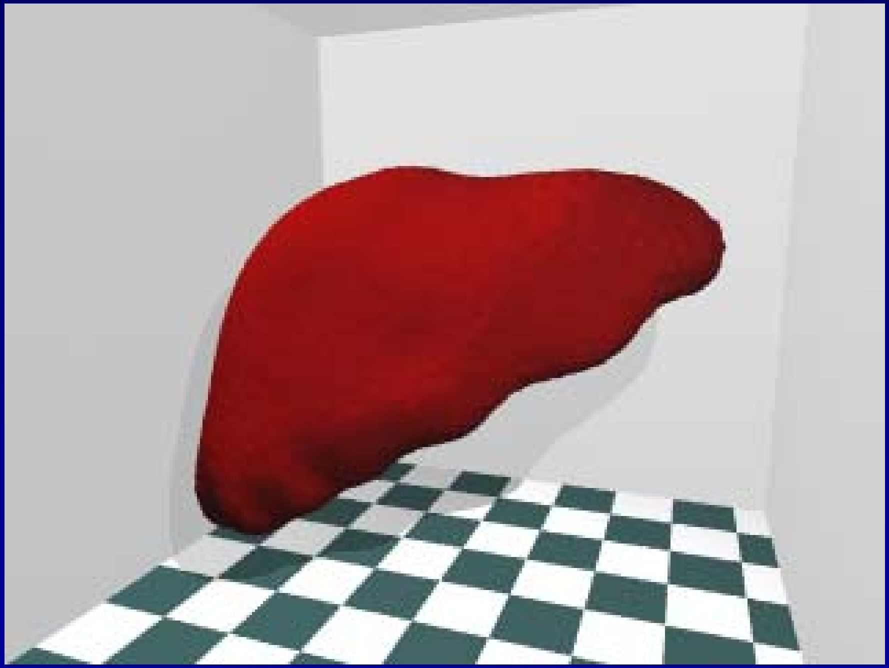
# Level Set Illustration



# Geometric Applications

- Shape design
- Object deformation
- Model reconstruction





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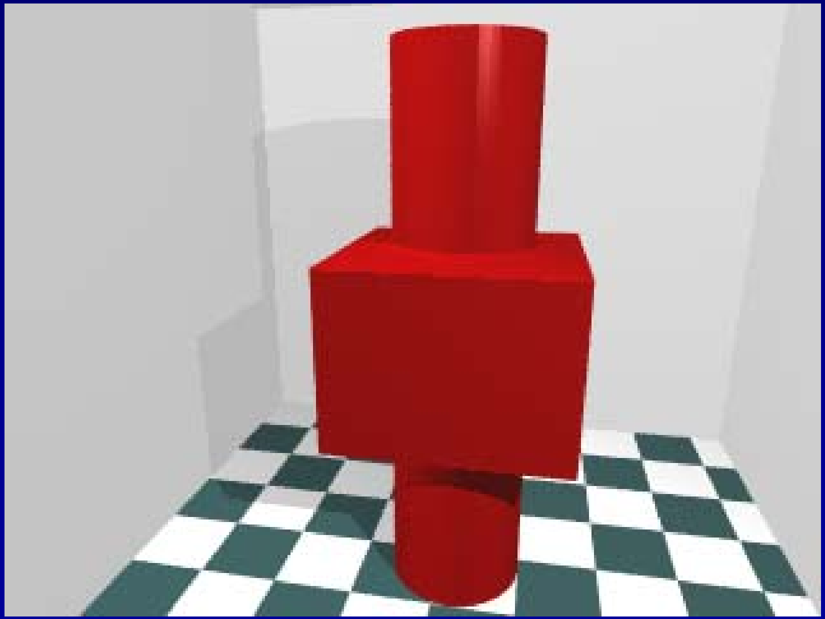
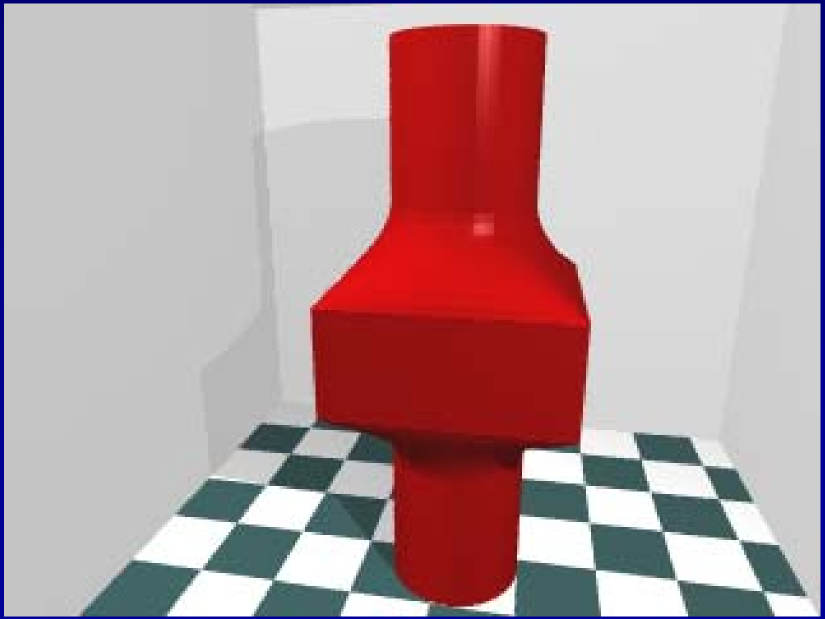
# Geometric Applications

- Shape design
- Object deformation
- Model reconstruction
- Shape blending ……





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# A PDE Example

- PDE (Partial Differential Equation)

$$\sum_{n=0}^{r} \sum_{l,m \geq 0}^{l+m=n} \alpha_{l,m}(u,v) \frac{\partial^n}{\partial u^l \partial v^m} f(u,v) = g(u,v)$$

  - Order $r$
  - $\alpha_{l,m}(u,v)$, $g(u,v)$ : control functions
  - $f(u,v)$: unknown function of $u,v$

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# Related Work of Physics-based Modeling

[Terzopoulos *et al.* 87]

[Terzopoulos and Fleisher 88]

[Celniker and Gossard 91]

[Qin and Terzopoulos 94, 96]

[Koch *et al.* 96]

[Mandal *et al.* 98, 99]

[Dachille *et al.* 99]

……

# Background Summary

- Geometric PDE techniques
- Level set method
- Diffusion equations with applications
- PDE-based simulation and image processing
- Implicit models
- Physics-based techniques
- Medial axis extraction

# PDE Techniques for Graphics

- Using differential properties
- Various applications
  - Image processing

[Bertalmio *et al.* 00]

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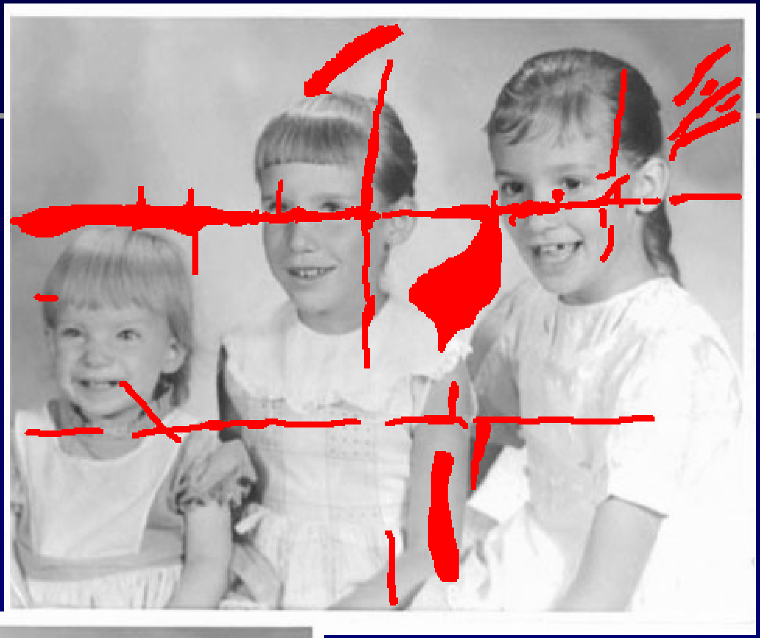
# Image Inpainting

- Inpainting:
  - Modify images in an undetectable way
  - Damage recovery, selected area removal
- Use gradient information, especially around the boundary of selected regions
- Propagate information from the surrounding areas using certain PDEs of gradient vectors



[Bertalmio *et al.*00]

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# Modeling Fracture

[O'Brien and Hodgins99]

# Fluid Dynamics

- Navier-Stokes equations

$$\begin{cases} \nabla \cdot \mathbf{u} = 0 & \text{(a)} \\ \dfrac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \dfrac{1}{\rho}\nabla \mathbf{p} + \nu \nabla^2 \mathbf{u} + \mathbf{f} & \text{(b)} \end{cases}$$

  - $\mathbf{u}$: velocity field
  - $\mathbf{p}$: pressure field
  - $\rho$: density
  - $\nu$: kinematic viscosity of the fluid
  - $\mathbf{f}$: external force
  - $\nabla = (\partial/\partial x, \partial/\partial y), \nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$

# Applications of Fluid Dynamics

- Gas simulation
- Water simulation
- Explosions
- Nature texturing

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# Simulating Gaseous Phenomena

[Foster and Metaxas97b]

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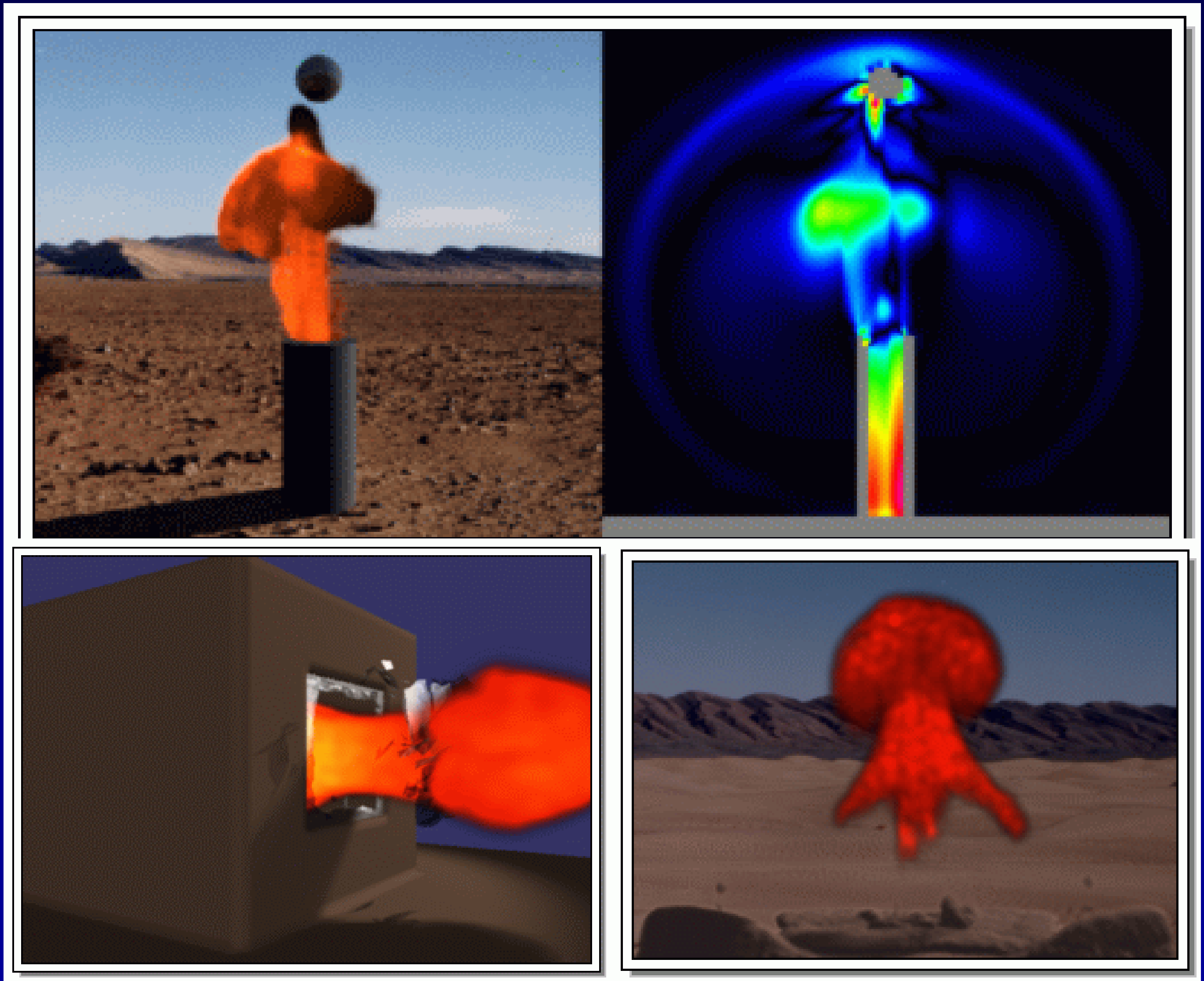
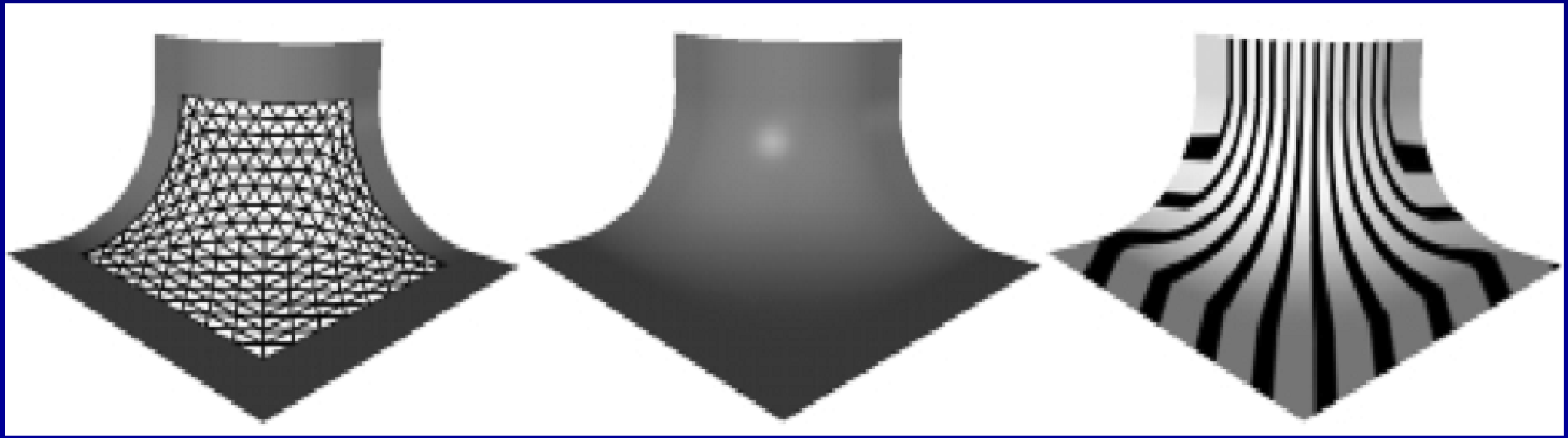
# Animating Explosions



[Yngve *et al.*00]

# PDE Techniques for Graphics

- Using differential properties
- Various applications
  - Image processing
  - Simulation
  - Visualization
  - Geometric modeling

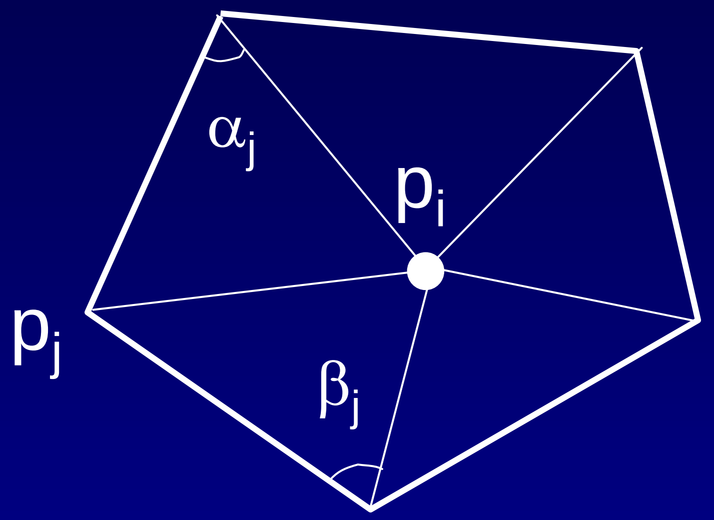


[Schneider and Kobbelt 00]

# Surface Fairing

- Curvature flow

$$\dot{\mathbf{x}} = -\kappa\,\mathbf{n}$$




$$\kappa\mathbf{n} = \frac{1}{4A}\sum_j (\cot\alpha_j + \cot\beta_j)(\mathbf{p}_i - \mathbf{p}_j)$$

# PDE Approach for Surface Fairing

- PDE approach can solve the fairing problem directly $\Delta_B H = 0$
- Fairing based on geometric invariants
- Construct surfaces based on discrete data with subdivision connectivity of regular patches

$$\Delta_B = \frac{1}{\lambda(u,v)^2}\left(\frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2}\right) = \frac{1}{\lambda(u,v)^2}\Delta^2$$

# Surface Fairing

- Taubin

$$\mathbf{P}_{new} = \mathbf{P}_{old} - (\mu - \lambda)\mathbf{U}(\mathbf{P}_{old}) - \mu\lambda\mathbf{U}^2(\mathbf{P}_{old}) \tag{4}$$

where $\mu > \lambda > 0$.

- Membrane or thin-plate energy

$$E_{membrane}(X) = \frac{1}{2}\int_{\Omega} X_u^2 + X_v^2 \, dudv$$

$$E_{thin\ plate}(X) = \frac{1}{2}\int_{\Omega} X_{uu}^2 + 2X_{uv}^2 + X_{vv}^2 \, dudv.$$

$$L(X) = X_{uu} + X_{vv}$$

$$L^2(X) = L \circ L(X) = X_{uuuu} + 2X_{uuvv} + X_{vvvv}.$$

# Outline

- Motivation and contributions
- Related work
- PDE-based geometric modeling system
  - Physics-based PDE surfaces/displacements
  - PDE-based arbitrary mesh modeling
  - Implicit elliptic PDE model
  - PDE-based free-form modeling and deformation
- Conclusion

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# Motivation: Why PDE Techniques?

- Formulate natural physical process
- Satisfy continuity requirements
- Minimize energy functionals
- Define objects using boundary information
- Provide intuitive and natural control
- Unify geometric and physical attributes
- Employ powerful numerical techniques

# Motivation: Limitations of Prior Work

- Indirect manipulation for PDE objects
- Limit constraints for geometric PDE objects
- Lack of local control for regional shape sculpting
- No intuitive manipulation with physical properties
- Limitations of acceptable shape representations for PDE models
- Lack of integration framework of different types of PDEs
- Limit applications of geometric PDE modeling system

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# Motivation: A General PDE Framework

- General modeling framework for geometric objects of different data formats
- Direct manipulation and interactive sculpting with global/local control
- Integration of physical properties for realistic modeling
- Comprehensive toolkits for various modeling functionalities

  Design, reconstruction, abstraction, manipulation ……

# Contributions: Overview

**Modeling Representations and Techniques:**

| Parametric Surfaces | Arbitrary Meshes | Implicit Functions | Dynamic Model | Free-form Solids |
|---|---|---|---|---|

**PDE-based Geometric Modeling System**

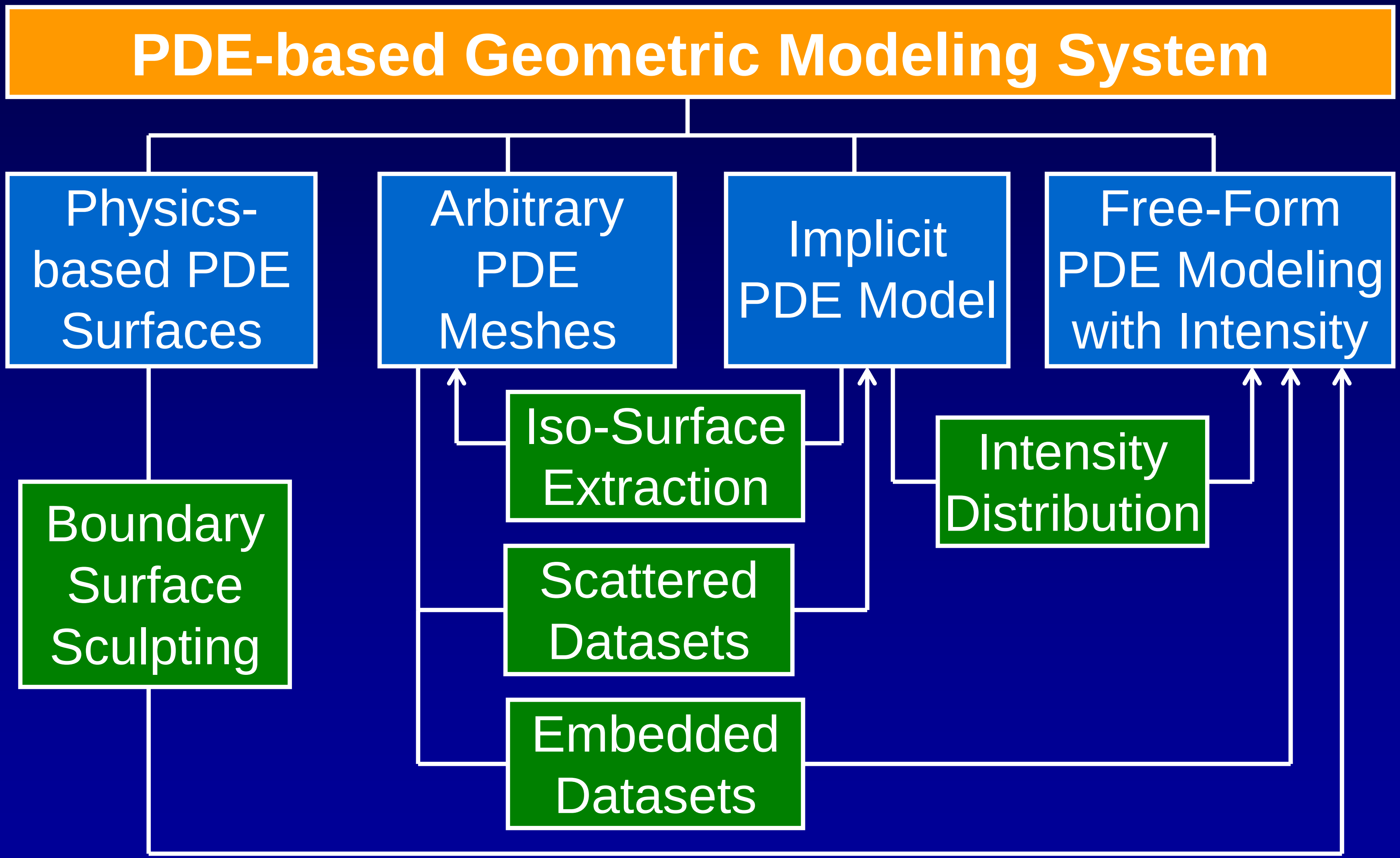
**Applications:**

| Shape Design | Object Reconstruction | Shape Sculpting | Model Abstraction | Morphing | …… |
|---|---|---|---|---|---|

# Contributions: System Functions

- PDE-based geometric modeling system
  - Physics-based PDE surfaces/displacements
  - PDE-based arbitrary mesh modeling
  - Implicit PDE shape design and manipulation
  - PDE-based free-form modeling and deformation

# Contributions: System Components

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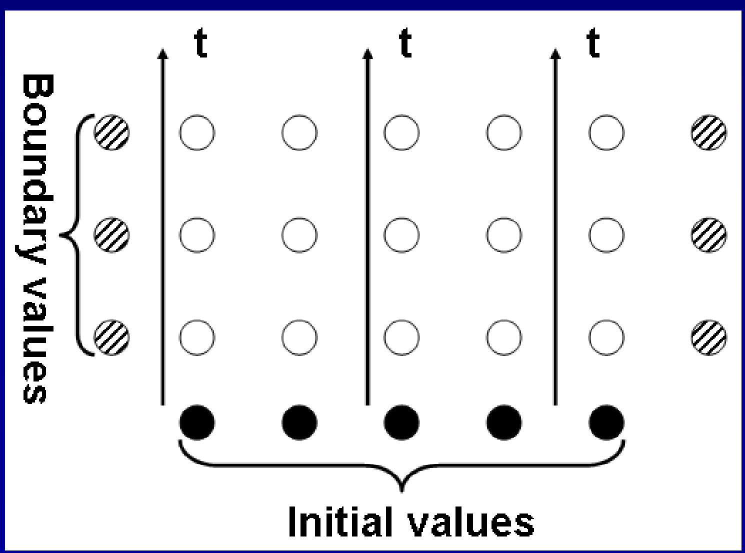
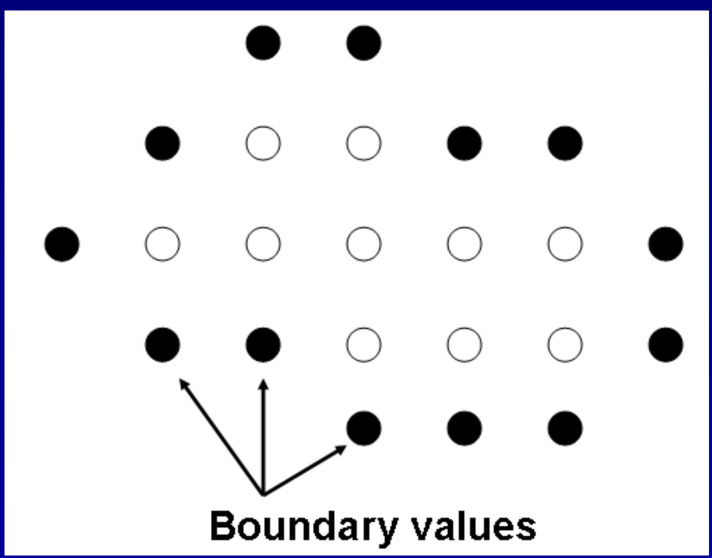
# Another Classification of PDEs

- Initial value problem
  - Given information at $t_0$ , the solution will propagate forward in time
- Boundary value problem
  - Given boundary information of the region of interest of variables
  - Solution will be a static function within the region

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# Summary of PDE Classifications

| PDE types | Hyperbolic PDE | Parabolic PDE | Elliptic PDE |
|---|---|---|---|
| Initial value problem | Wave equation | Diffusion equation | |
| Boundary value problem | | | Poisson equation |

- Poisson equation:

  Geometric modeling, image processing……

- Diffusion equation:

  Texture synthesis, image processing, ……

- Wave equation:

  Fluid simulation, nature texturing, ……

# PDE-based Geometric Modeling

- PDE surfaces and solids:
  - Blending problem [Bloor and Wilson 89]
  - Free-form surfaces [Bloor and Wilson 90b]
  - B-spline approximation [Bloor and Wilson 90a]
  - Functionality design [Lowe *et al.* 90]
  - PDE solids [Bloor and Wilson 93]
  - Interactive design [Ugail *et al.* 99]
- Variational models:

  Surface fairing [Schneider and Kobbelt 00]

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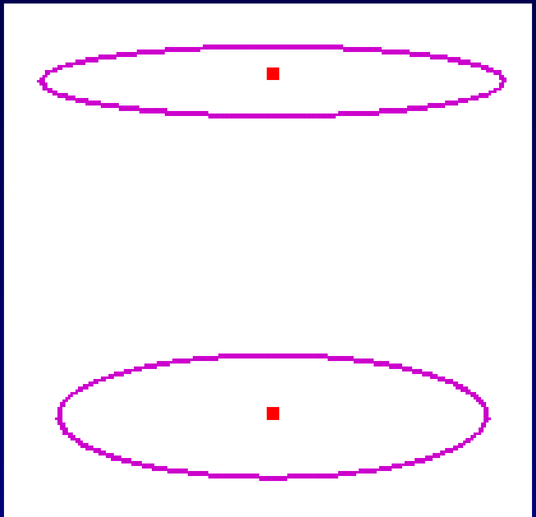
# PDE Surfaces and Solids

- PDE surface formulation

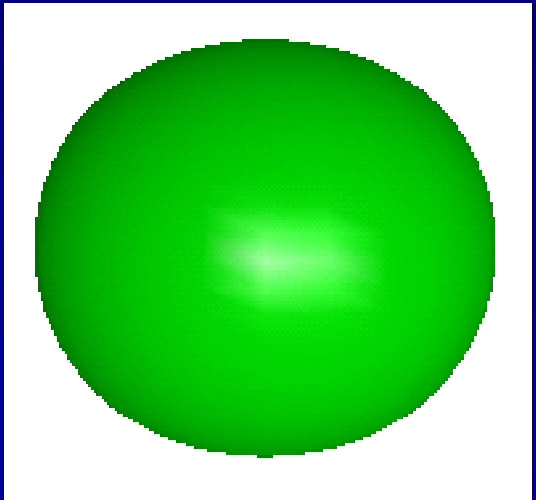
$$\left(\frac{\partial^2}{\partial u^2}+a^2\frac{\partial^2}{\partial v^2}\right)^2\mathbf{X}(u,v)=\mathbf{0}$$

Biharmonic equation if $a=1$



- PDE solid formulation

$$\left(\frac{\partial^2}{\partial u^2}+\frac{\partial^2}{\partial v^2}+\frac{\partial^2}{\partial w^2}\right)\mathbf{X}(u,v,w)=\mathbf{0}$$



# Level Set Method

- Originally defined for front propagation

$$\phi_t - F|\nabla \phi| = 0, F \text{ is the speed function;}$$

$$\phi(x, t = 0) = \pm d(x), d(x) \text{ is the distance from } x \text{ to } \Gamma_0;$$

$$\text{Moving front is the zero-level set} : \Gamma_t = \{x \mid \phi(x,t) = 0\}.$$

- Prior work of level set method
  - Front propagation
    [Osher and Sethian88], [Adalsteinsson and Sethian95]
  - Shape reconstruction
    [Zhao *et al.*00], [Zhao *et al.*01]
  - Shape transformation
    [Breen and Whitaker01]
  - Shape modeling and editing
    [Barentzen and Christensen02], [Museth *et al.*02]

  [SIGGRAPH’02 Course Notes 10]

  ……

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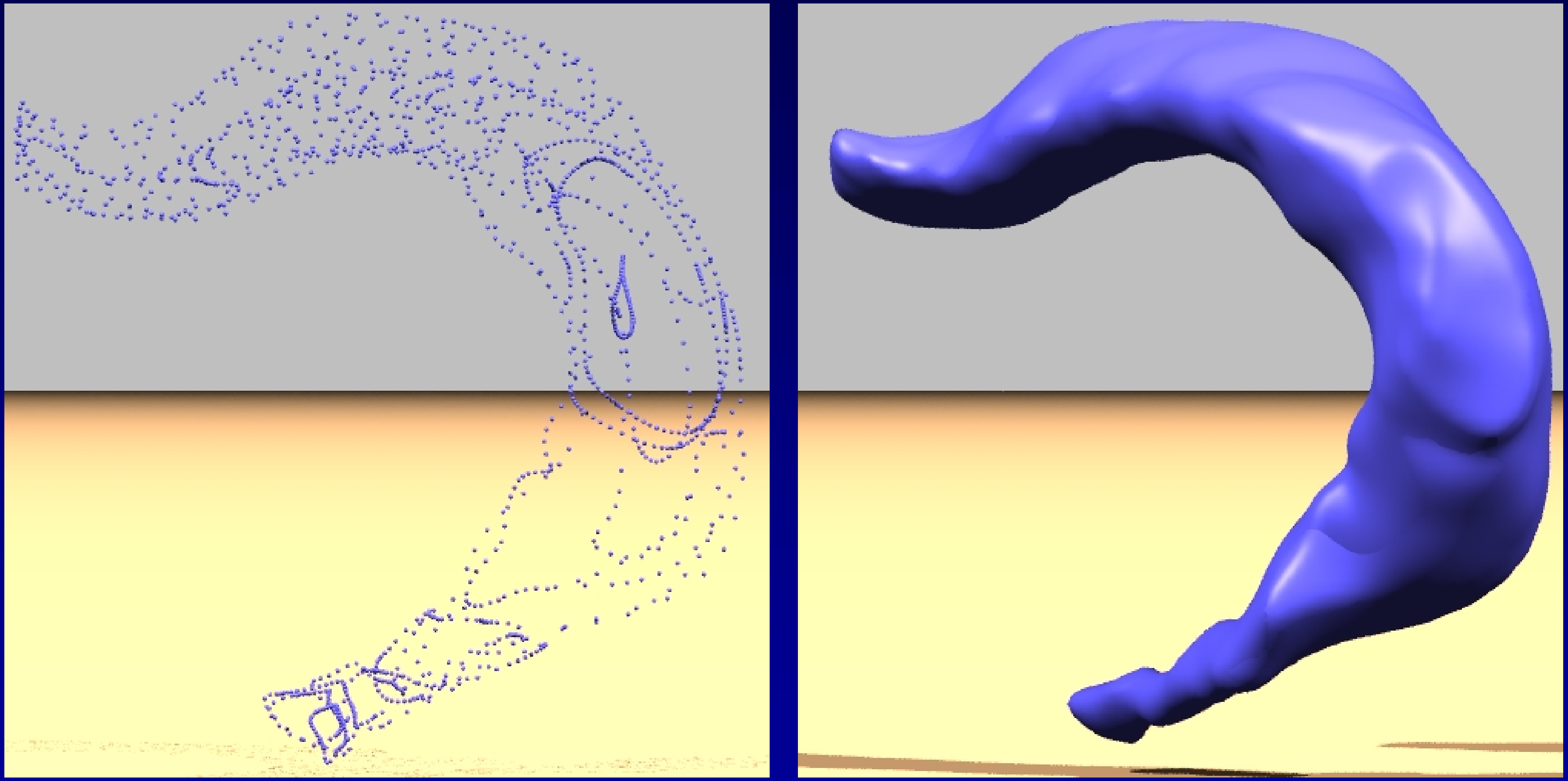
# Classification of PDEs

$$A\frac{\partial^2 u}{\partial x^2} + 2B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + Fu = G$$

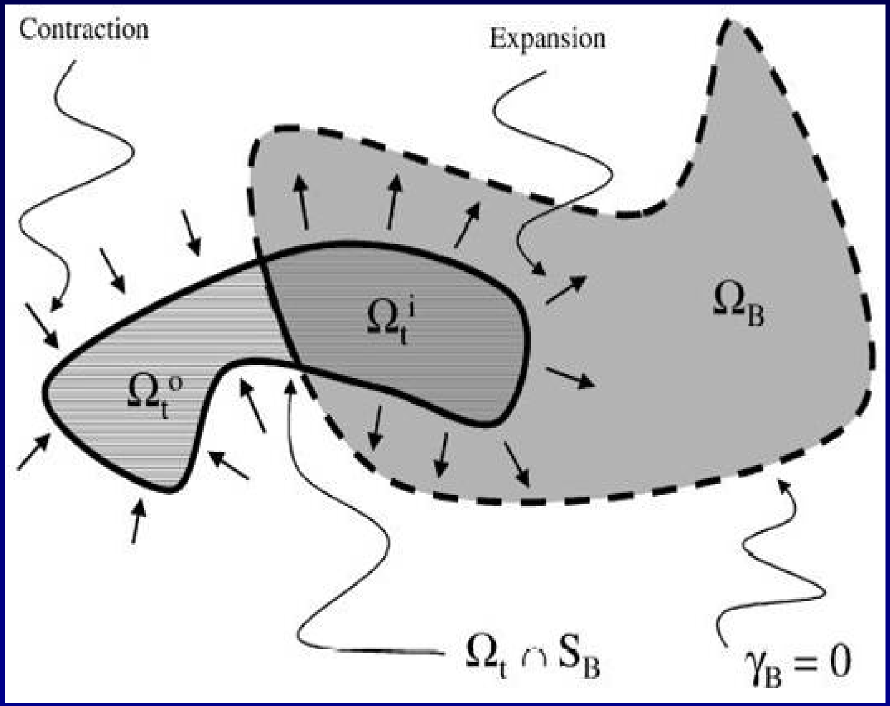
- 3 types of PDEs based on *characteristics*
  - $B^2 - AC > 0$: *hyperbolic*
    - Wave equation $\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2}$
  - $B^2 - AC = 0$: *parabolic*
    - Diffusion equation $\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left( D \frac{\partial u}{\partial x} \right)$
  - $B^2 - AC < 0$: *elliptic*
    - Poisson equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \rho(x, y)$

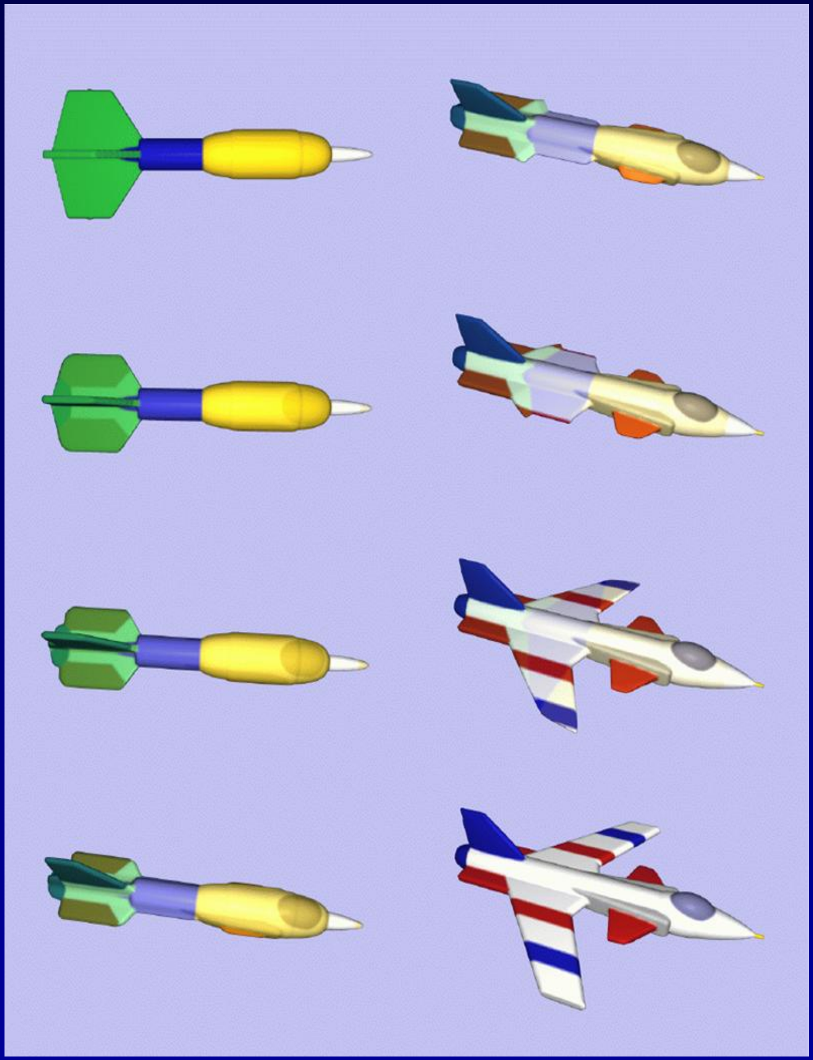
# Shape Reconstruction Using Level Set Method



[Zhao *et al.*00]

# Shape Morphing Using Level Set Method



[Breen and Whitaker 01]

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# Diffusion Equations

- Reaction-diffusion textures

  [Witkin and Kass91], [Turk91]
- Tensor field visualization

  [Kindlmann *et al.*00]
- Vector field visualization

  [Diewald *et al.*00]
- Image processing

  Image enhancement, filtering,……

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# Reaction-Diffusion Texture

- Synthesizing natural textures
- Reaction-diffusion system
  - Diffusion of morphogens
  - Nonlinear PDEs
  - Biological patterns

# Reaction-Diffusion Equation

- Diffusion, dissipation, reaction
- Reaction-diffusion equation:

$$\dot{C} = a^2 \nabla^2 C - bC + R,$$

$\dot{C}$ is the time derivative of $C$,

$$\nabla^2 C = \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2},$$

$a$ is the rate constant for diffusion,

$b$ is the rate constant for dissipation,

$R$ is the reaction function.

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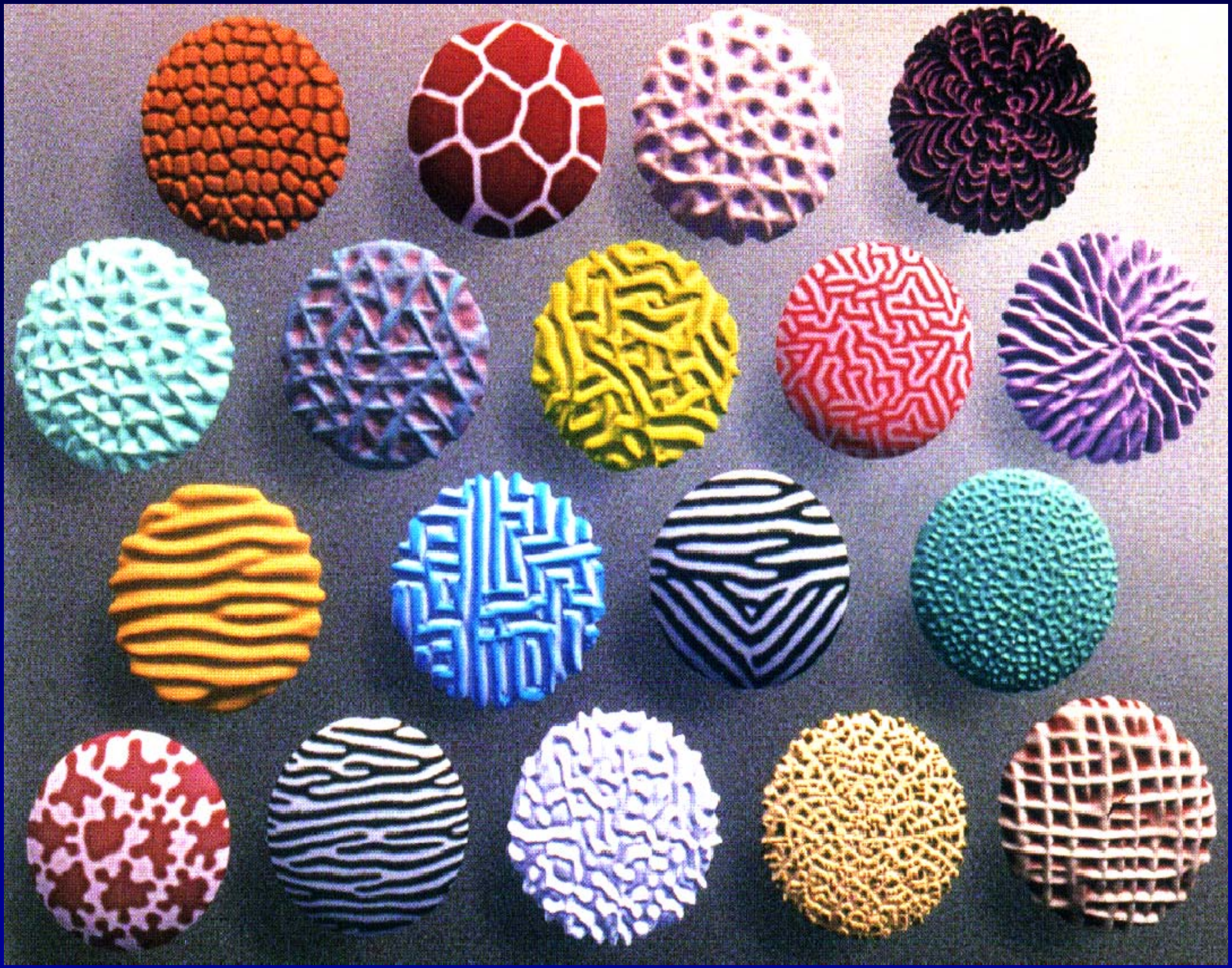
# Reaction-diffusion texture buttons



1. reptile, giraffe, coral, scalloped.
2. spiral, triweave, twisty maze, replication, purple thing
3. sand, maze, zebra haunch, radial
4. space giraffe, zebra, stucco, beats us, weave

**By type**: Isotropic, multi-orientation, and diffusion mapped

[Witkin and Kass 91]

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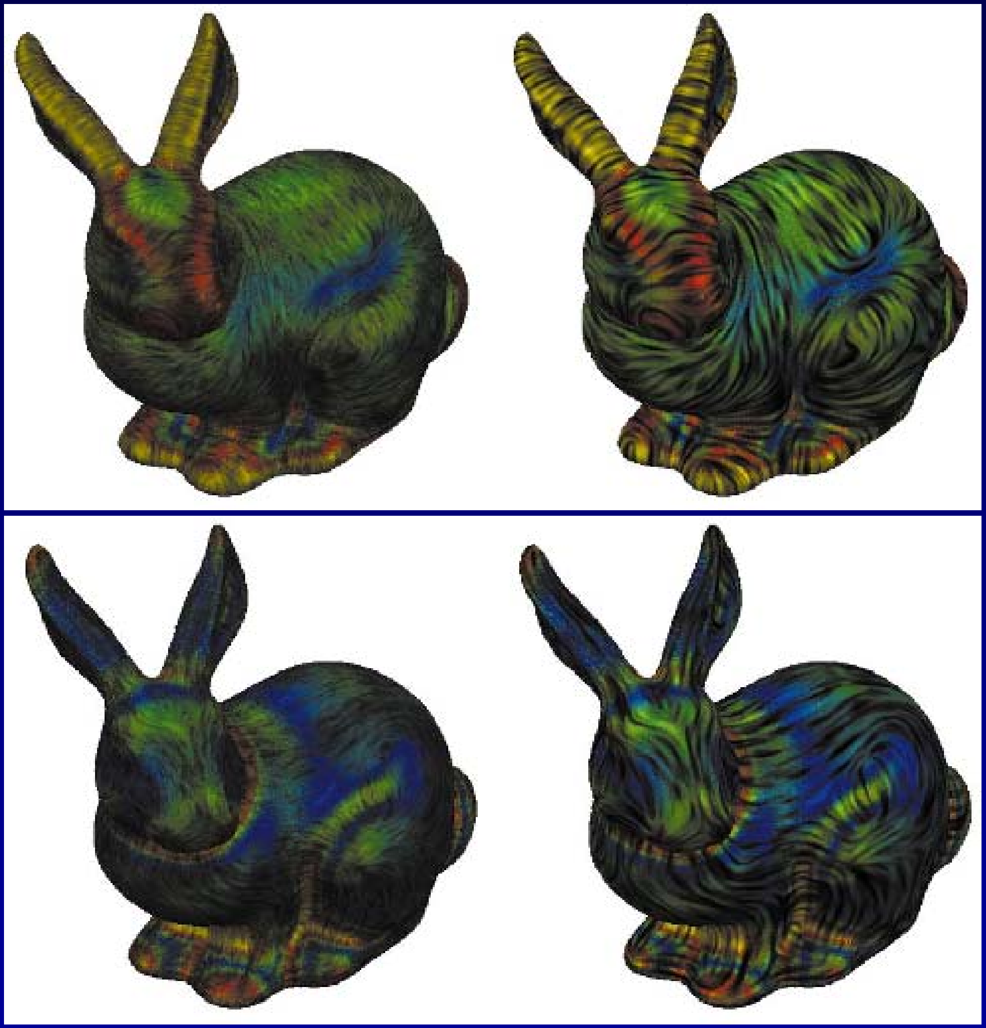
# Vector Field Visualization



[Diewald *et al.* 00]

Different time steps of the anisotropic diffusion for both principal curvature directions

# Other Applications of PDEs

- Modeling fracture
  [O'Brien and Hodgins99], [O'Brien00]
- Simulating gas
  [Foster and Metaxas97b]
- Water simulation
  [Kass and Miller90], [Foster and Metaxas96,97a], [Stam99]
- Modeling explosion
  [Yngve *et al.*00]
- Image processing
  [Bertalmio *et al.*00], [Pérez *et al.*03]

…….

# Implicit Model

- Implicit surfaces and solids:

  $\{(x,y,z)|\, f(x,y,z)=c\}$, $\{(x,y,z)|\, f(x,y,z)\leq c\}$

- Techniques to model implicit objects
  - Particle based implicit surface sculpting

    [Witkin and Heckbert 94]
  - Trivariate B-splines for implicit models

    [Raviv and Elber 99], [Hua and Qin01,02]
  - Level set method

    [Zhao *et al.*00,01]
  - Variational implicit functions

    [Turk and O'Brien 99,02], [Morse *et al.*01]

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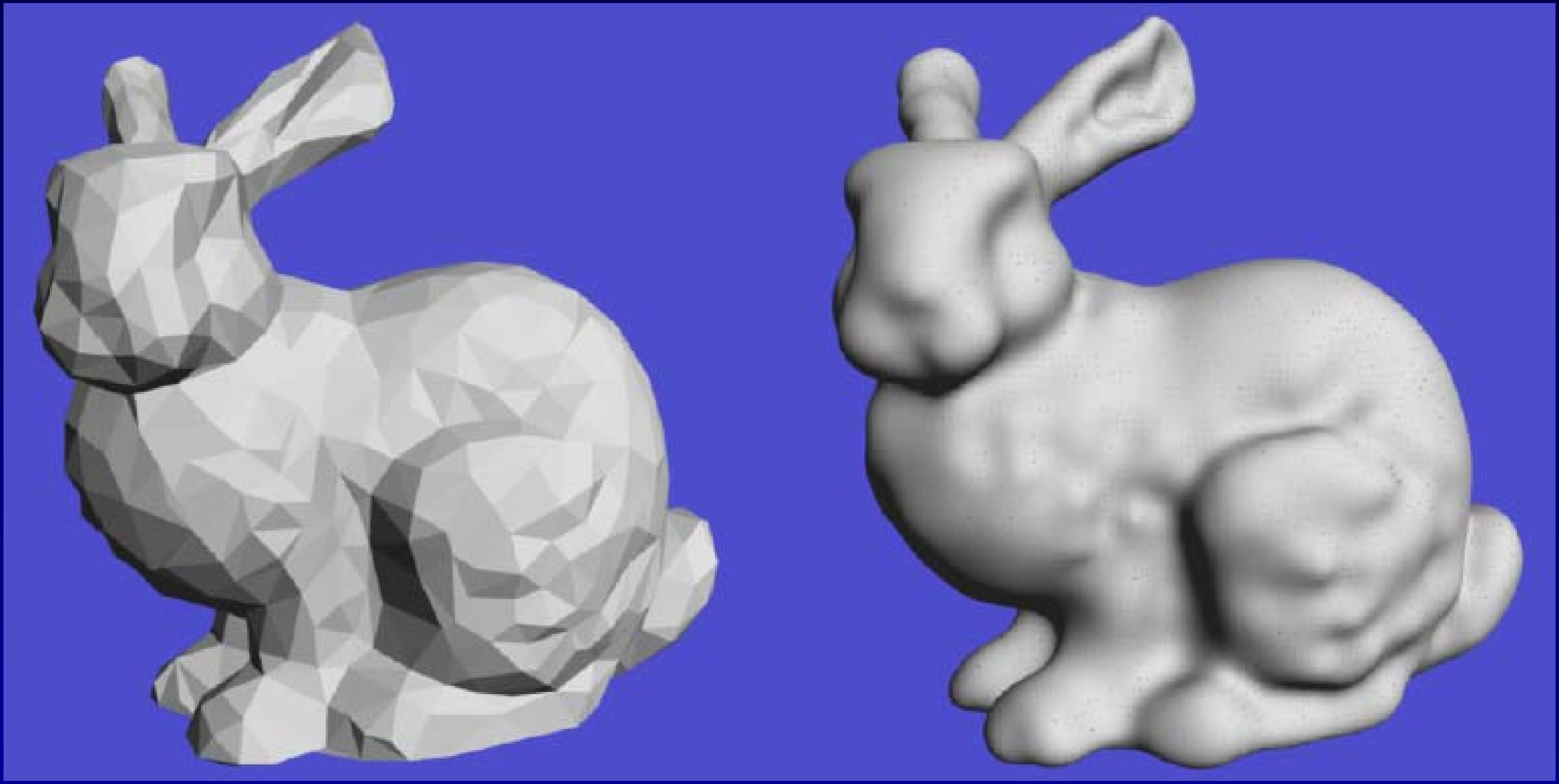
# Example of Implicit Models



[Turk and O'Brien02]

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# Physics-based Modeling

- Combines physical properties with geometric models
- Leads to *deformable models*
- Allows direct manipulation of objects via forces
- Creates natural-looking motions through simulation
- Can be integrated with general PDE framework

# Deformable Models

- Controlled by Lagrangian equations of motion

$$\frac{\partial}{\partial \mathrm{t}}\left(\mu \frac{\partial \mathbf{r}}{\partial t}\right)+\gamma \frac{\partial \mathbf{r}}{\partial t}+\frac{\delta \varepsilon(\mathbf{r})}{\partial \mathbf{r}}=\mathbf{f}(\mathbf{r}, t)$$

  - $\mathbf{r}(\mathbf{a},t)$: position of particle $\mathbf{a}$ at time $t$
  - $\mu(\mathbf{a})$: mass density of the body at $\mathbf{a}$
  - $\gamma(\mathbf{a})$: damping density
  - $\mathbf{f}(\mathbf{r}, t)$: external force
  - $\varepsilon(\mathbf{r})$: measures the potential energy of the elastic deformation of the body

# Discretized Mass-Spring Model

$$\mathbf{M}\ddot{\mathbf{p}} + \mathbf{D}\dot{\mathbf{p}} + \mathbf{K}\mathbf{p} = \mathbf{f}$$

- $\mathbf{M}$: mass matrix
- $\mathbf{D}$: damping matrix
- $\mathbf{K}$: stiffness matrix
- $\mathbf{f}$: external force
- $\mathbf{p}$: discrete sample points

# Applications of Physics-based Modeling

- Dynamics NURBS (DNURBS)
  [Qin and Terzopoulos 94, 95, 96]
- Physics-based subdivision
  [Mandal *et al.* 98, 99], [McDonnell *et al.* 00, 01]
- Cloth simulation and animation
  [Carignan *et al.* 92], [Baraff and Witkin 98]
- Facial simulation
  [Lee *et al.* 95], [Koch *et al.* 96]
- Physics-based implicit functions
  [Jing and Qin 01,02]

……

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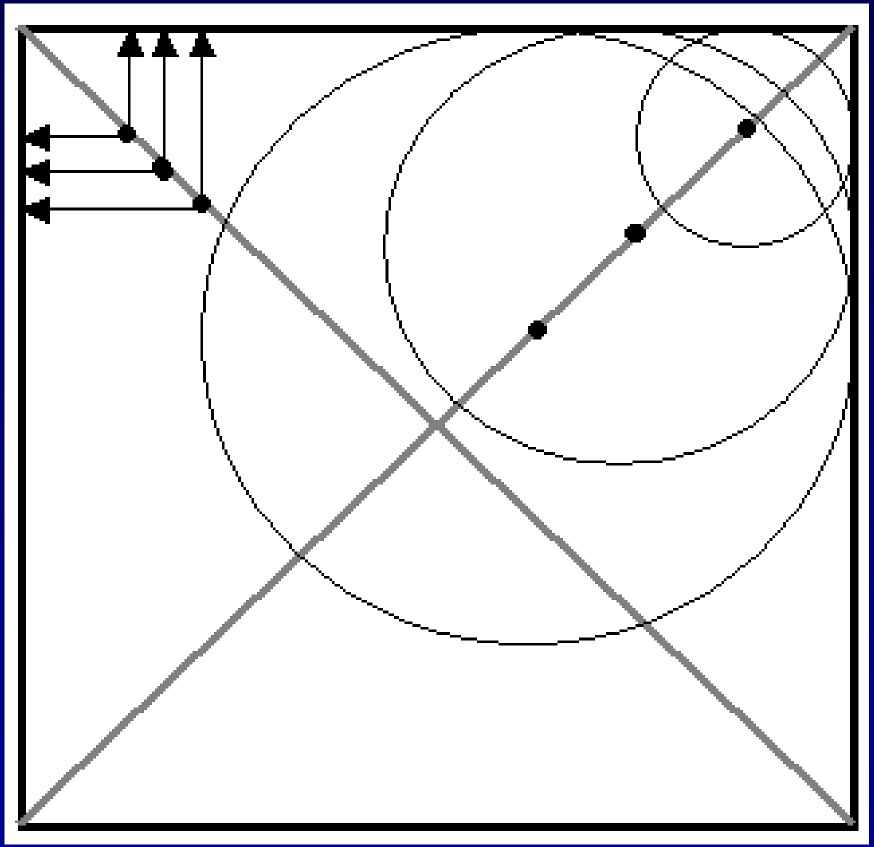
# Medial Axis Extraction

- Locus of all centers of circles/spheres inside the object
- Collection of points with more than one closest points on the boundary
- Set of singularities of signed distance function from boundary



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# Medial Axis Extraction Techniques

- Thinning
  [Arcelli and Baja85][Lee and kashyap94][Manzanera *etal.*99]
- Distance functions
  [Arcelli and Baja92][Leymarie and Levine92][Bitter *et al.*01]
- Voronoi skeletons
  [Goldak *et al.*91][Ognievicz93][Amenta *et al.*01]
- Level set method
  [Kimmel *et al.*95][Ma *et al.*03]
- Direction testing
  [Bloomenthal and Lim99]
- Hybrid techniques
  [Bouix and Siddiqi00]

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# Outline

- Motivation and contributions
- Related work
- PDE-based geometric modeling system
  - Physics-based PDE surfaces/displacements
  - PDE-based arbitrary mesh modeling
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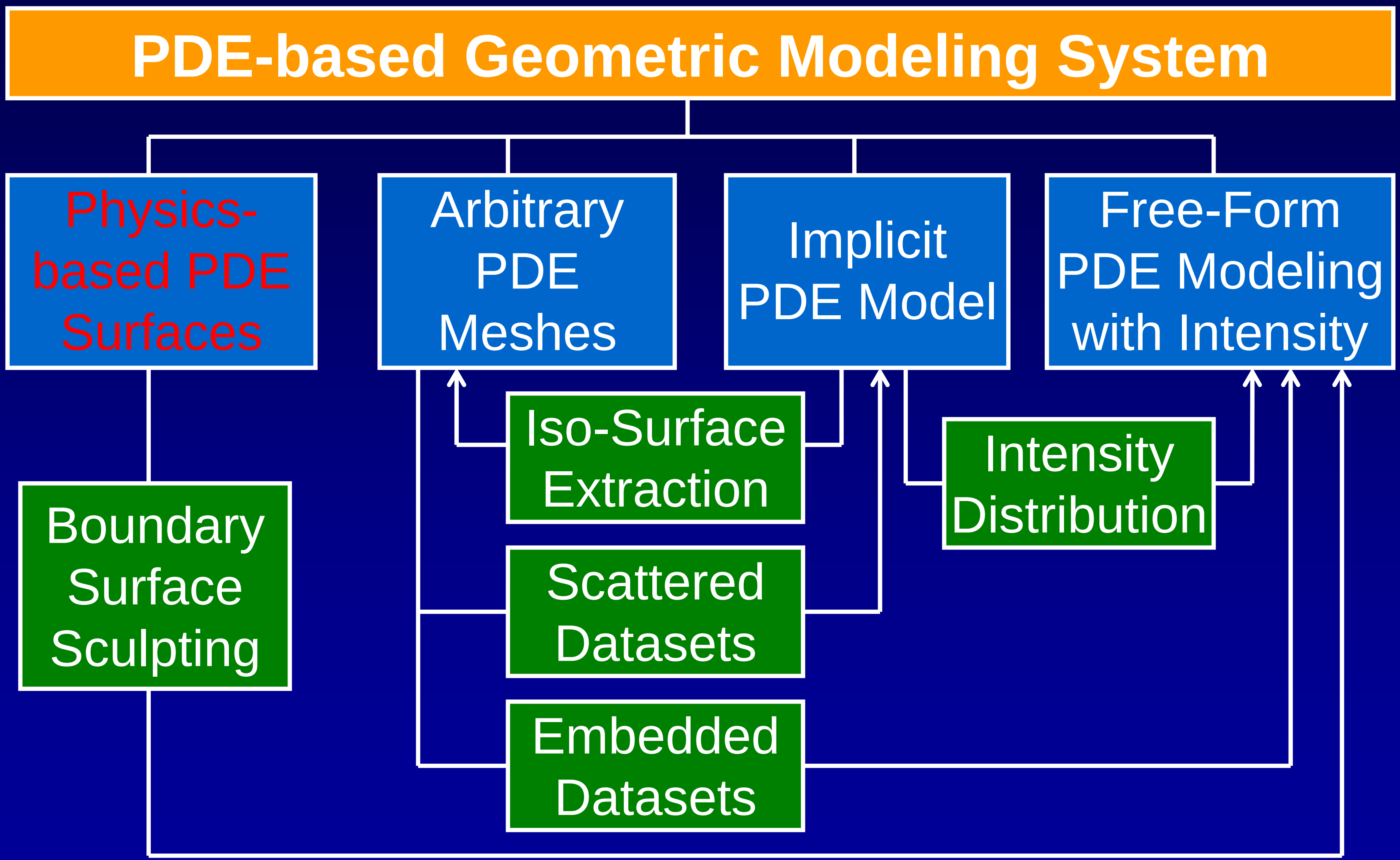
# System Outline

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# PDE Surfaces

- PDE surface formulation:

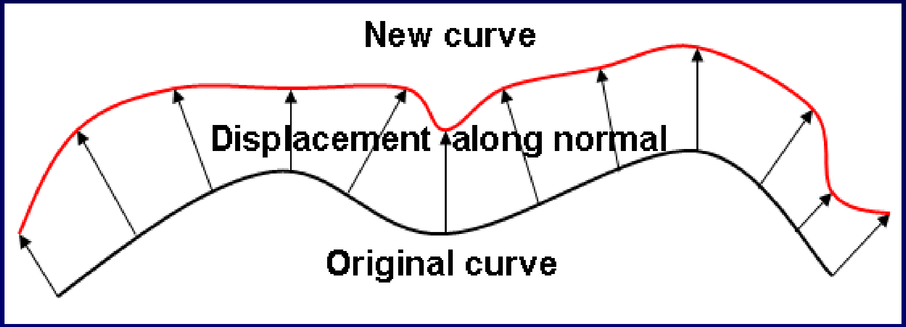
$$\left(\frac{\partial^2}{\partial u^2} + a^2(u,v)\frac{\partial^2}{\partial v^2}\right)^2 \mathbf{X}(u,v) = 0$$

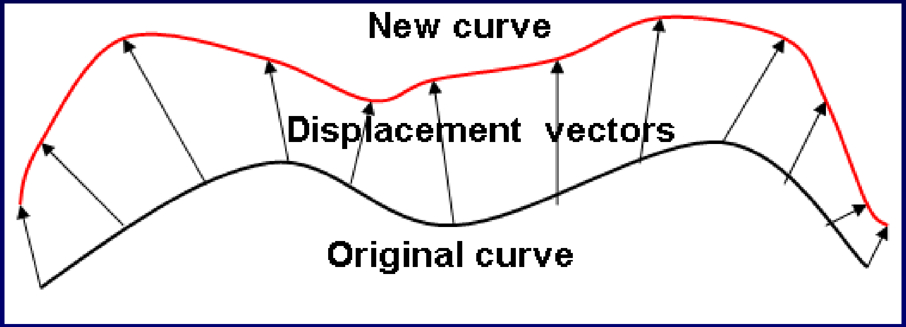
$$\mathbf{X}(u,v) = \begin{bmatrix} x(u,v) & y(u,v) & z(u,v) \end{bmatrix}^{\mathrm{T}}$$

  – $a(u,v)$: blending coefficient function controlling the contributions of the parametric directions

# PDE Surface Displacements

- Displacements




- PDE surface displacement formulation:

$$\mathbf{X}(u,v) = \mathbf{X}^0(u,v) + \mathbf{O}(u,v),$$

$$\left(\frac{\partial^2}{\partial u^2} + a^2(u,v)\frac{\partial^2}{\partial v^2}\right)^2 \mathbf{O}(u,v) = \mathbf{0}$$

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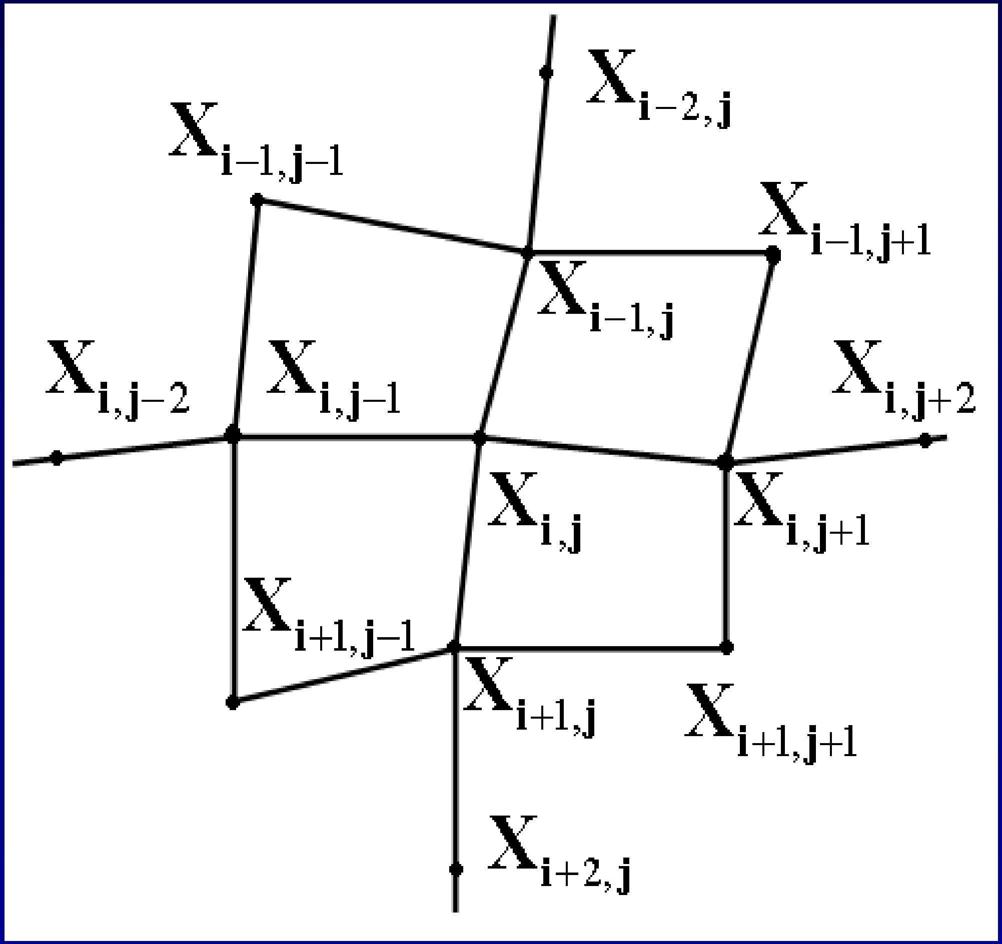
# Finite Difference Method

- Divides the working space into discrete grids

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# Finite Difference Method

- Divides the working space into discrete grids
- Samples the PDE at grid points with discretized approximations

$$\frac{\partial^4 \mathbf{X}_{i,j}}{\partial u^4} = \frac{\mathbf{X}_{i-2,j} + \mathbf{X}_{i+2,j} - 4\mathbf{X}_{i-1,j} - 4\mathbf{X}_{i+1,j} + 6\mathbf{X}_{i,j}}{\Delta u^4},$$

$$\frac{\partial^4 \mathbf{X}_{i,j}}{\partial v^4} = \frac{\mathbf{X}_{i,j-2} + \mathbf{X}_{i,j+2} - 4\mathbf{X}_{i,j-1} - 4\mathbf{X}_{i,j+1} + 6\mathbf{X}_{i,j}}{\Delta v^4},$$

$$\frac{\partial^4 \mathbf{X}_{i,j}}{\partial u^2 v^2} = \frac{\mathbf{X}_{i-1,j-1} + \mathbf{X}_{i+1,j-1} + \mathbf{X}_{i-1,j+1} + \mathbf{X}_{i+1,j+1} - 2(\mathbf{X}_{i-1,j} + \mathbf{X}_{i+1,j} + \mathbf{X}_{i,j-1} + \mathbf{X}_{i,j+1}) + 4\mathbf{X}_{i,j}}{\Delta u^2 \Delta v^2}$$

$$\left(\frac{\partial^2}{\partial u^2} + a_{i,j}^2 \frac{\partial^2}{\partial v^2}\right)^2 \mathbf{X}_{i,j} = 0$$

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# Finite Difference Method

- Divides the working space into discrete grids
- Samples the PDE at grid points with discretized approximations
- Forms a set of algebraic equations

$$\mathbf{H}\mathbf{X} = \mathbf{z}$$

- Enforcing additional constraints

$$\mathbf{H}_c\mathbf{X} = \mathbf{z}_c$$

- Physics-based discrete PDE model:

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{D}\dot{\mathbf{X}} + (\mathbf{K} + \mathbf{H}_c)\mathbf{X} = \mathbf{f} + \mathbf{z}_c$$

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# Discretized Approximations

- Displacement model:

$$\mathbf{H}_c \mathbf{O} = \mathbf{z}_c$$

$$\mathbf{X} = \mathbf{X}^0 + \mathbf{O}$$

- Iterative techniques and multi-grid algorithm to improve the performance
- Easy for local control

# Flexible Boundary Conditions

- Generalized boundary conditions

  $$\mathbf{X}(u_i, v) = \mathrm{f}_i(v)$$

  $$\mathbf{X}(u, v_j) = \mathrm{g}_j(u)$$

  where $0 \le u_i \le 1$ and $0 \le v_j \le 1$, and $\mathrm{f}_i(v)$ and $\mathrm{g}_j(u)$ are isoparametric curves

  – Hermite-like boundary constraints

  – Coons-like boundary constraints

  – Gordon-like boundary constraints

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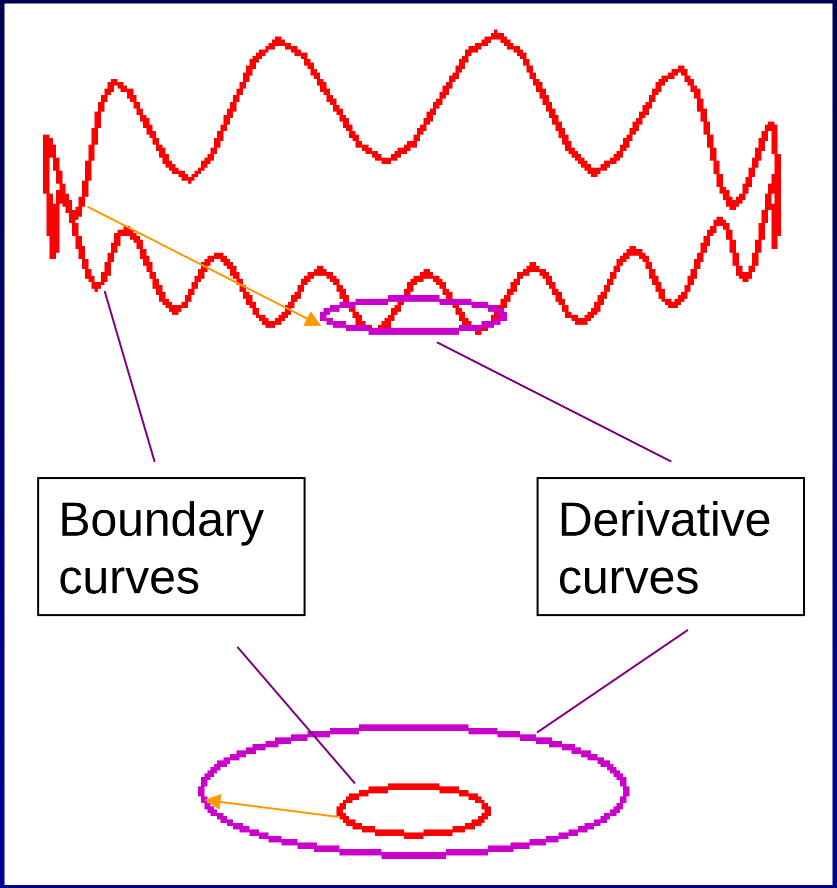
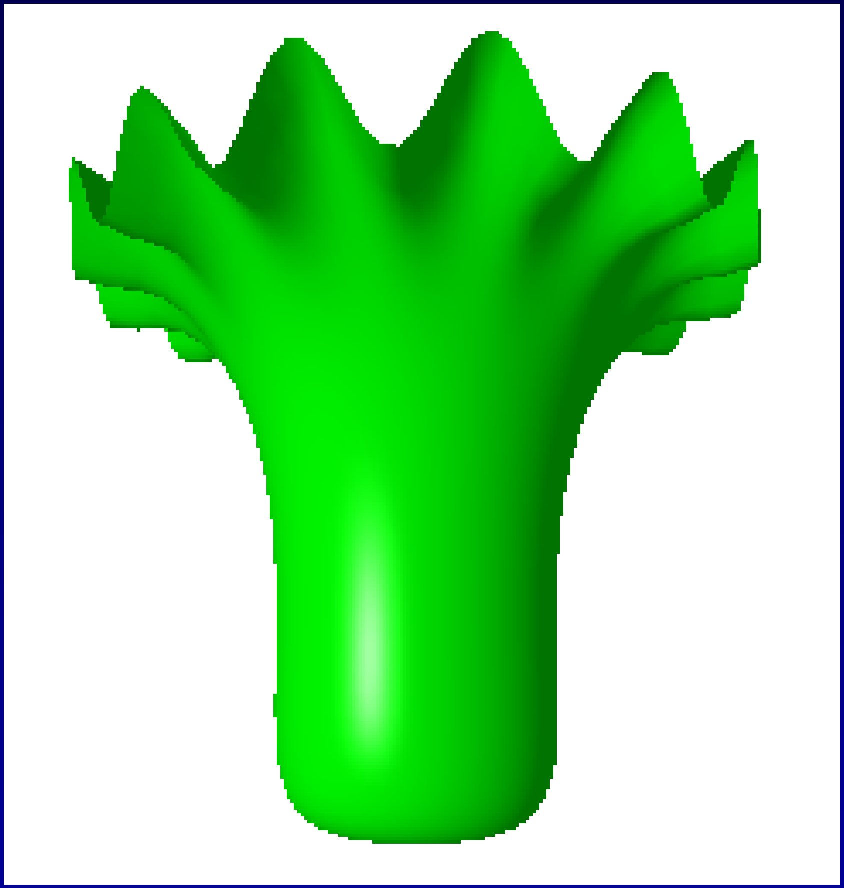
# Hermite-like Boundary Constraints



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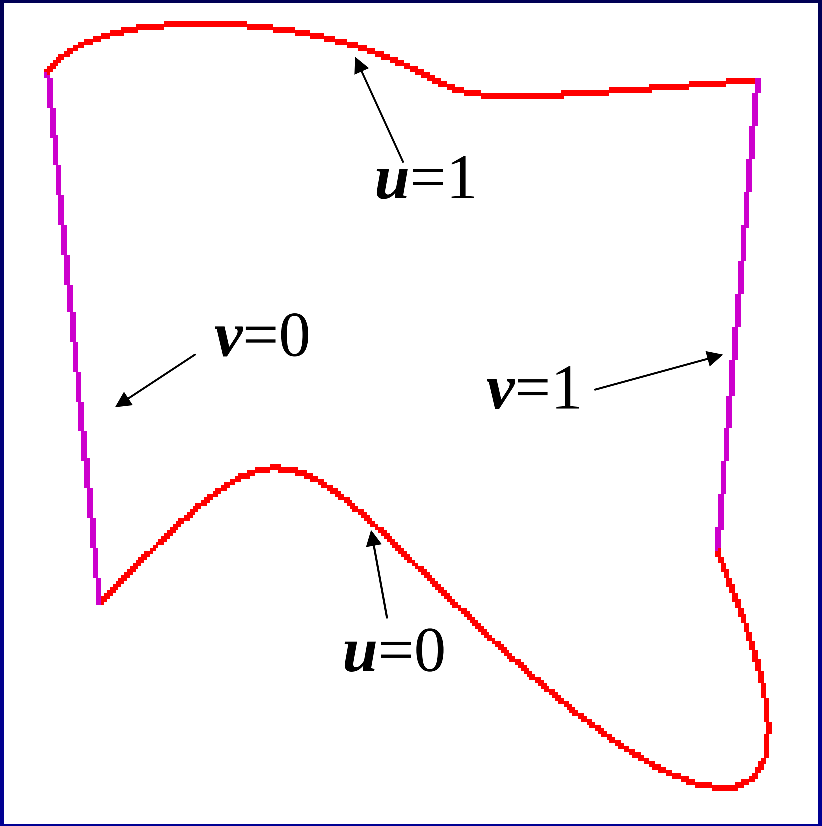
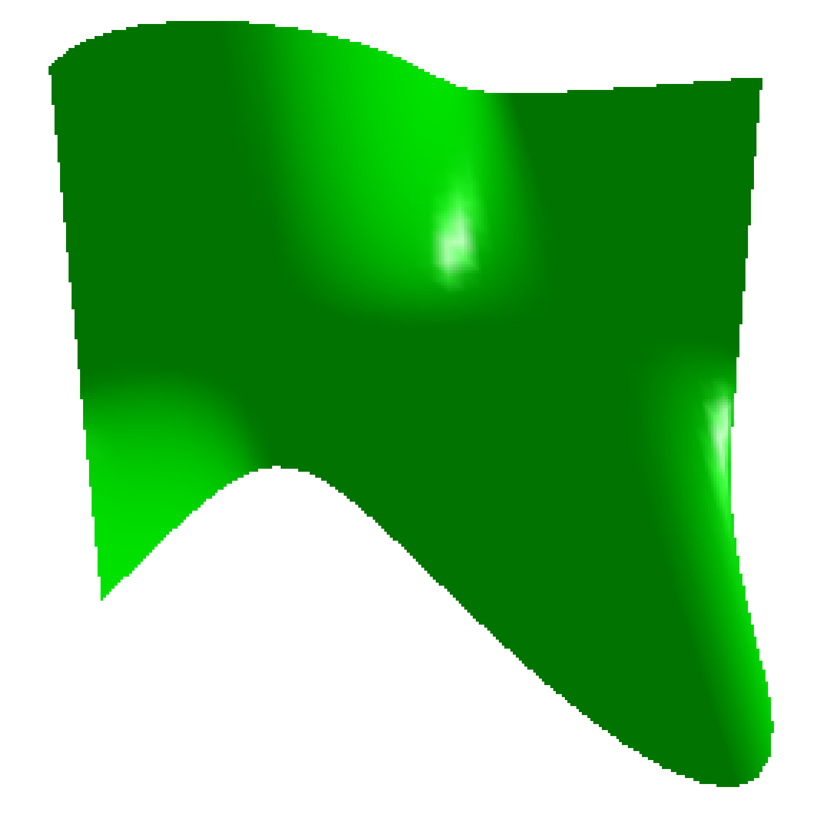
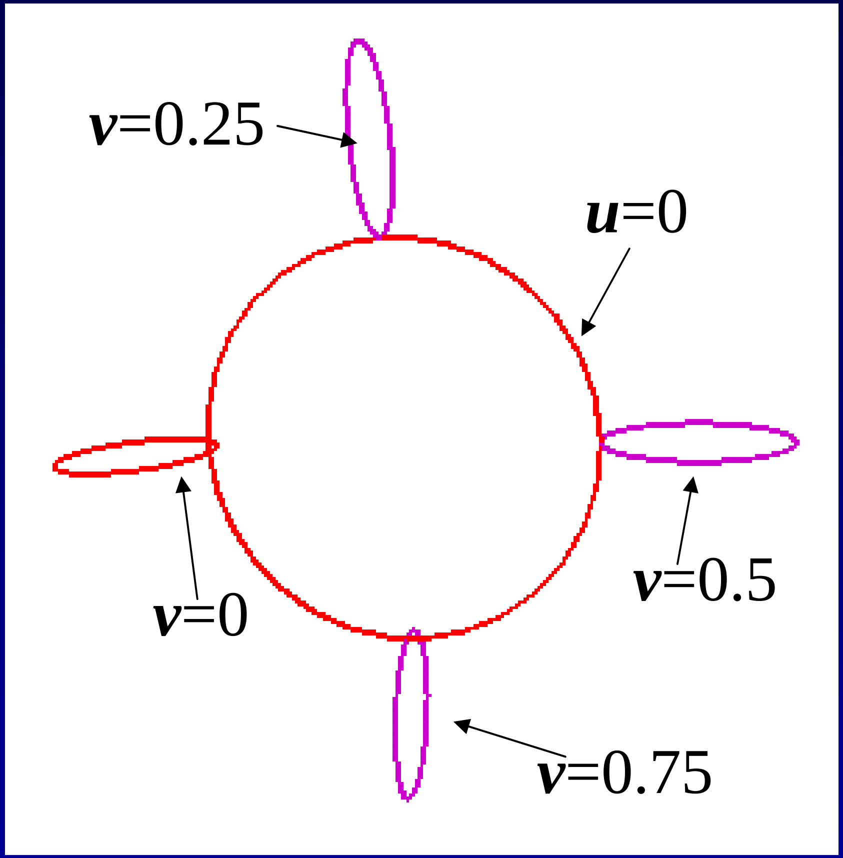
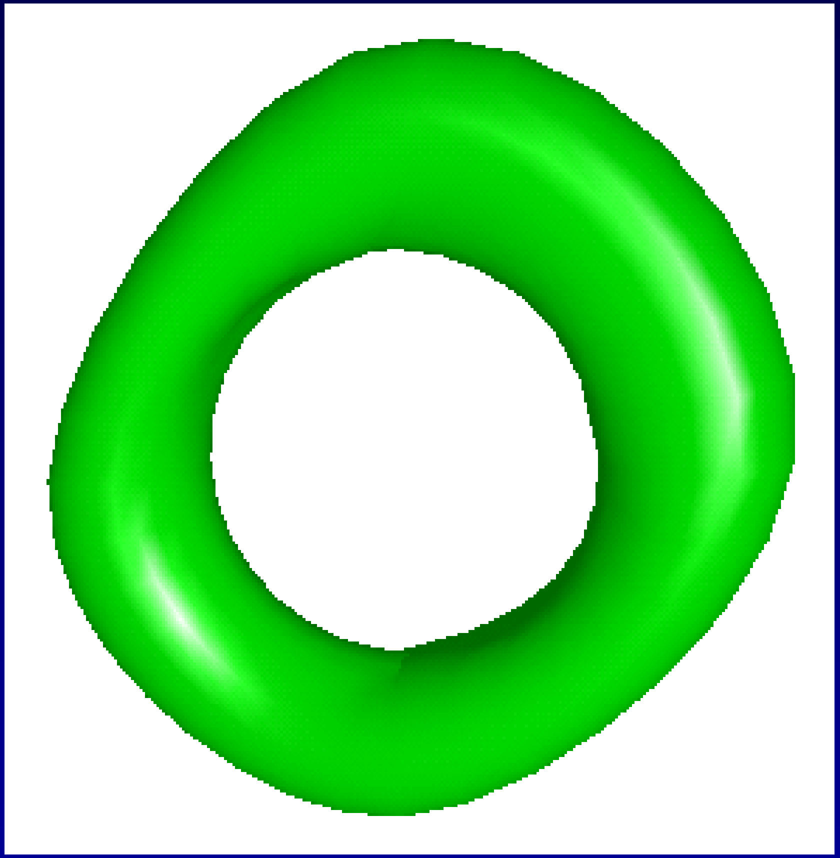
# Coons-like Boundary Constraints



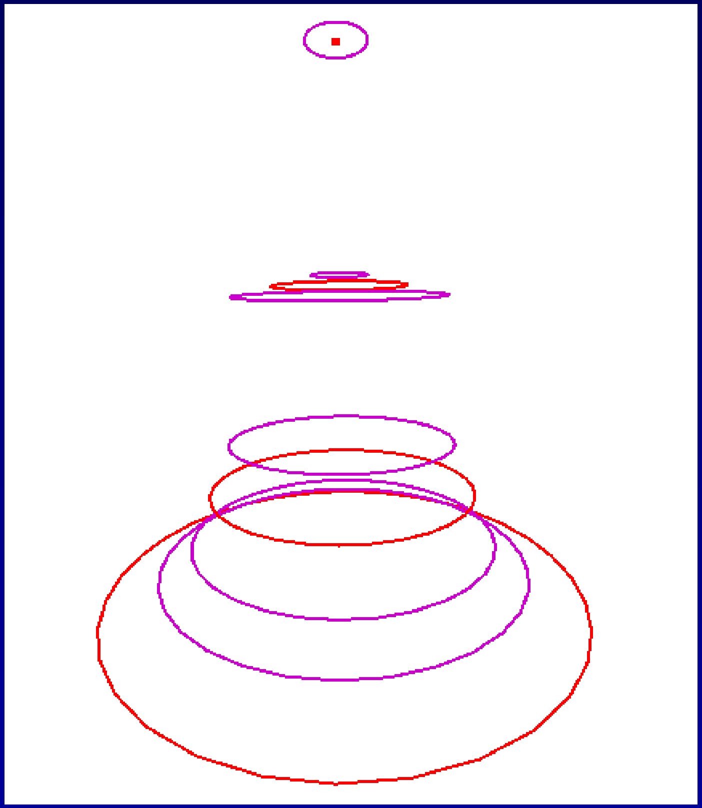
# Gordon-like Boundary Constraints



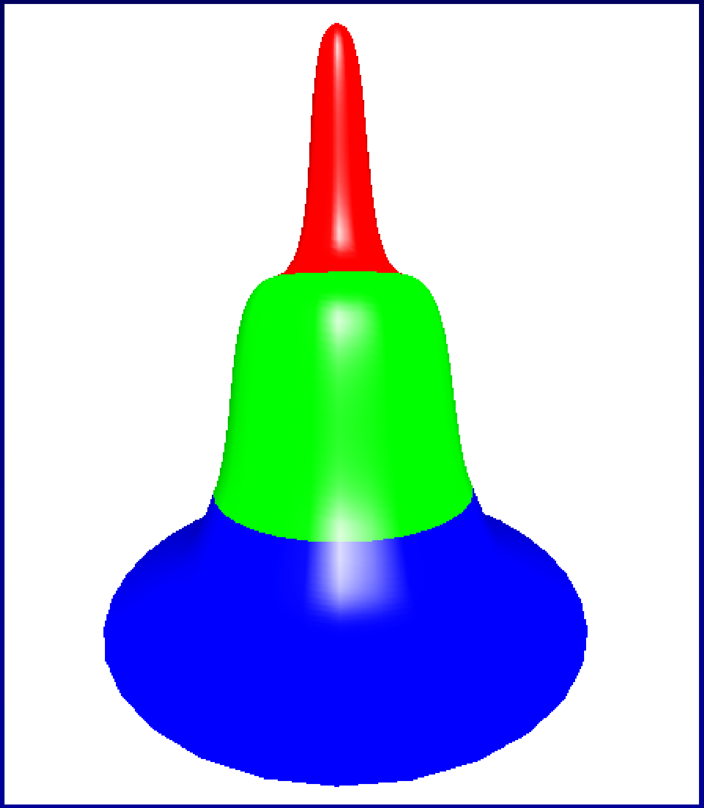



# Joining Multiple PDE Surfaces

- Boundary curves:



- Three connected surfaces:



# PDE Surface Manipulations

- Boundary sculpting
- Blending coefficient control
- Direct manipulation
  - Point based sculpting: position, normal, curvature
  - Curve deformation
  - Region manipulation
- Displacement deformation

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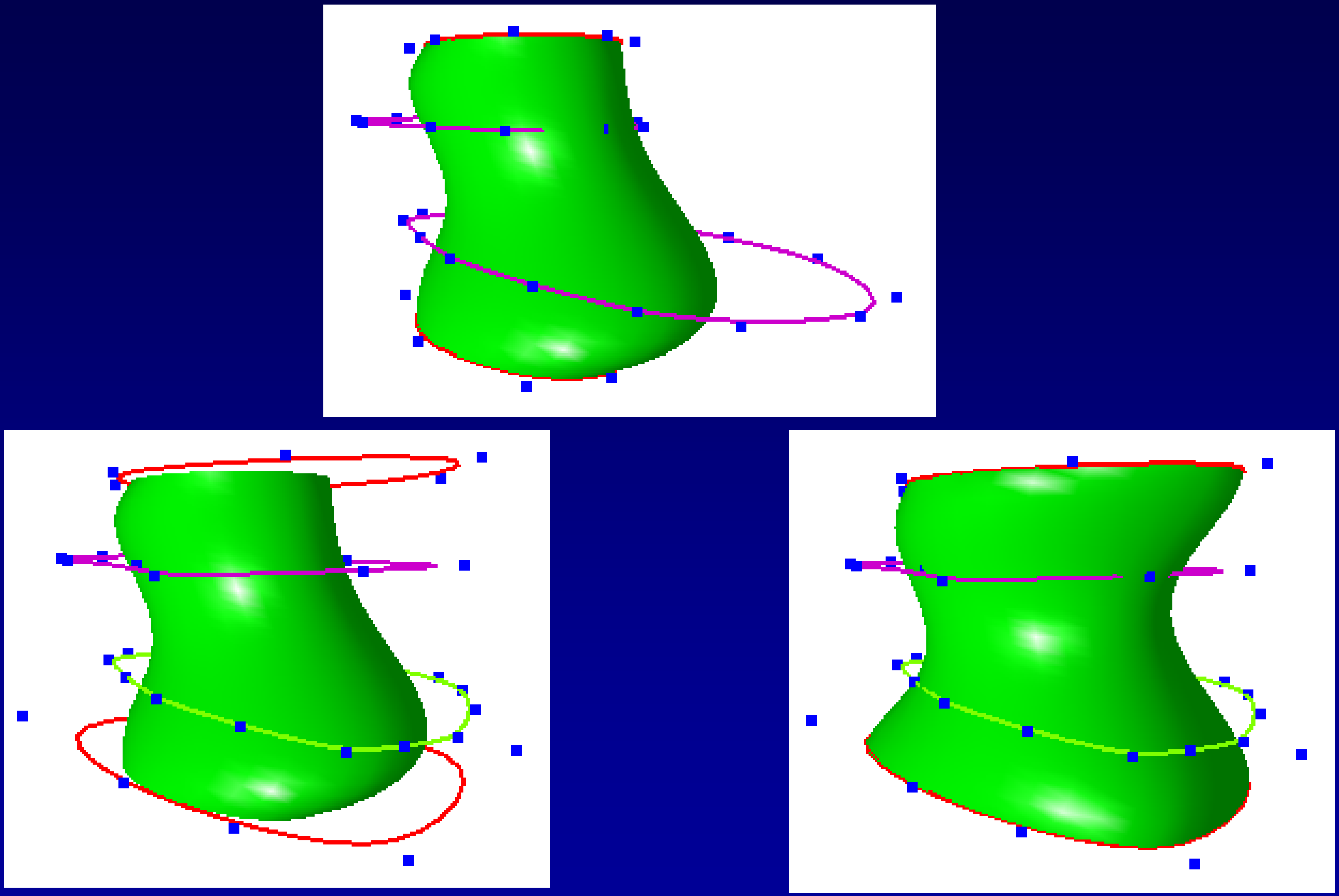
# Boundary Curve Sculpting



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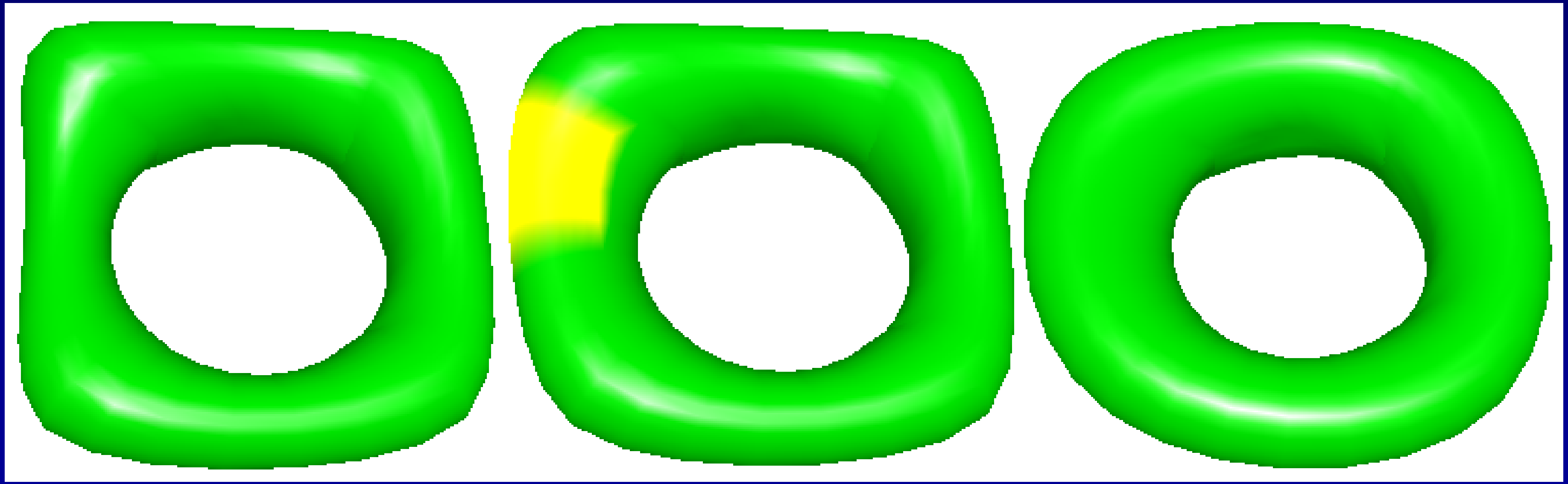
# Effect of a($u$,$v$)

- The value of $a(u_i, v_j)$ at each $(u_i, v_j)$ can be changed interactively.
  - **a($u$,$v$)=3.1**
  - **a($u$,$v$)=5.2** at yellow part
  - **a($u$,$v$)=5.2**



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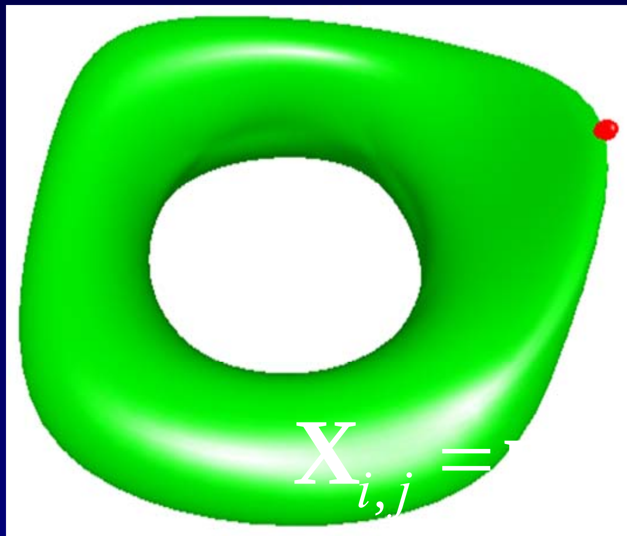
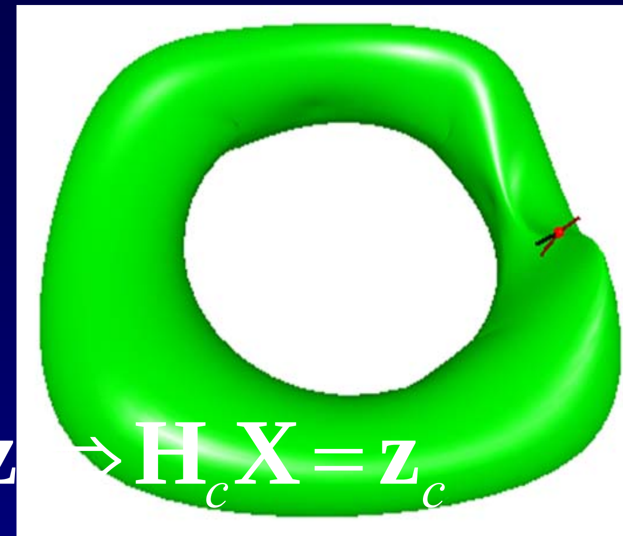
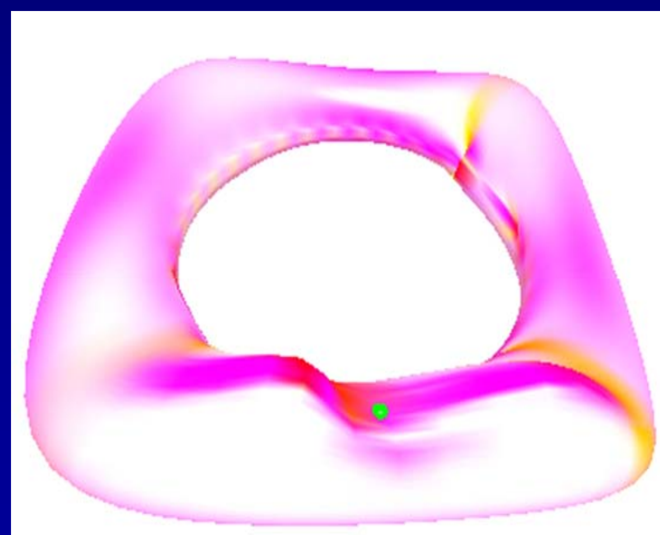
# Point-based Manipulation

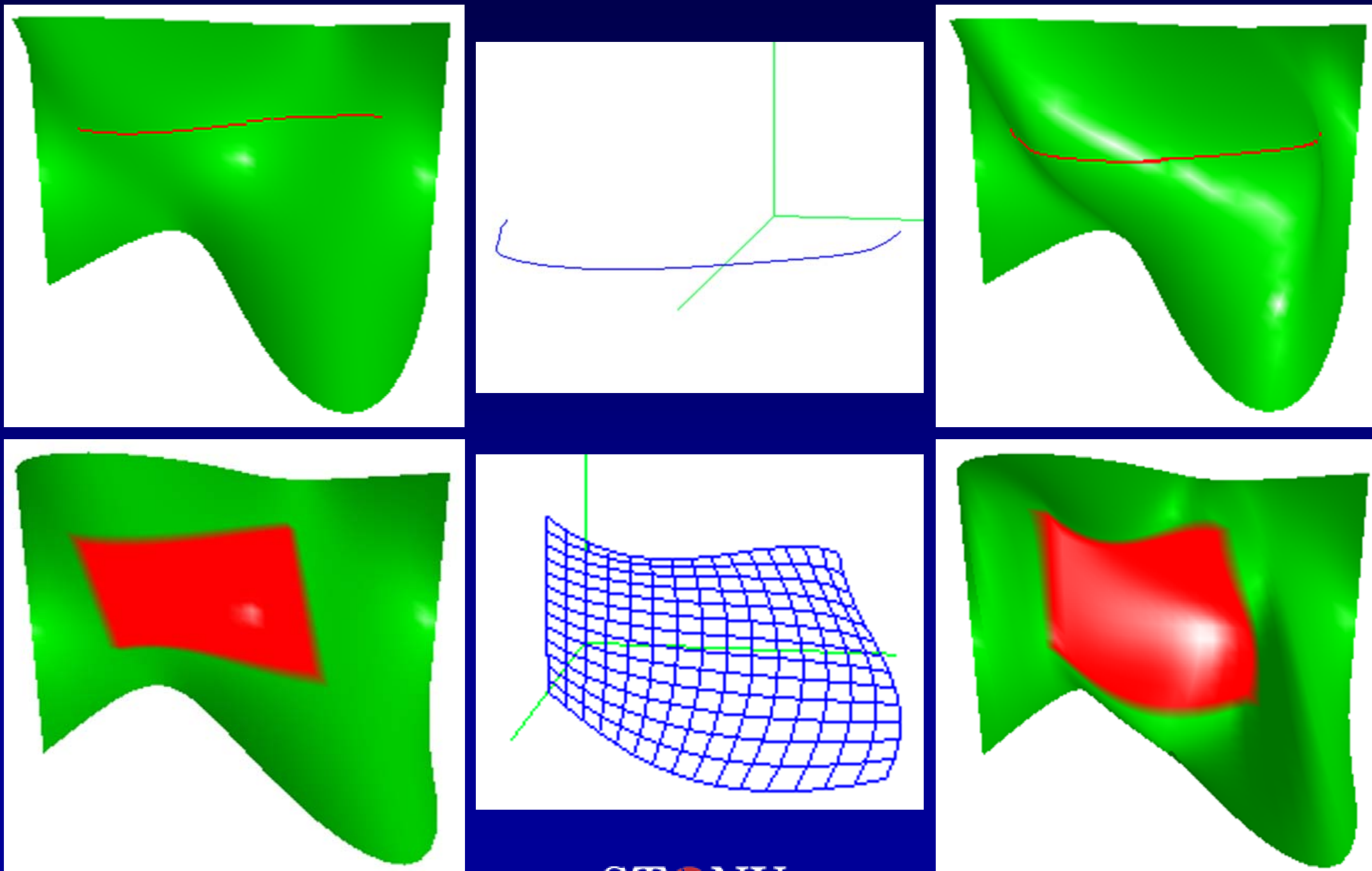




$$\mathbf{X}_{i,j} = \ldots \rightarrow \mathbf{H}\mathbf{X} = \mathbf{z} \rightarrow \mathbf{H}_c\mathbf{X} = \mathbf{z}_c$$



# Curve and Region Sculpting



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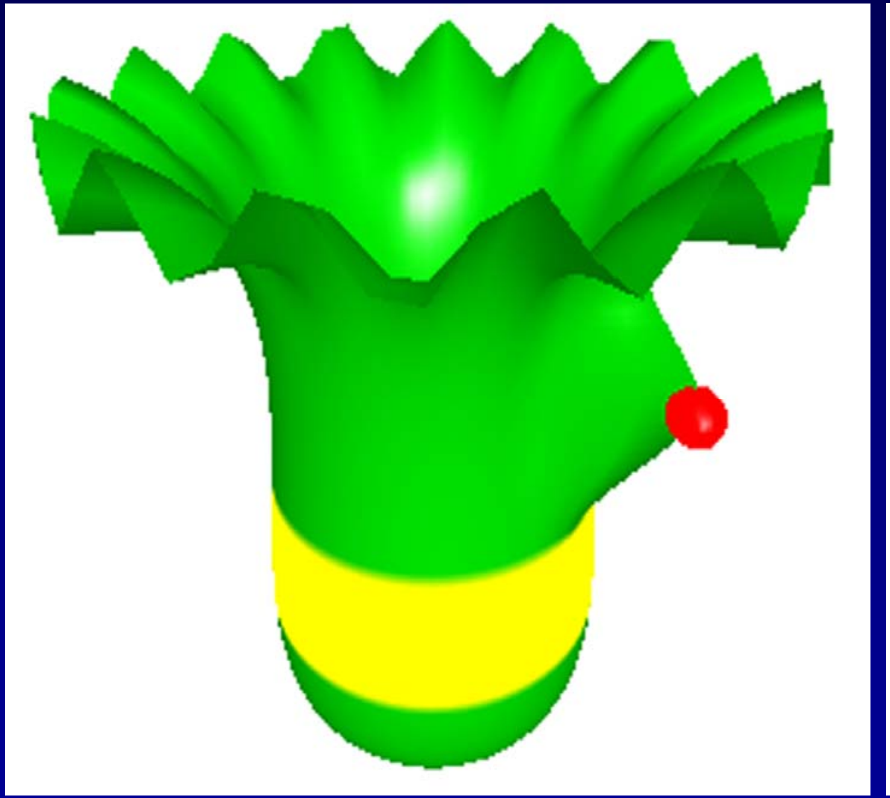
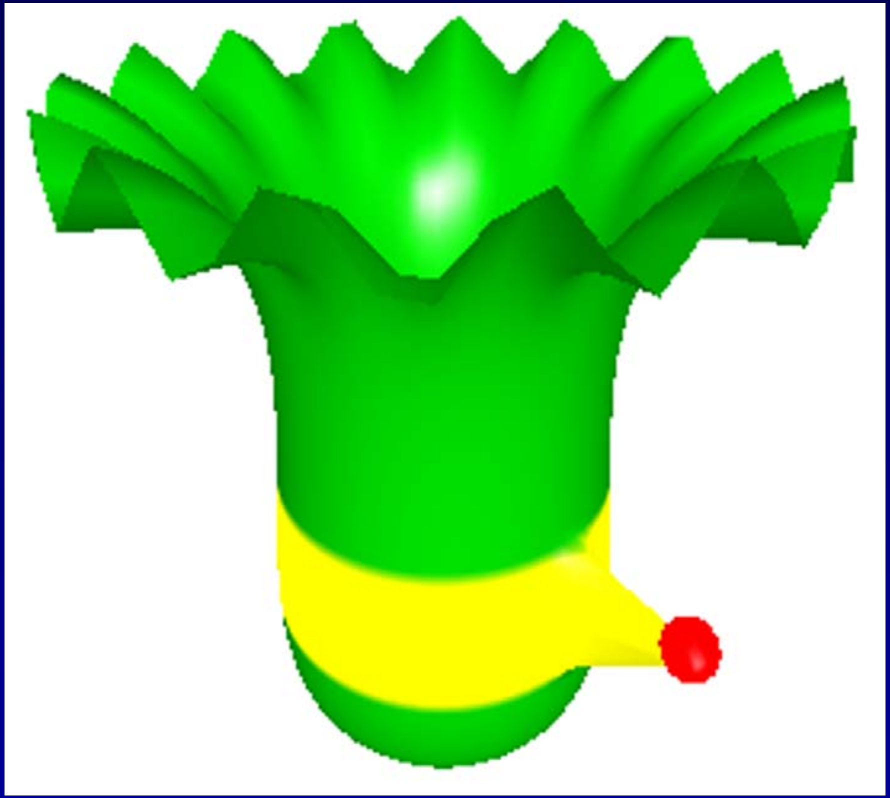
# Local Sculpting

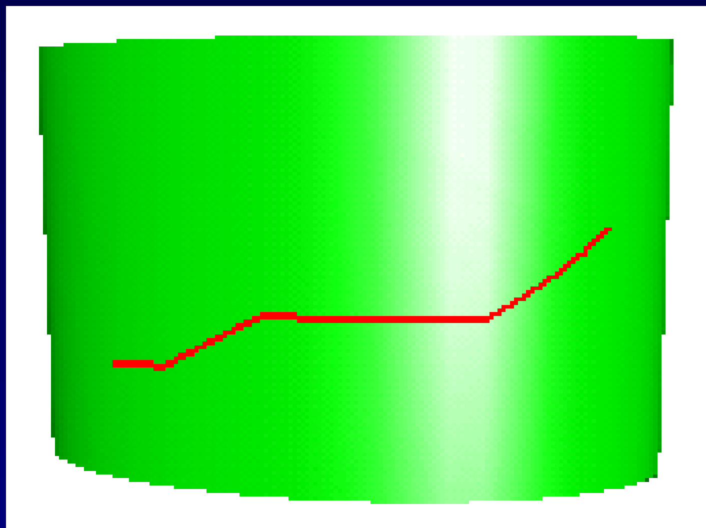


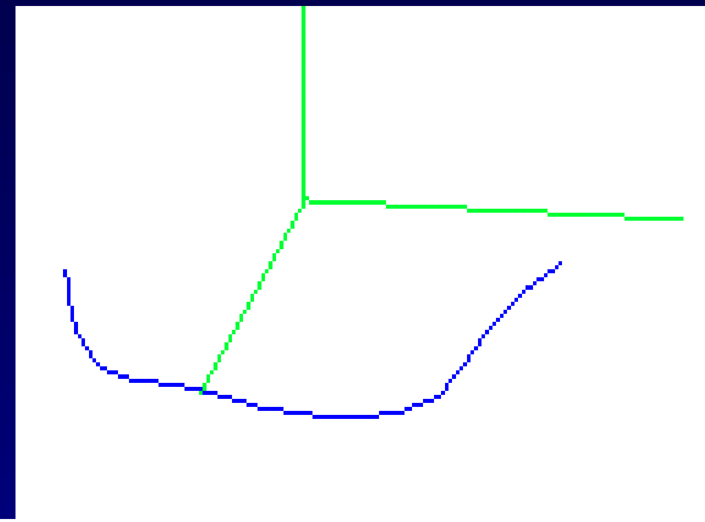


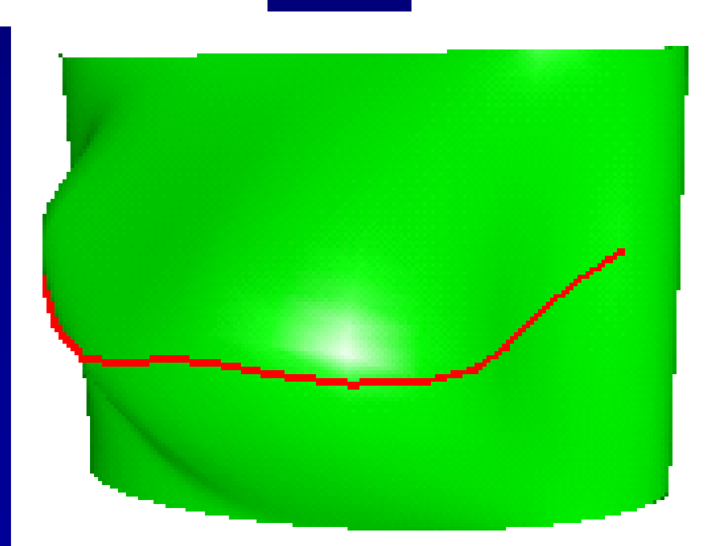
# Curve Editing

1. Select an arbitrary source curve on the PDE surface by picking points on the ***u-v*** domain;
2. Define a cubic B-spline curve with desired shape as the destination curve;
3. Map the source curve to the shape of the destination curve, i.e. put the constraints into the system;
4. Solve the constrained equations to get the new surface.

# Curve Editing







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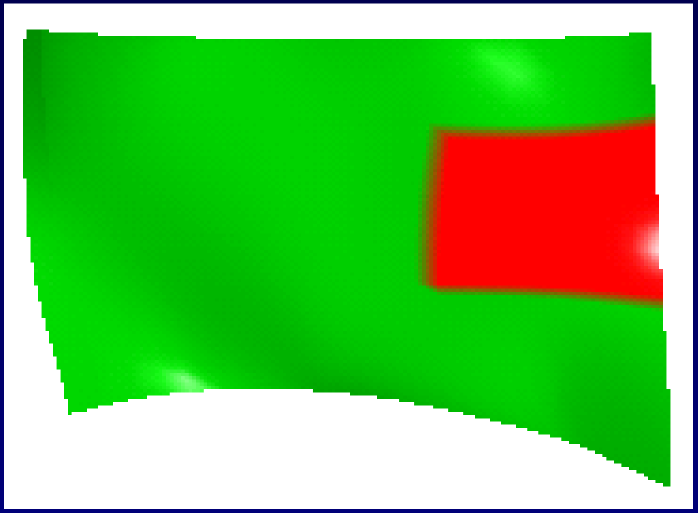
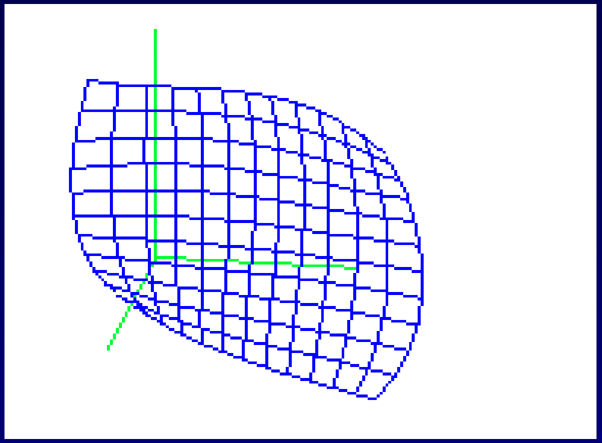
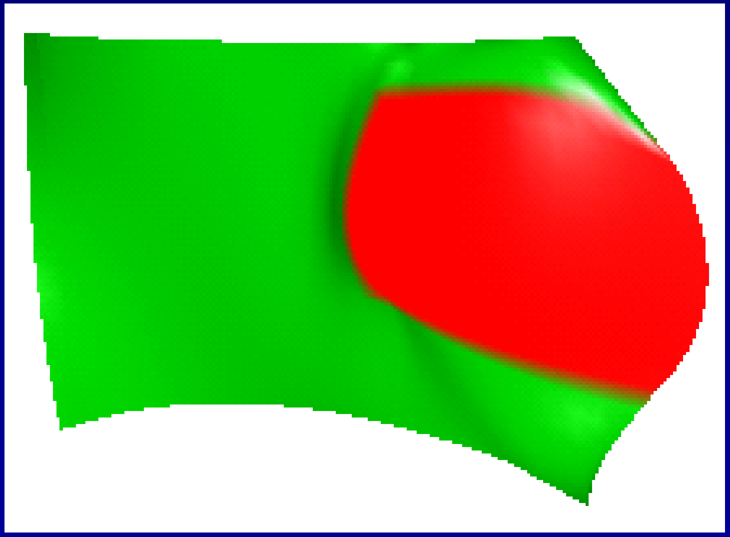
# Region Sculpting

1. Select an area on the PDE surface;
2. Define a cubic B-spline patch with the same number of sample points of the source region;
3. Map the source region to the shape of the destination patch, i.e. put those constraints into the linear equation system;
4. Solve the constrained equations to get the new surface.

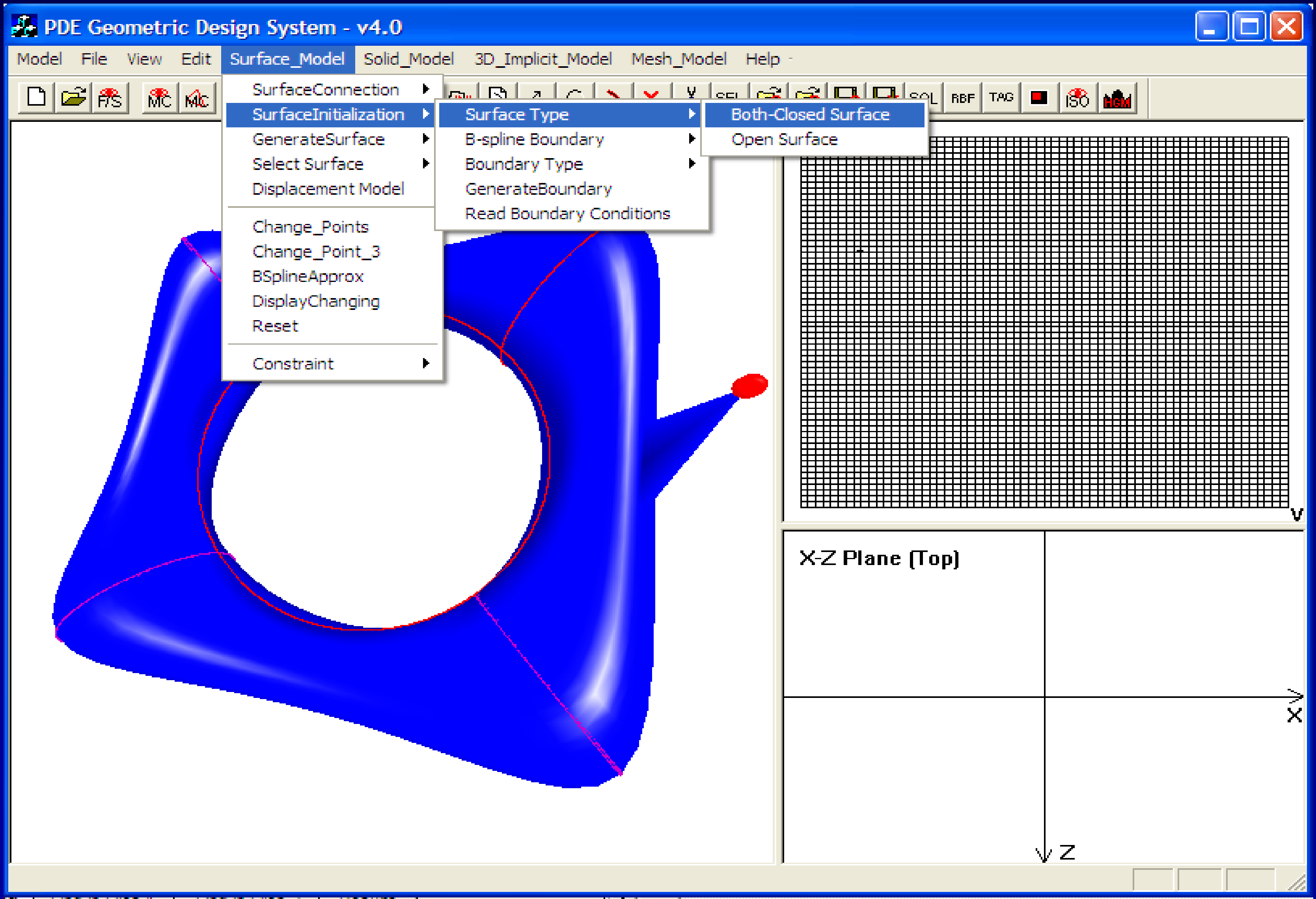
# Region Sculpting



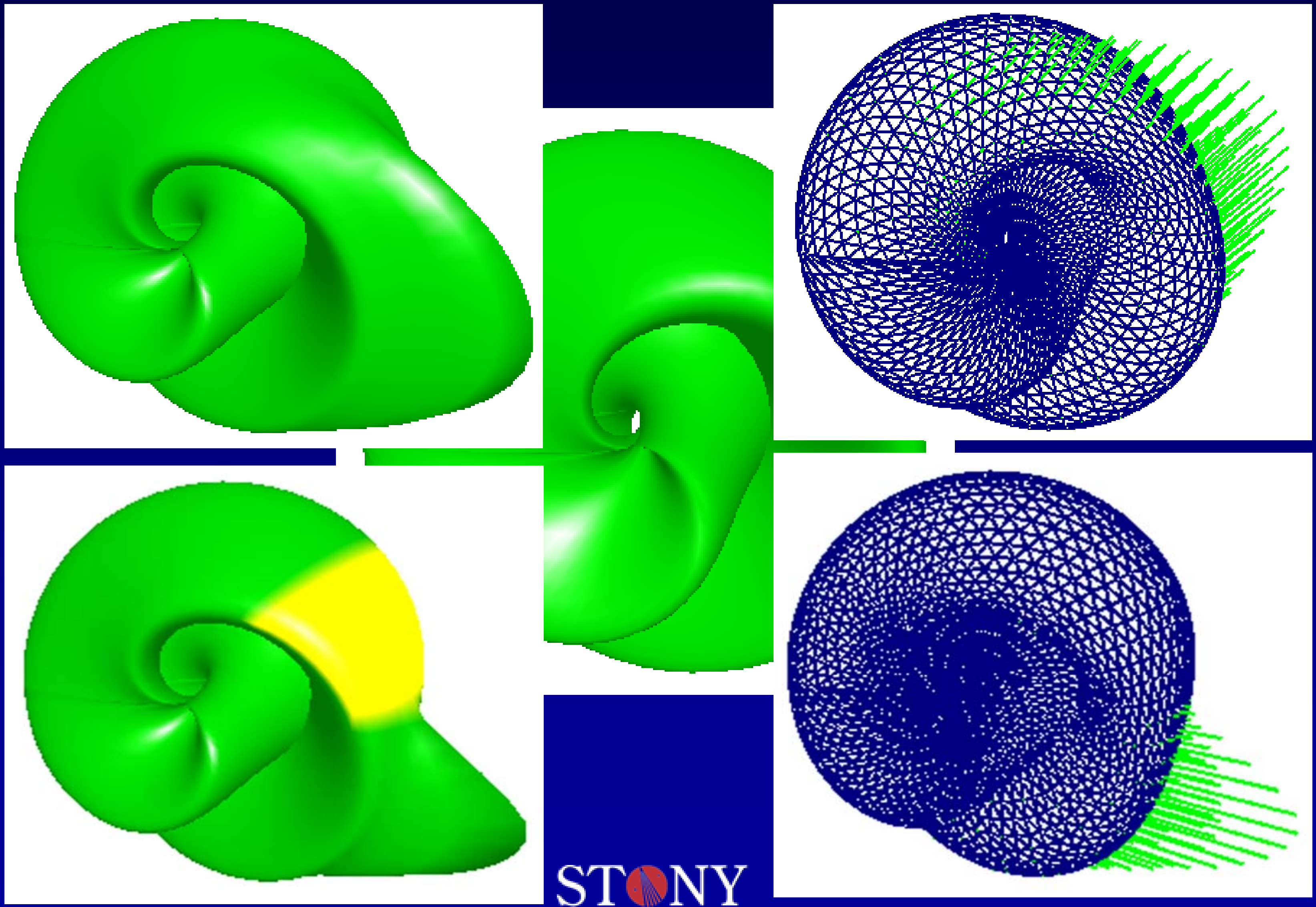




# Interface of PDE Surfaces



# Point Editing on Displacements



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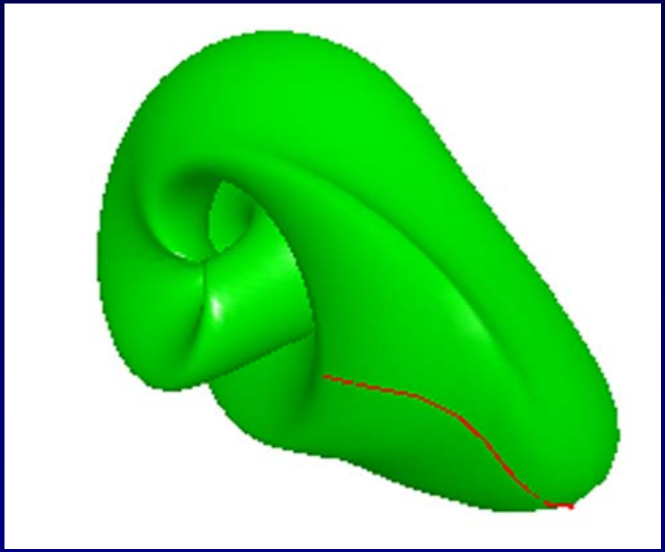
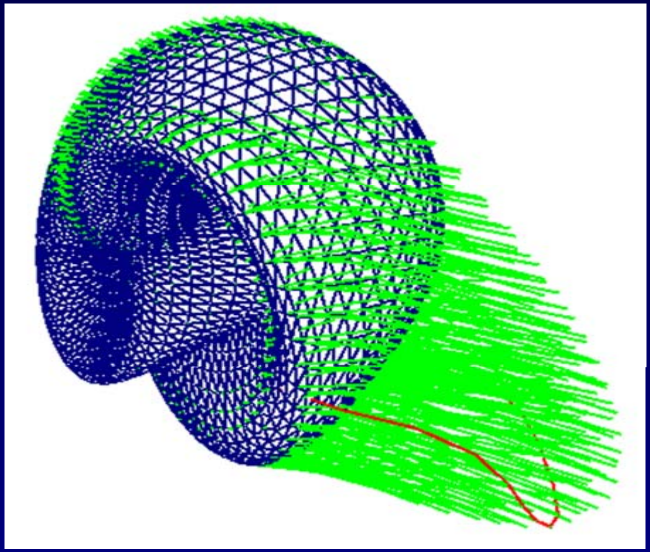
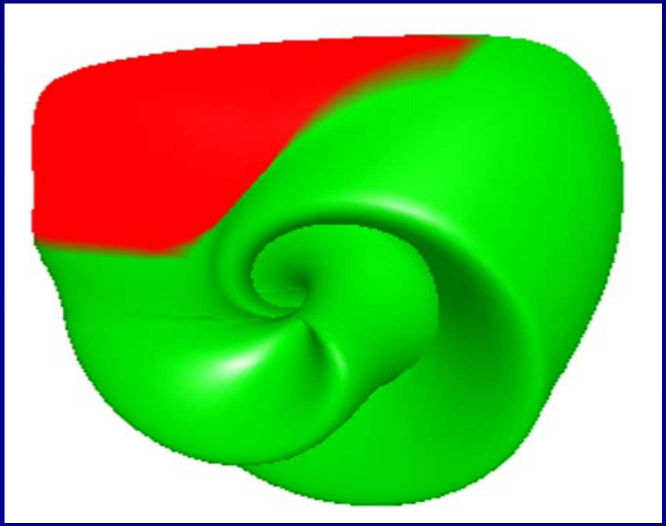
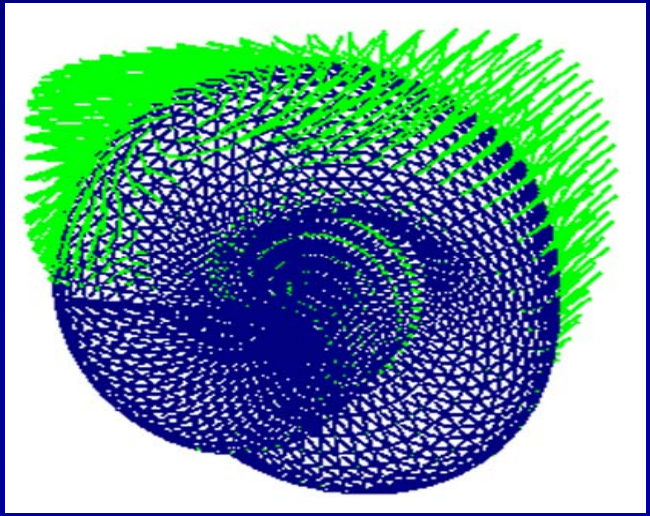
# Displacement Curve and Region Sculpting









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# B-spline Approximation

- B-spline surfaces

$$\mathbf{X}(u,v)=\sum_{i=1}^{k}\sum_{j=1}^{l}B_{i,c}(u)B_{j,d}(v)\mathbf{p}_{i,j}$$

- Obtain B-spline control mesh from PDE surfaces

$$\mathbf{BP}=\mathbf{X}$$

$$\mathbf{B}^{\mathrm{T}}\mathbf{BP}=\mathbf{B}^{\mathrm{T}}\mathbf{X}$$

- B-spline approximation for dynamic models

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# B-Spline Formulation

- B-Spline curve and surface:

$$\mathbf{s}(u) = \sum_i \mathbf{d}_i N_i^n(u)$$

$$\mathbf{s}(u,v) = \sum_i \sum_j \mathbf{d}_{i,j} N_i^n(u) N_j^m(v)$$

- B-Spline basis function:

$$N_i^r(u) = \begin{cases} 1, \text{ if } r = 0 \text{ and } u \in [u_i, u_{i+1}); \\ 0, \text{ if } r = 0 \text{ and } u \notin [u_i, u_{i+1}); \\ (u - u_i)\dfrac{N_i^{r-1}(u)}{u_{i+r} - u_i} + (u_{i+r+1} - u)\dfrac{N_{i+1}^{r-1}(u)}{u_{i+r+1} - u_{i+1}}, r > 0 \end{cases}$$

Knots sequence: $[u_0, u_1, \cdots]$

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# NURBS Formulation

$$\mathbf{C}(u) = \frac{\sum_{i=0}^{n} w_i \mathbf{P}_i N_i^p(u)}{\sum_{i=0}^{n} w_i N_i^p(u)}$$

$$\mathbf{S}(u,v) = \frac{\sum_{i=0}^{n} \sum_{j=0}^{m} w_{i,j} \mathbf{P}_{i,j} N_i^p(u) N_j^q(v)}{\sum_{i=0}^{n} \sum_{j=0}^{m} w_{i,j} N_i^p(u) N_j^q(v)}$$

Weights: $w_i, w_{i,j}$

# Free-Form Splines

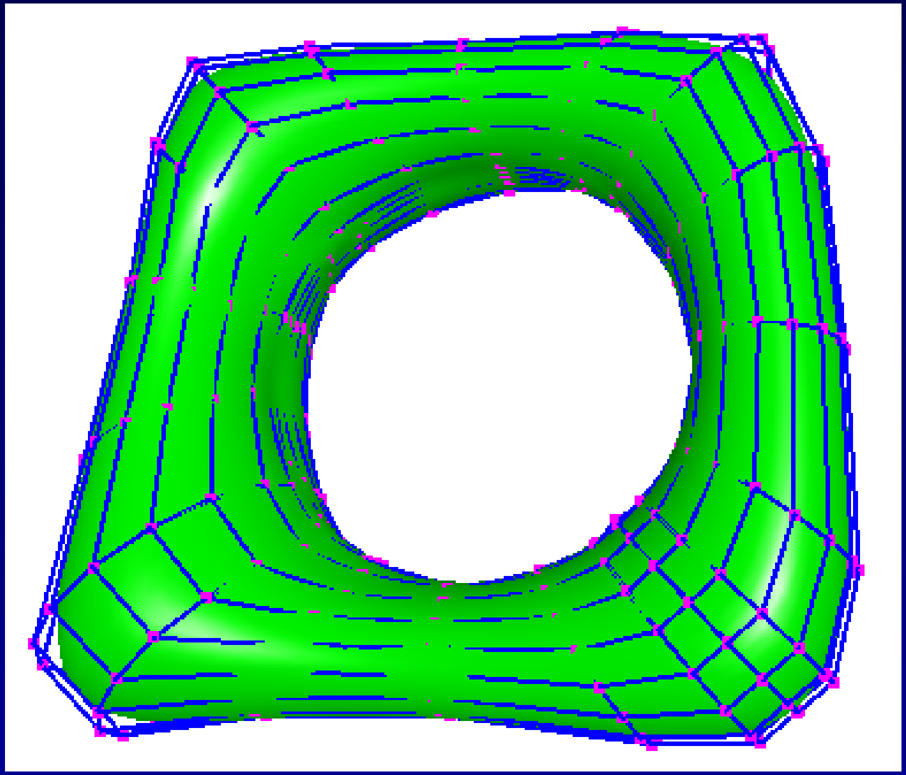
- Piecewise polynomials with certain differentiability constraints
- Local control and extra DOF for manipulation
- NURBS
  - Non-Uniform Rational B-Splines
  - Industrial standard

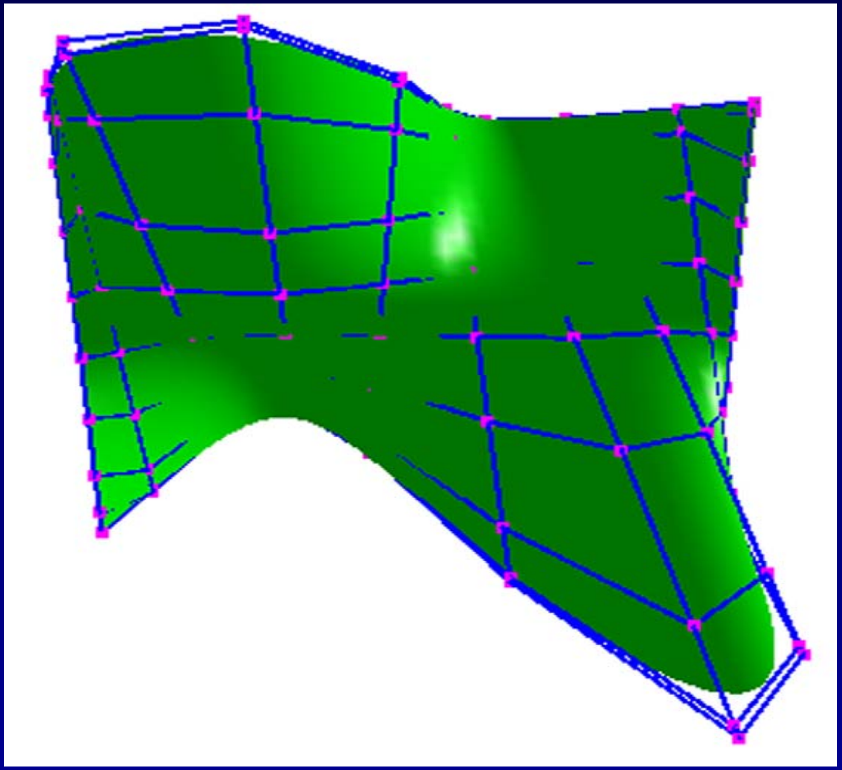
# Pros and Cons of NURBS

- Model both analytic and free-form shapes
- Local control
- Clear geometric interpretations
- Smooth objects
- Powerful modeling toolkits
- Invariant under various manipulations

- Extra storage for traditional objects
- Too many degrees of freedom
- Difficult to model intersection, overlapping
- Less natural and counter-intuitive
- Strong mathematics
- Difficult for arbitrary topology

# Examples of B-spline Approximation





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# Physics-based PDE Surfaces

- Physics-based PDE surface and displacement model
- Flexible boundary conditions
- Global manipulations
  Joining multiple surfaces, boundary sculpting
- Direct local sculpting
  Coefficient control, point, curve, region sculpting, displacement manipulation, material property modification
- B-spline approximation

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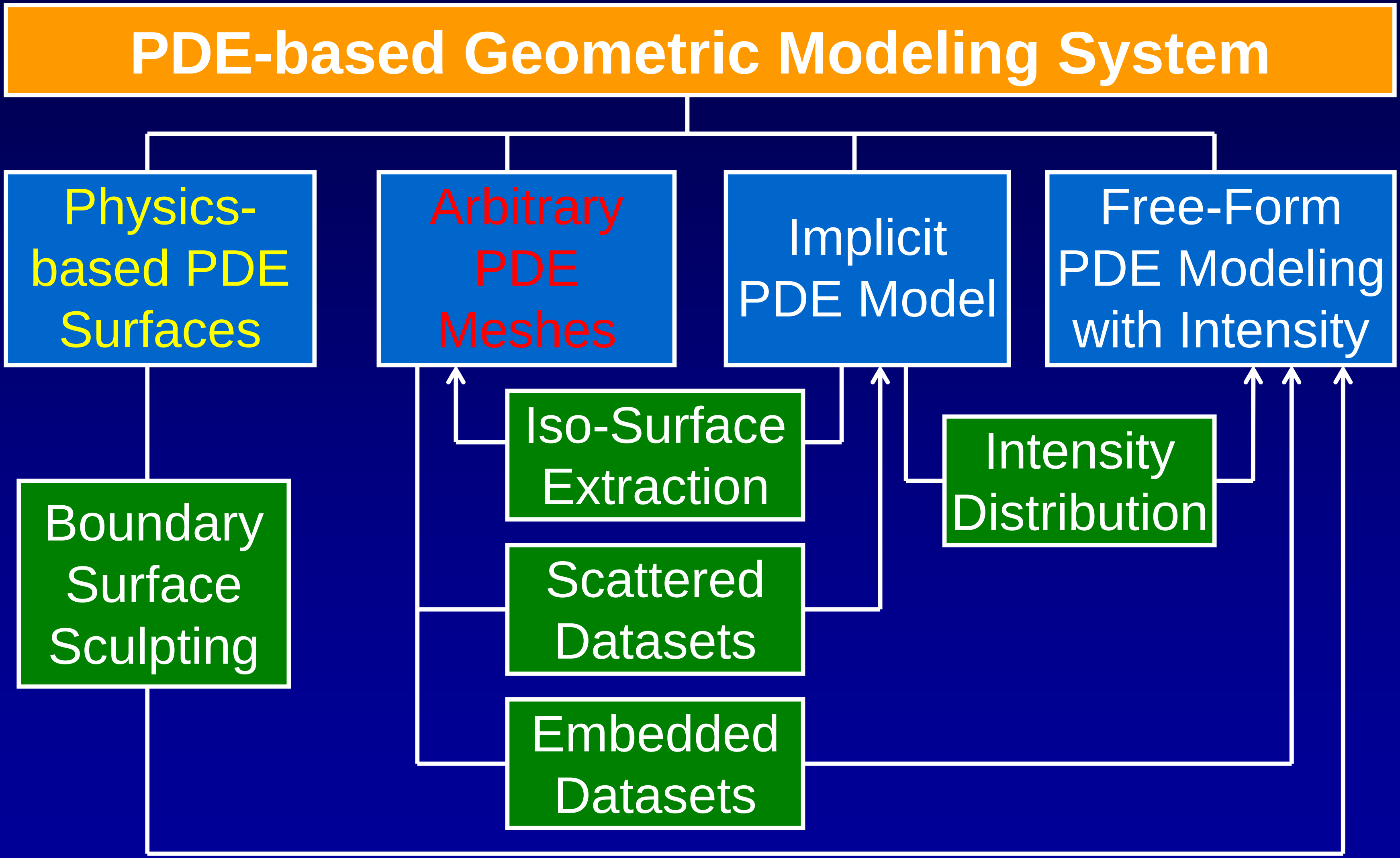
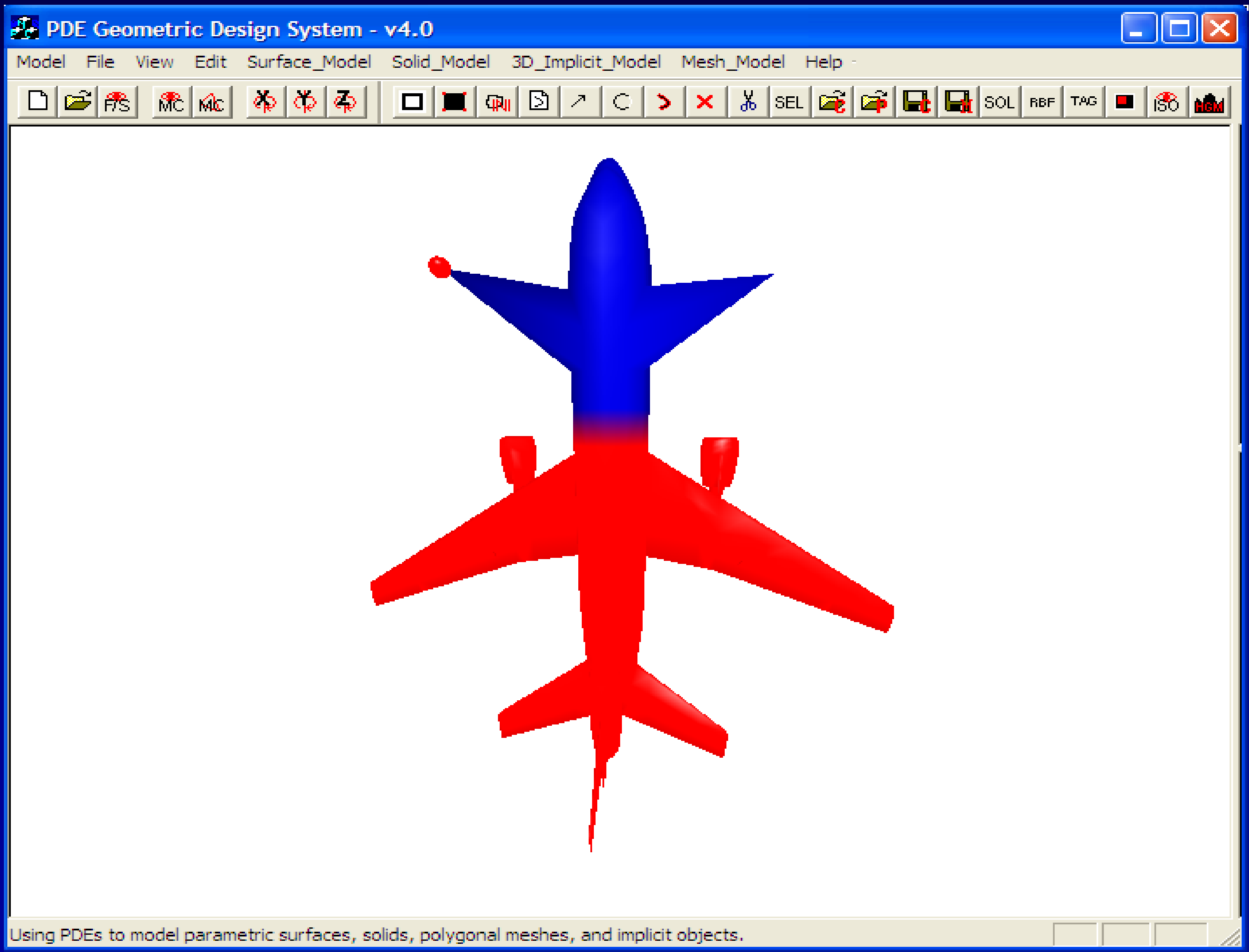
# System Outline



# Outline

- Motivation and contributions
- Background review
- PDE-based geometric modeling system
  - Physics-based PDE surfaces/displacements
  - PDE-based arbitrary mesh modeling
  - Implicit elliptic PDE model
  - PDE-based free-form modeling and deformation
- Conclusion

# Interface

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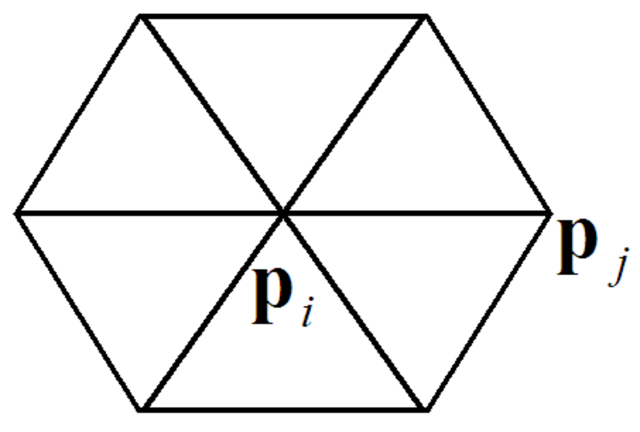
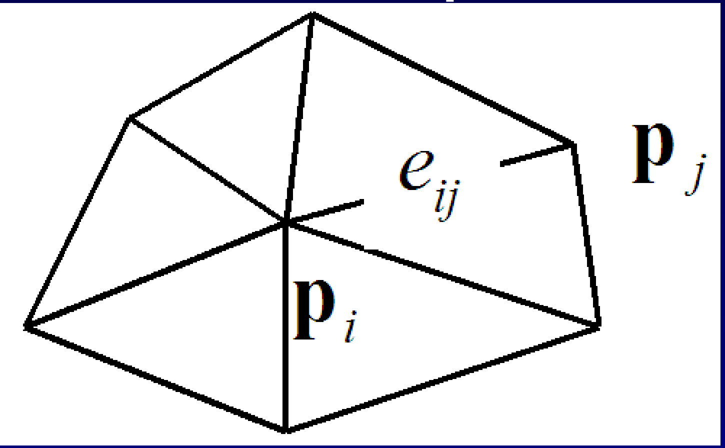
# PDE-based Arbitrary Mesh Model

- Traditional PDE surfaces
  - Defined on regular domain
  - Difficult to model arbitrary topological surfaces
- Polygonal meshes
  - Define surfaces as collection of points and their relations
  - Shape of arbitrary topology
- Goal: use PDE techniques to model arbitrary polygonal meshes

# PDE Approximation of Arbitrary Meshes

- Umbrella operator for discrete Laplacian:




$$\nabla^2 \mathbf{p}_i \approx \frac{1}{n}\sum_{j\in N_1(i)} \mathbf{p}_j - \mathbf{p}_i, \qquad \nabla^2 \mathbf{p}_i \approx \frac{2}{E}\sum_{j\in N_1(i)} (\mathbf{p}_j - \mathbf{p}_i)/e_{i,j}$$

- Approximating formulations:

$$\nabla^2 \mathbf{p} = \mathbf{F}, \nabla^4 \mathbf{p} = \mathbf{F}$$

$$\nabla^4 \mathbf{p}_i = \nabla^2(\nabla^2 \mathbf{p}_i) \approx \frac{1}{n}\sum_{j\in N_1(i)} \nabla^2 \mathbf{p}_j - \nabla^2 \mathbf{p}_i$$

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# Direct Manipulation of Arbitrary Meshes

- Take input meshes as general constraints
- Use umbrella operators to approximate the PDEs
- Point-based manipulation for shape deformation
- Local control by selecting regions of interests
- Possible to integrate with subdivision models

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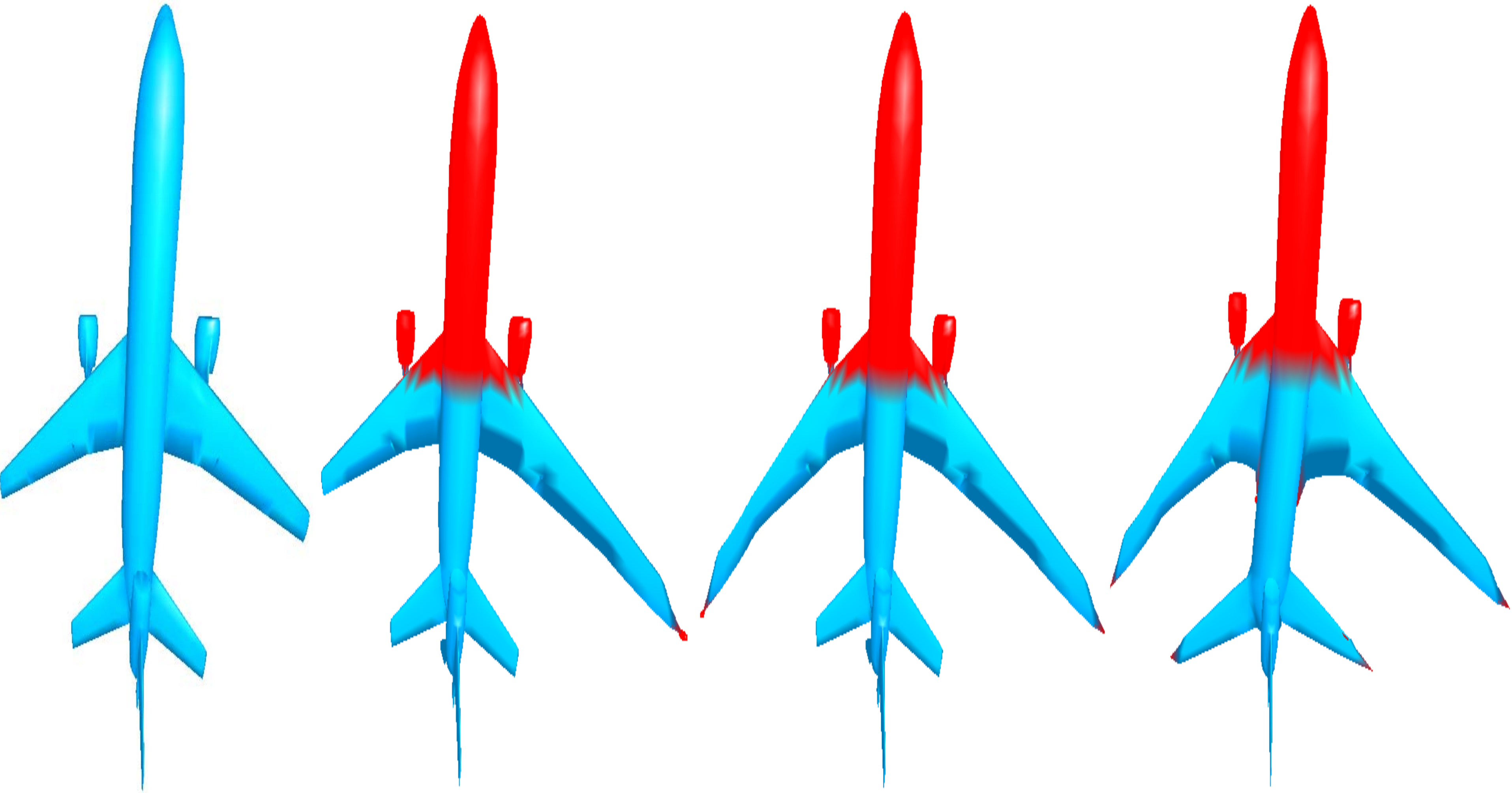
# Direct Manipulation of Arbitrary Meshes



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# PDE-based Medial Axis Extraction

- Compact representation of arbitrary polygonal meshes
- Diffusion-based equations to simulate grassfire process
- Approximates medial axes for manipulation purposes
- Facilitates skeleton-based shape sculpting for arbitrary meshes

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# Formulations

- Diffusion-based PDE

$$\frac{\partial \mathbf{S}(\mathbf{p},t)}{\partial t} = \mathbf{D}(\mathbf{N},\kappa)\nabla^2\mathbf{S}$$

- Normal approximation

$$\mathbf{N}_i = \mathbf{t}_1 \times \mathbf{t}_2 = \sum_{j=0}^{n-1} \cos\frac{2\pi j}{n}\mathbf{p}_j \times \sum_{j=0}^{n-1} \sin\frac{2\pi j}{n}\mathbf{p}_j$$

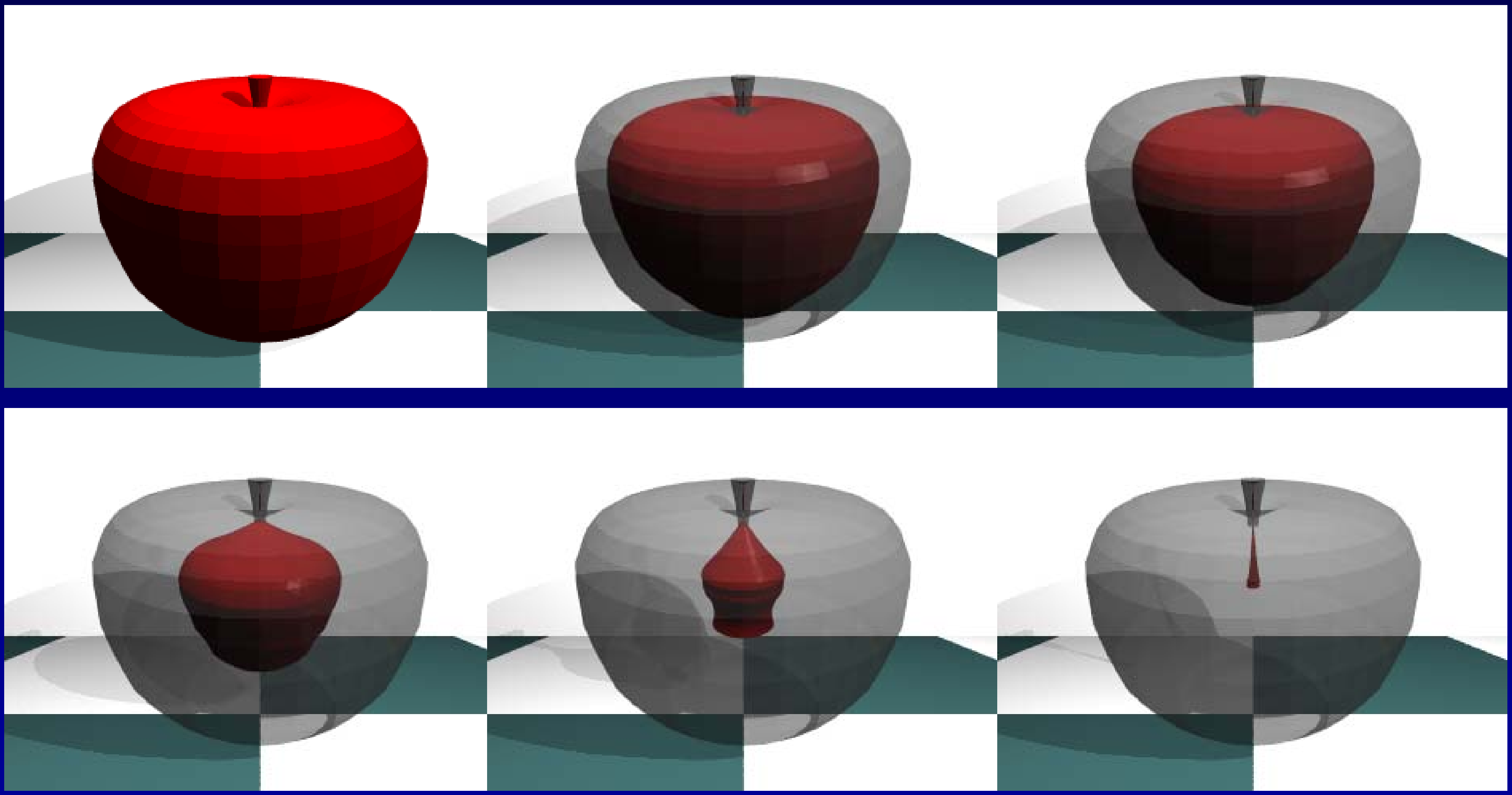
- Gaussian curvature approximation

$$\kappa_i = \frac{2\pi - \sum_{j=0}^{n-1}\phi_j}{\frac{1}{3}\sum_{j=0}^{n-1}A_j}, \kappa_i = \frac{\pi - \sum_{j=0}^{n-1}\phi_j}{\frac{1}{3}\sum_{j=0}^{n-1}A_j}$$

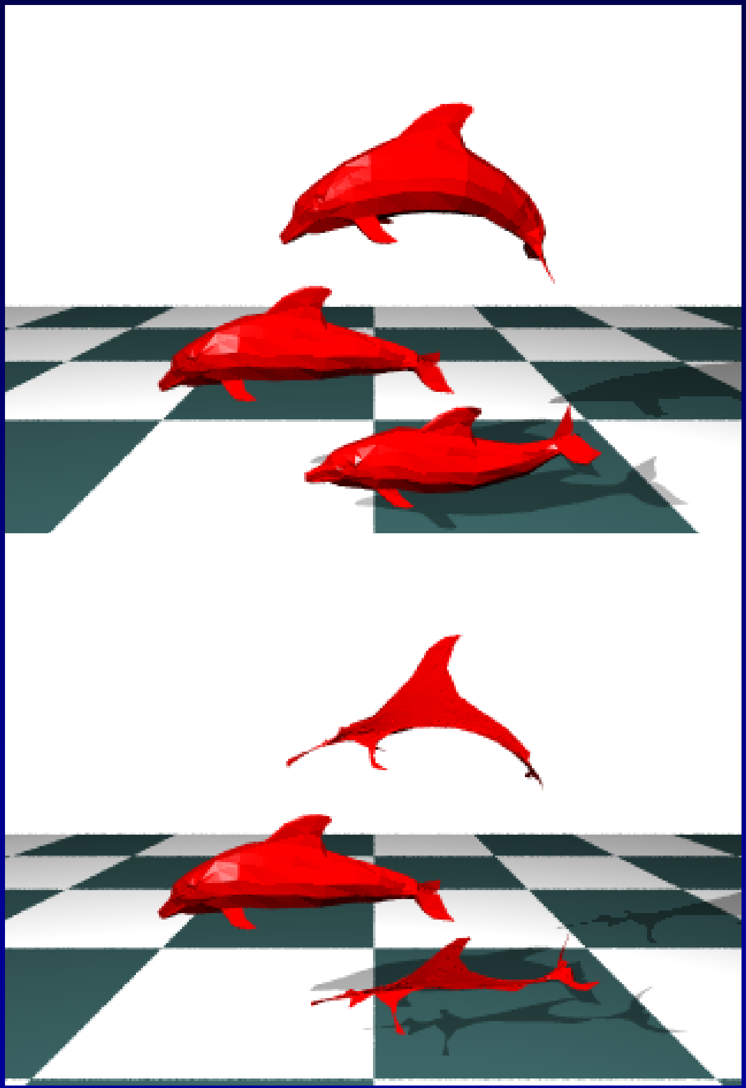
# Medial Axis Extraction Algorithm

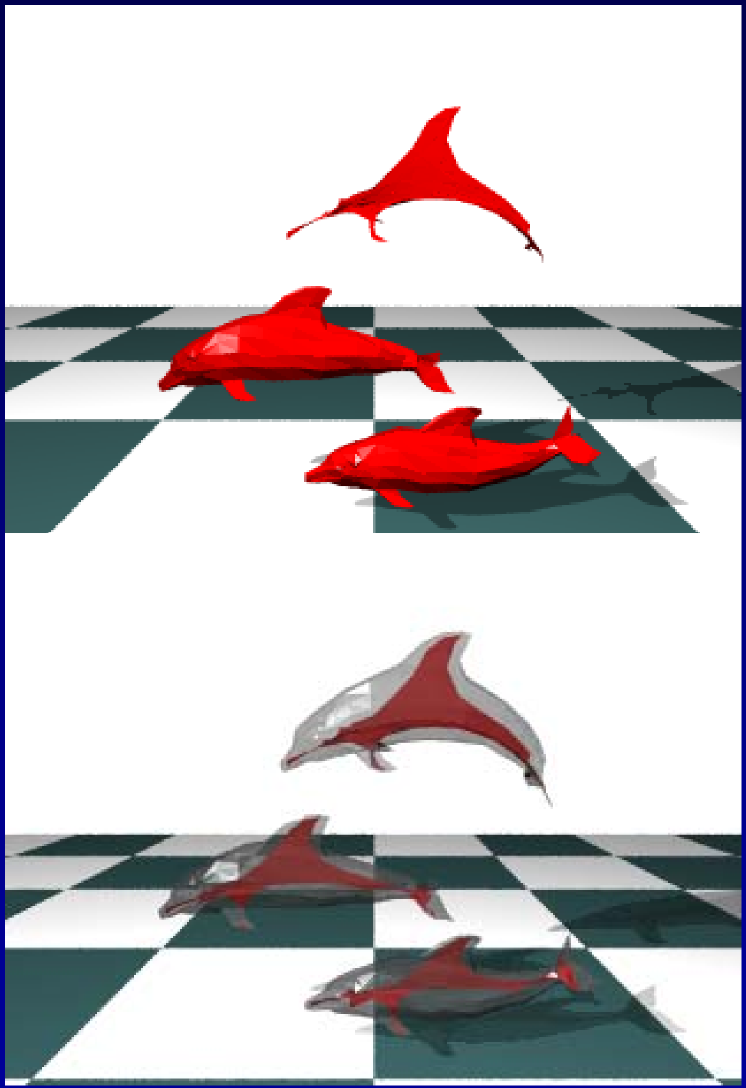
- Initialization
  - Approximate surface normal and other differential properties
- Skeletonization
  - Compute evolving surface
  - Collision detection to find skeletal points
  - Surface optimization
- User interaction
  - User-defined skeleton, local skeletonization

# Progressive Medial Axis Extraction



# Local Region Skeletonization





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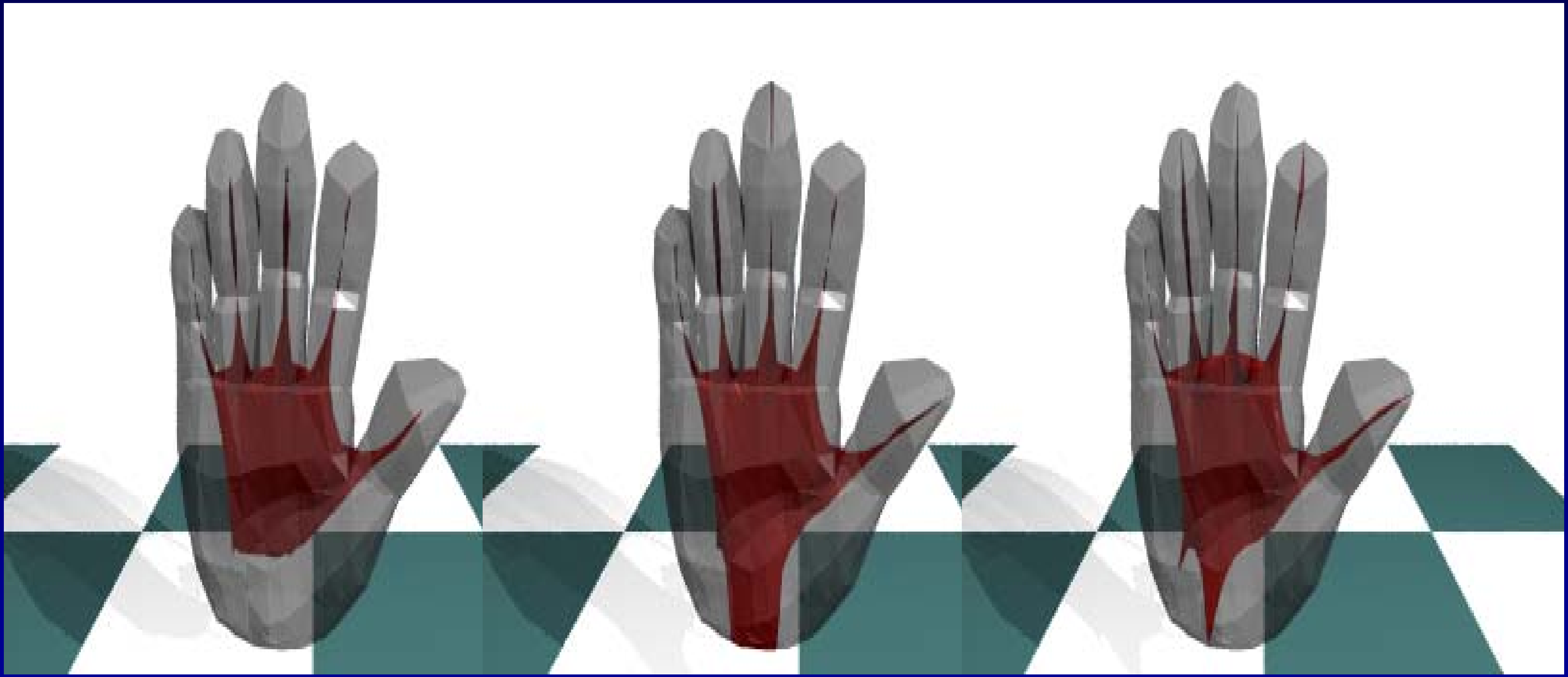
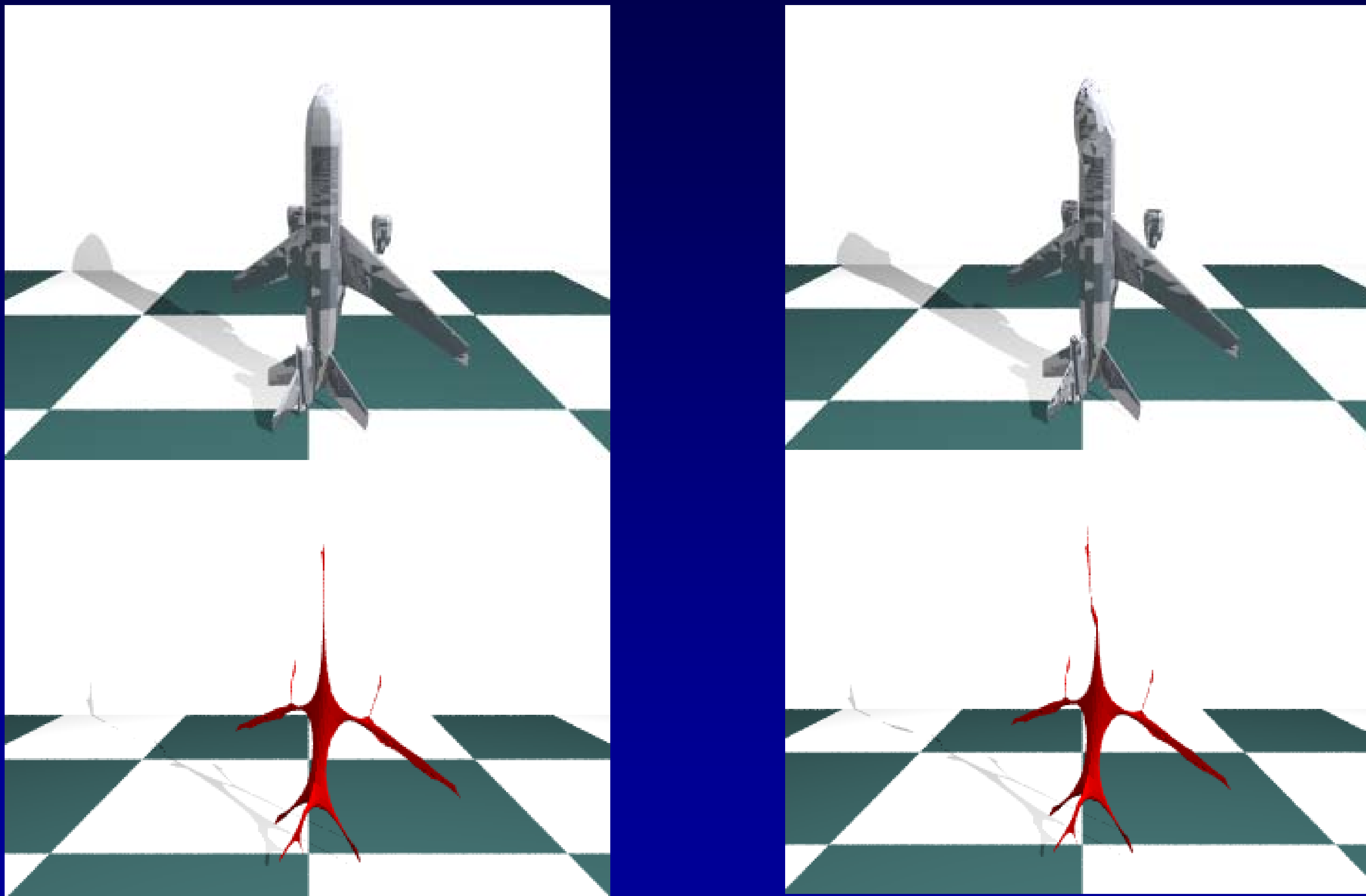
# User-defined Skeleton



# Skeleton-based Sculpting



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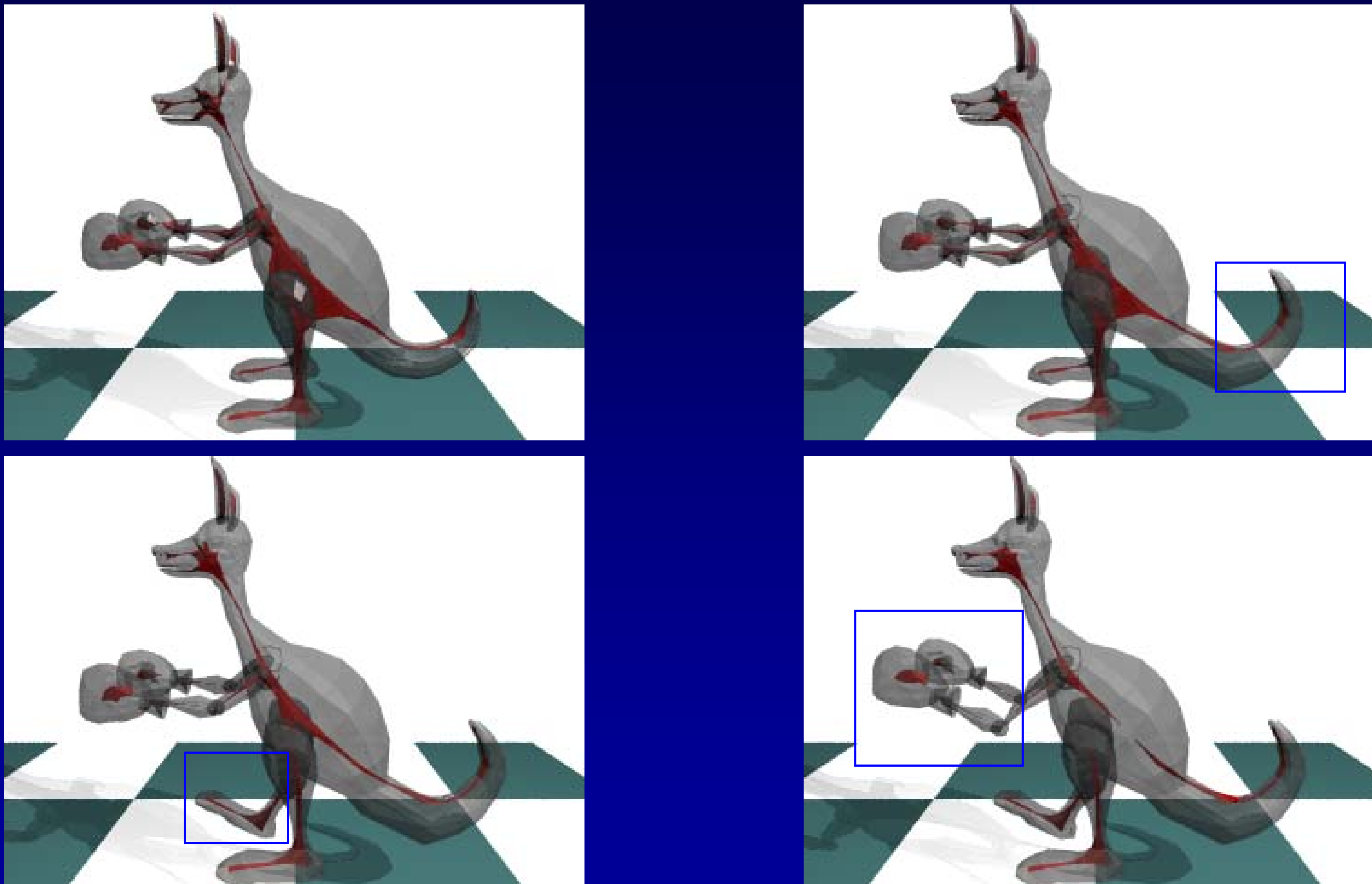
# Skeleton-based Sculpting



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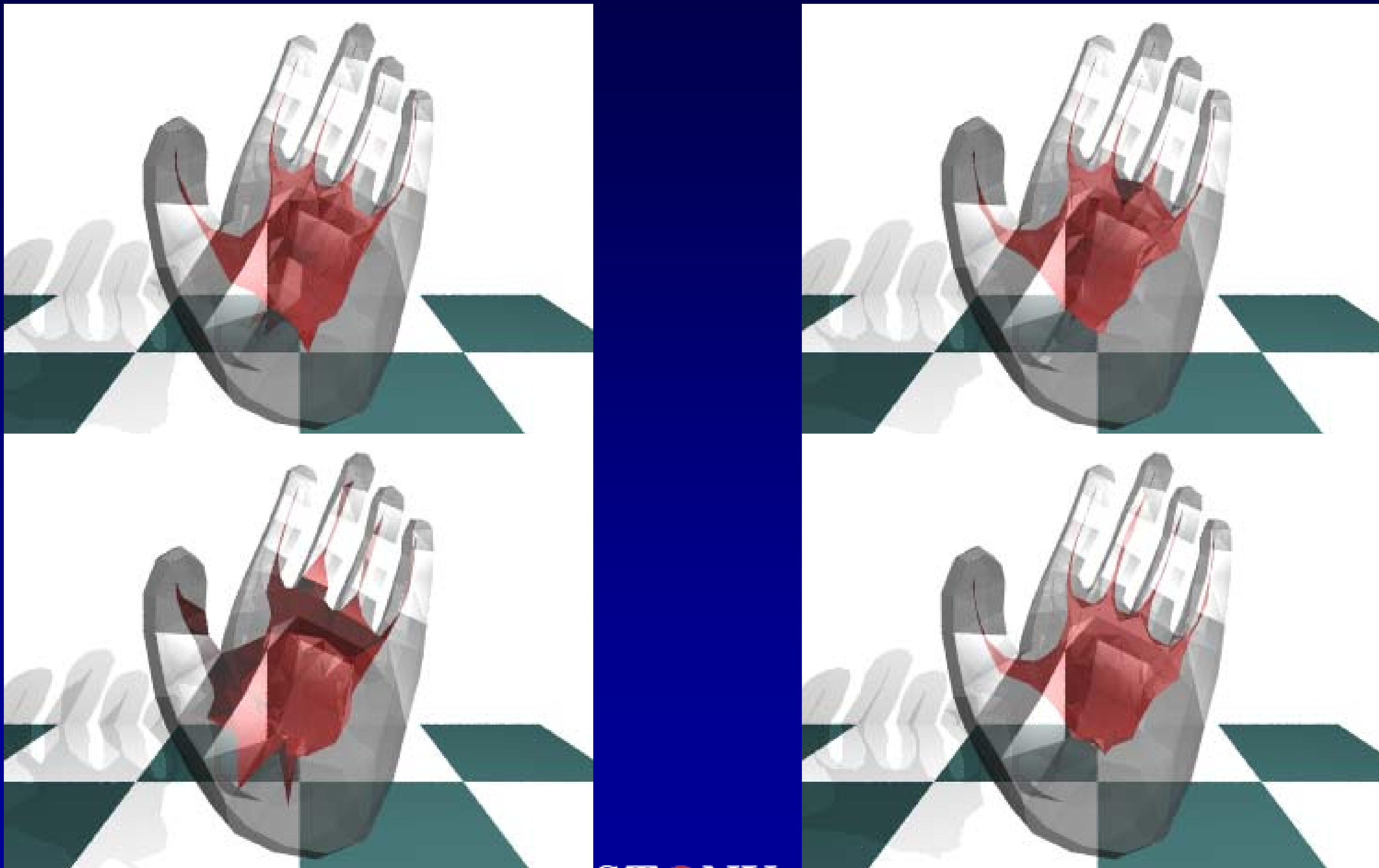
# Curvature Manipulation



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# PDE-based Arbitrary Mesh Modeling

- Direct manipulation on polygonal meshes
- Diffusion-based medial axis extraction
  - Progressive visualization
  - User interaction
  - Local region skeletonization
- Skeleton-based shape manipulation

# Outline

- Motivation and contributions
- Background review
- PDE-based geometric modeling system
  - Physics-based PDE surfaces/displacements
  - PDE-based arbitrary mesh modeling
  - Implicit elliptic PDE model
  - PDE-based free-form modeling and deformation
- Conclusion

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# Implicit PDE Modeling

- Implicit elliptic PDE formulation
- General boundary constraints
- Radial Basis Function (RBF) for initial guess
- Direct manipulation in the implicit working space
- Interactive sculpting of implicit PDE objects

# RBF Method (1)

- RBF: Radial Basis Function
- Solving interpolation problems by minimizing thin-plate energy in 3D

$$\iint f_{xx}^2 + 2 f_{xy}^2 + f_{yy}^2 \, dxdy$$

- Basis function:

$$\phi(\mathbf{x}) = |\mathbf{x}|^3$$

- Interpolation function:

$$f(\mathbf{x}) = \sum_{j=1}^{k} w_j \phi(\mathbf{x} - \mathbf{c}_j) + P(\mathbf{x})$$

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# RBF Method (2)

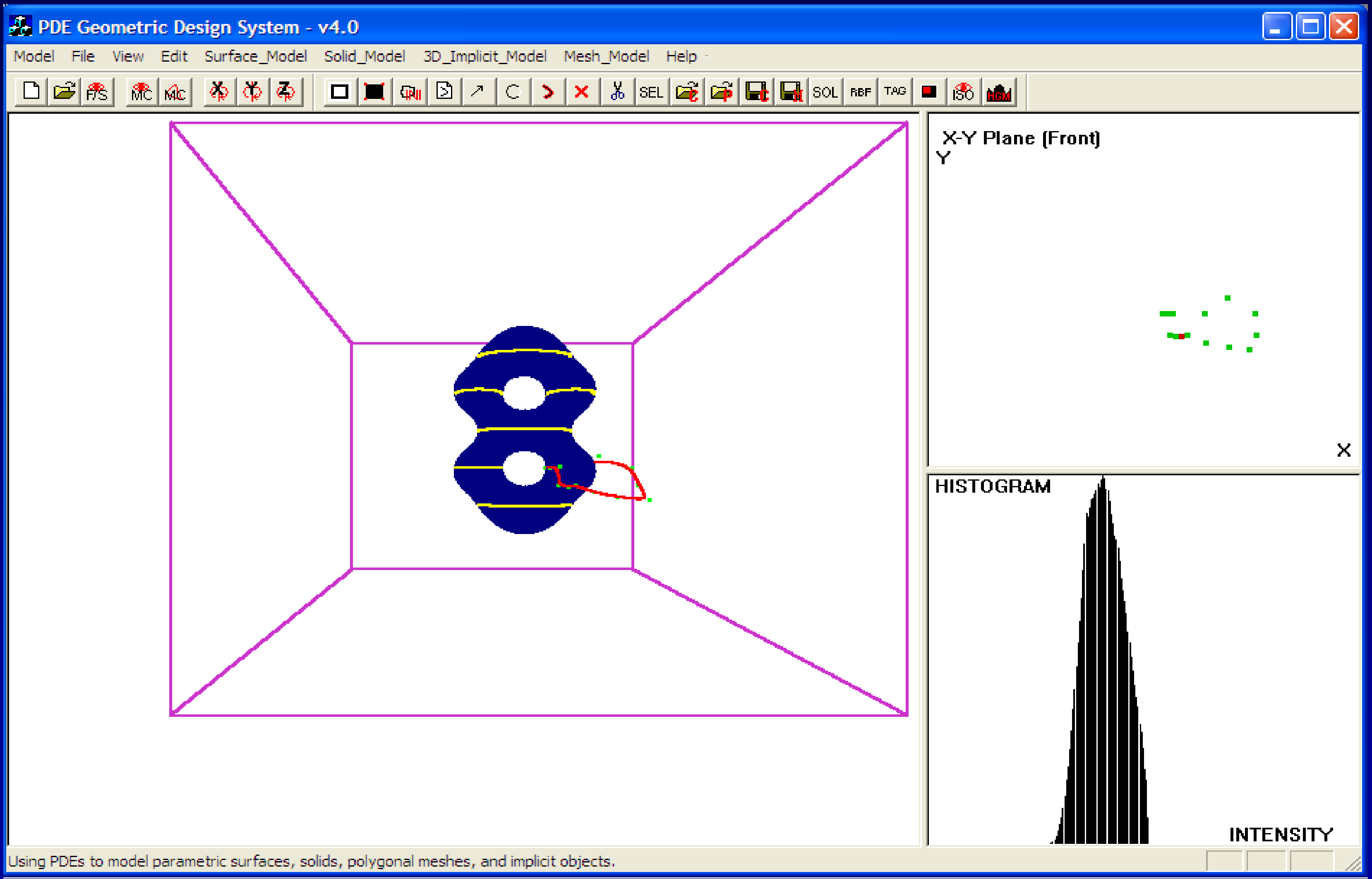
- Interpolation constraints:

$$h_i = f(\mathbf{c}_i) = \sum_{j=1}^{k} w_j \phi(\mathbf{c}_i - \mathbf{c}_j) + P(\mathbf{c}_i)$$

- Linear equation system:

$$\begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1k} & 1 & c_1^x & c_1^y & c_1^z \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2k} & 1 & c_2^x & c_2^y & c_2^z \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi_{k1} & \phi_{k2} & \cdots & \phi_{kk} & 1 & c_k^x & c_k^y & c_k^z \\ 1 & 1 & \cdots & 1 & 0 & 0 & 0 & 0 \\ c_1^x & c_2^x & \cdots & c_k^x & 0 & 0 & 0 & 0 \\ c_1^y & c_2^y & \cdots & c_k^y & 0 & 0 & 0 & 0 \\ c_1^z & c_2^z & \cdots & c_k^z & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_k \\ p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_k \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

# Interface of Implicit PDE Model

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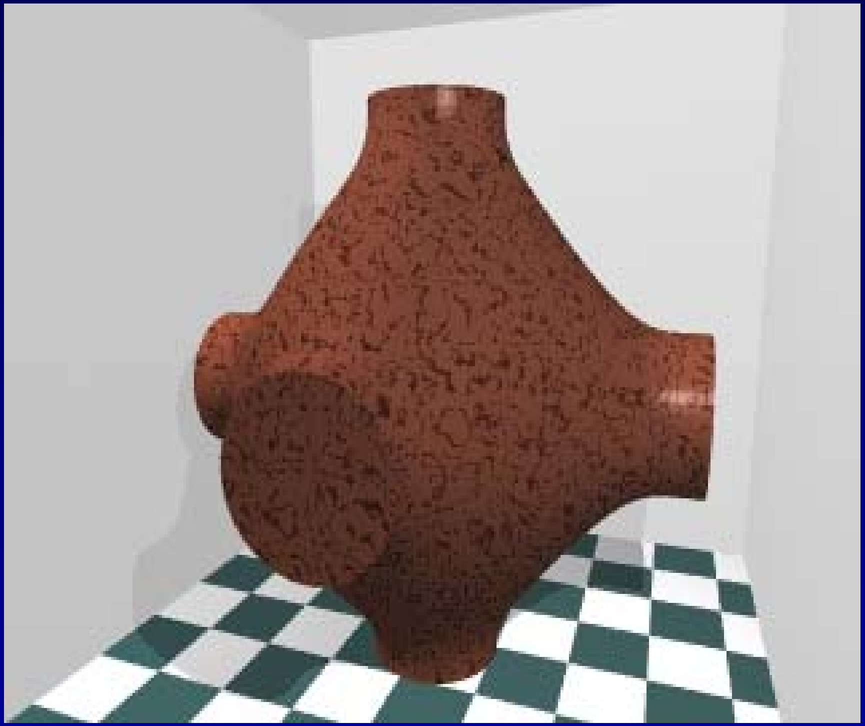
# Blending Coefficient Manipulation



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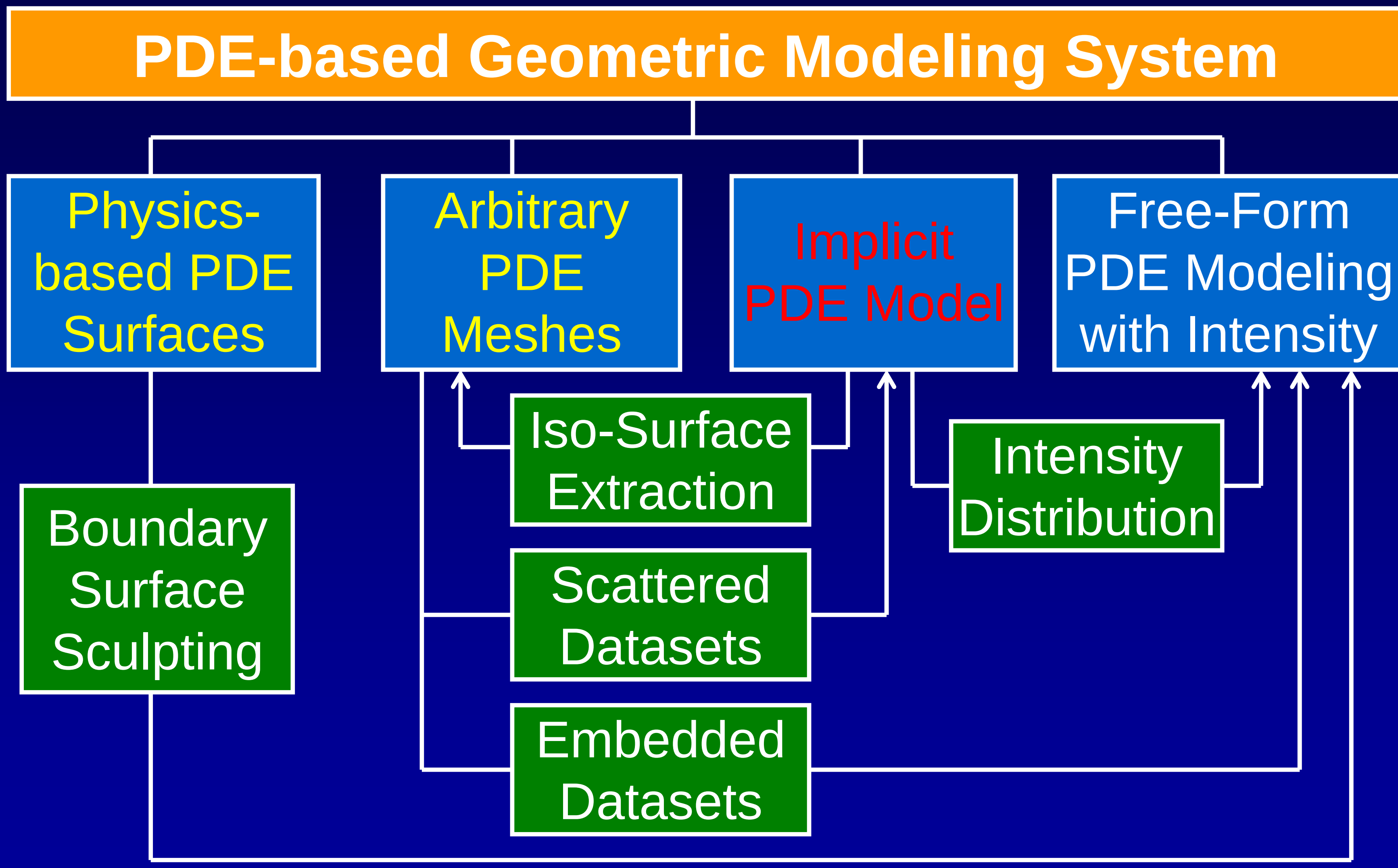
# System Outline

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# Implicit PDE Model

- Implicit PDE formulations:

$$\left( a^2(x,y,z)\frac{\partial^2}{\partial x^2} + b^2(x,y,z)\frac{\partial^2}{\partial y^2} + c^2(x,y,z)\frac{\partial^2}{\partial z^2} \right)^2 d(x,y,z) = 0$$

$$\left( a^2(x,y,z)\frac{\partial^2}{\partial x^2} + b^2(x,y,z)\frac{\partial^2}{\partial y^2} + c^2(x,y,z)\frac{\partial^2}{\partial z^2} \right) d(x,y,z) = 0$$

- Generalized boundary constraints
  - Initial guess
    - RBF (Radial Basis Function) interpolation
    - Distance field approximation
  - Smoothing

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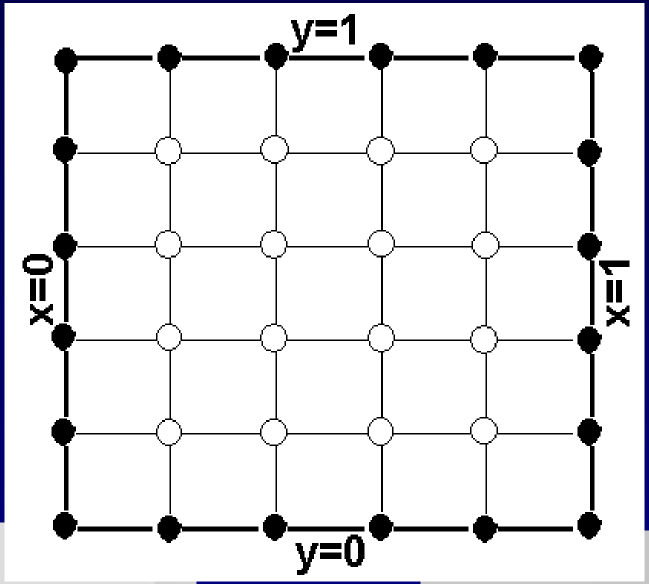
# Boundary Constraints for Implicit PDE

- Traditional boundary conditions (cross-sectional constraints)
- Boundary constraints for shape blending
- Arbitrary sketch curves
  - Initial guess: variational interpolation (RBF)
- Unorganized scattered data points
  - Initial guess: distance field approximation

# Traditional Boundary Conditions




# Traditional Boundary Conditions

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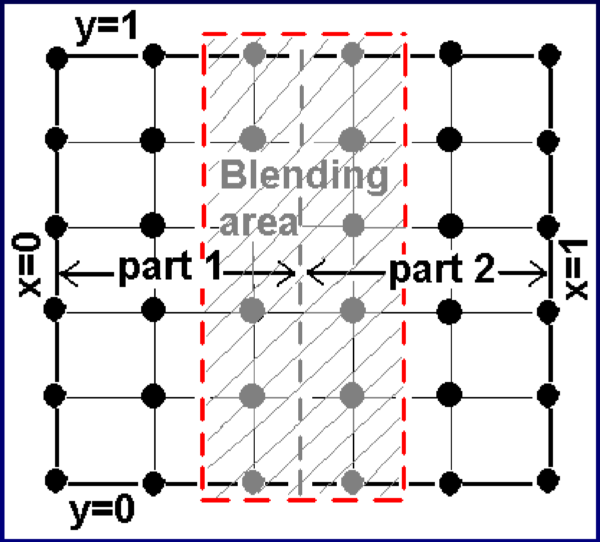
# Boundary Constraints for Shape Blending

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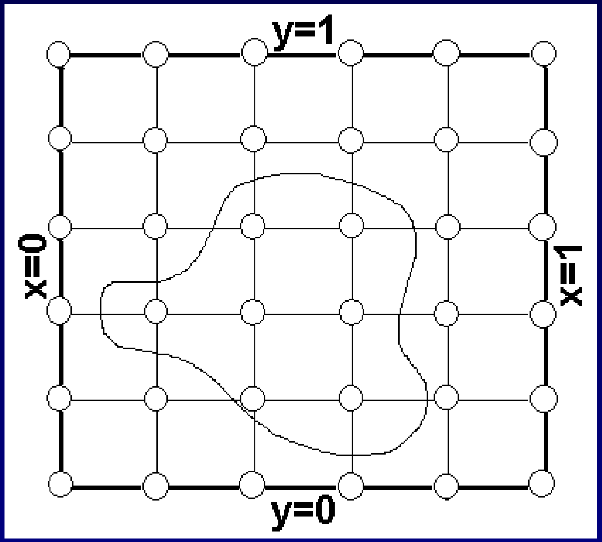
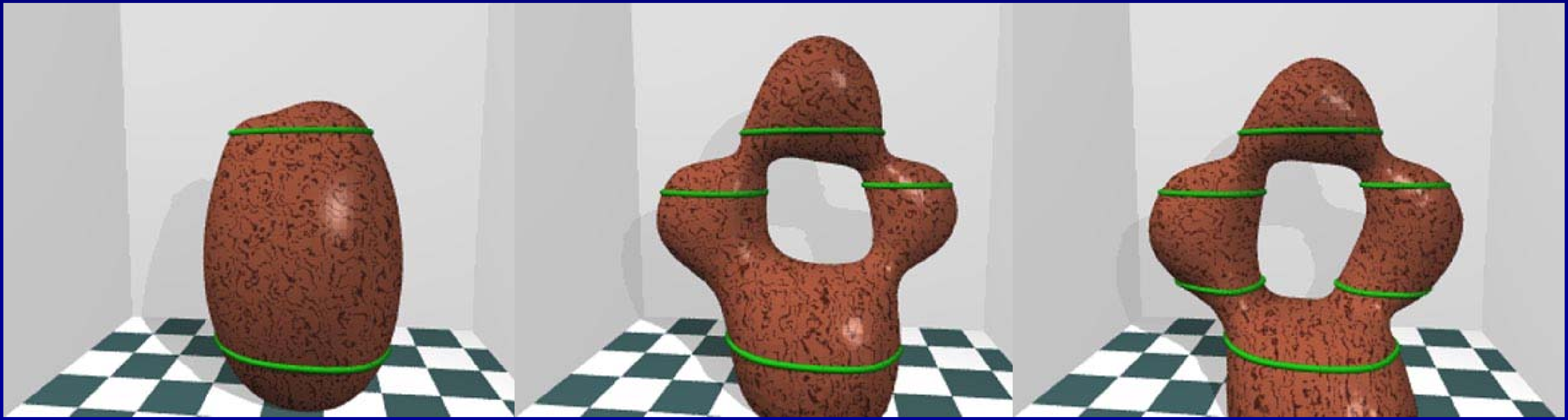
# Boundary Conditions of Sketch Curves



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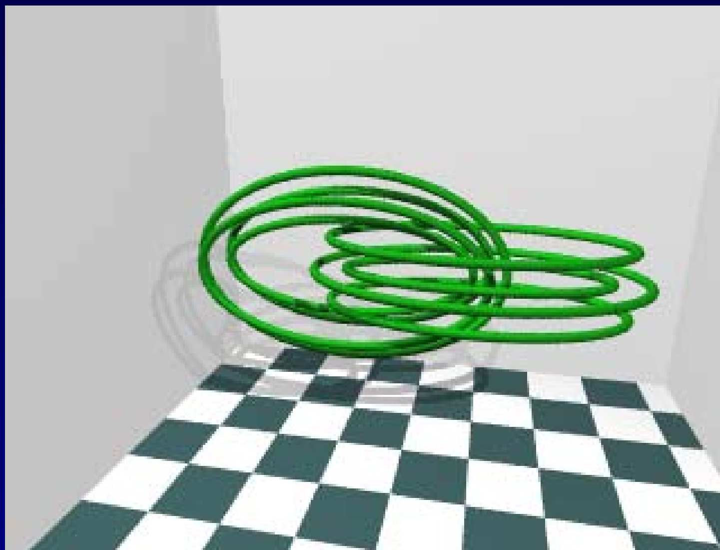
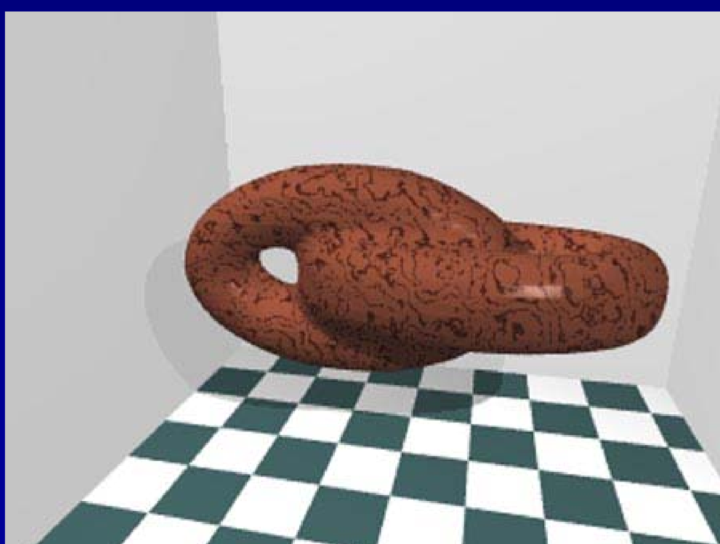
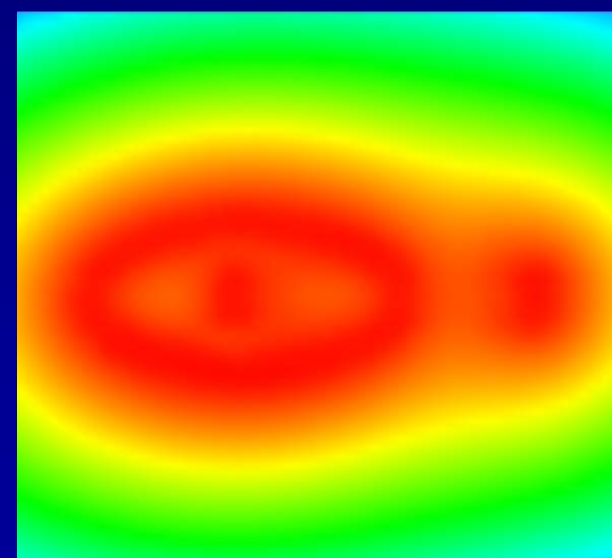
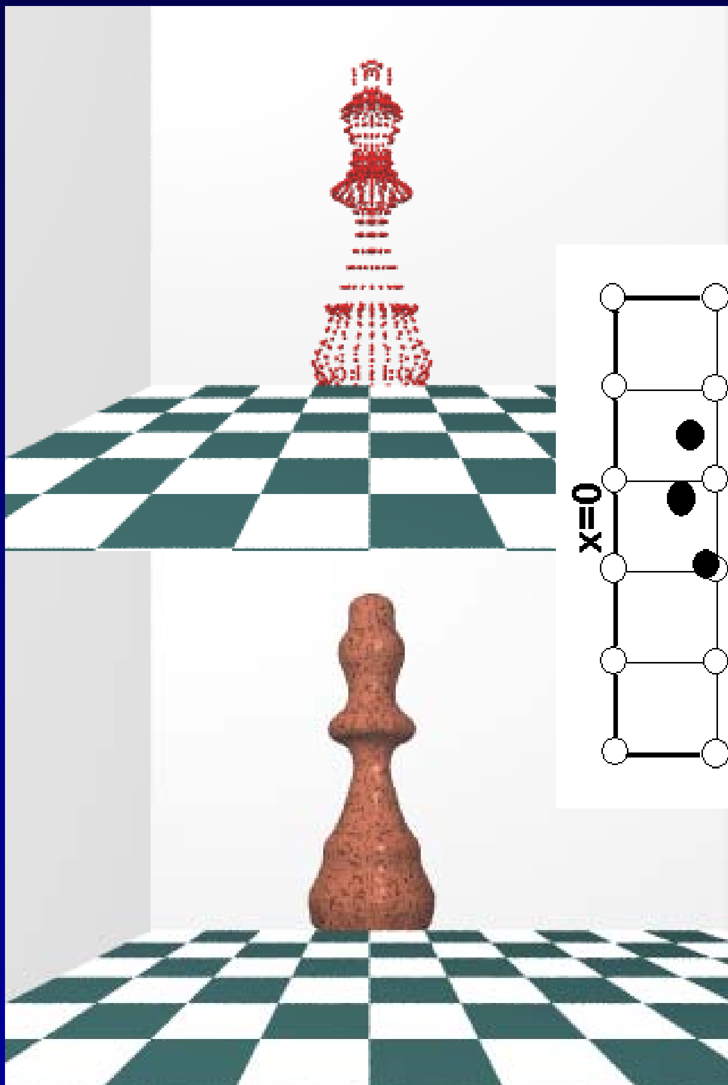
# Local RBF Method for Complex Model

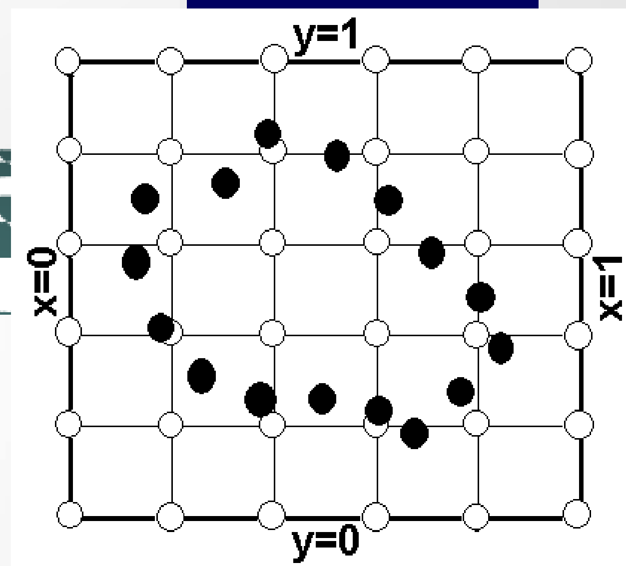


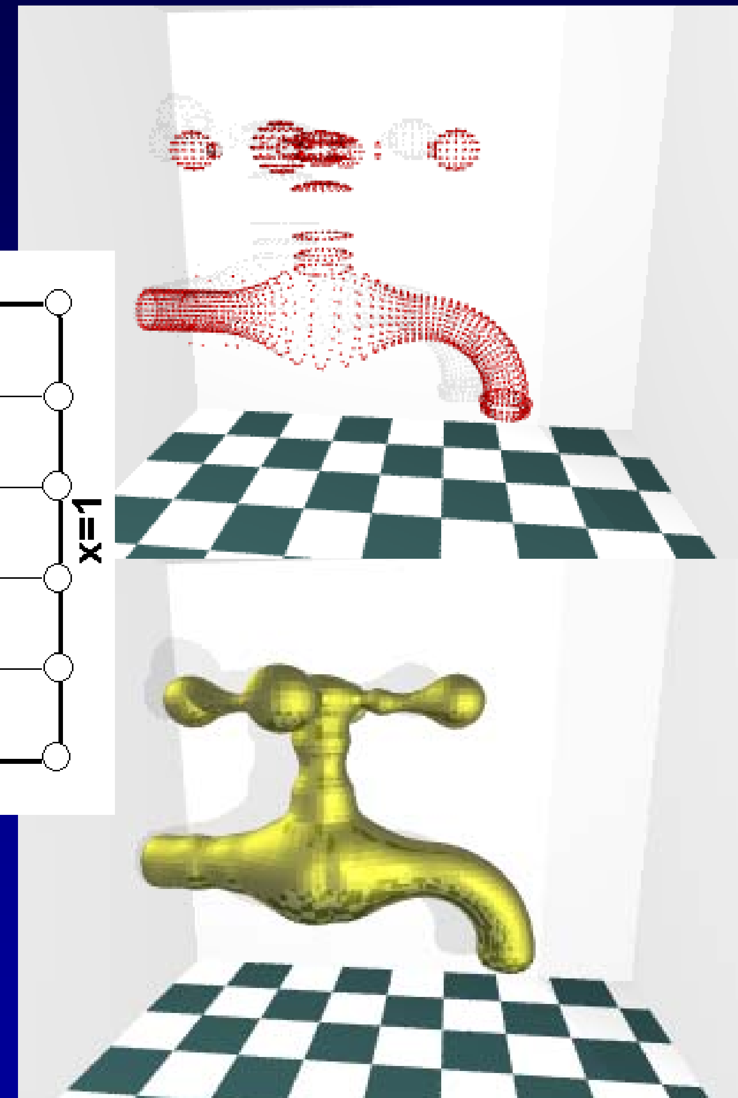




# Boundary Conditions of Scattered Points



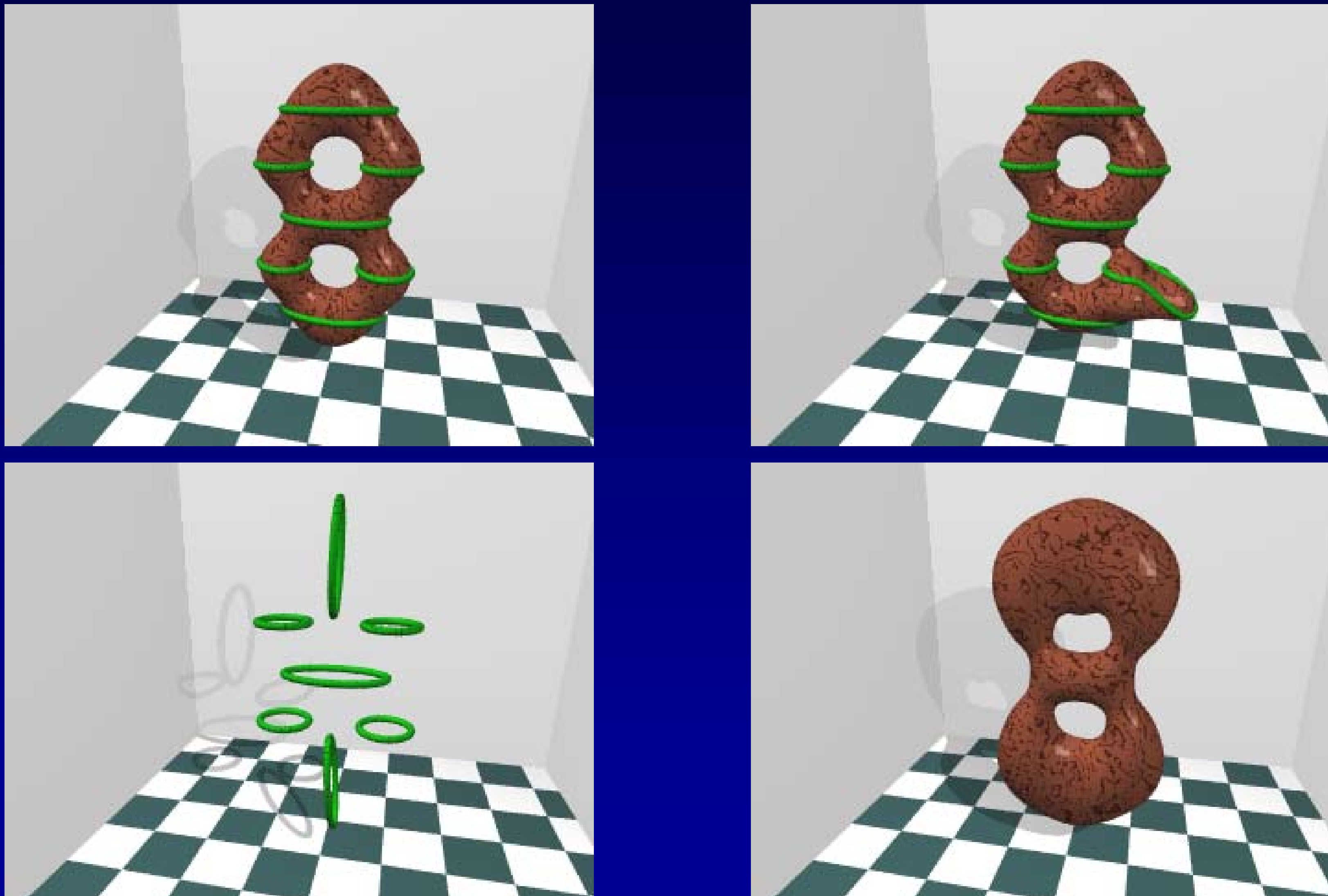


# Manipulation of Implicit PDE Objects

- Sketch curve sculpting
  - Shape, intensity, and gradient directions
- Blending control coefficient manipulation
- Direct manipulations
  - Iso-contour
  - Region intensity
  - CSG tools
  - Gradient sculpting
  - Curvature manipulation

# Sculpting of Sketch Curves



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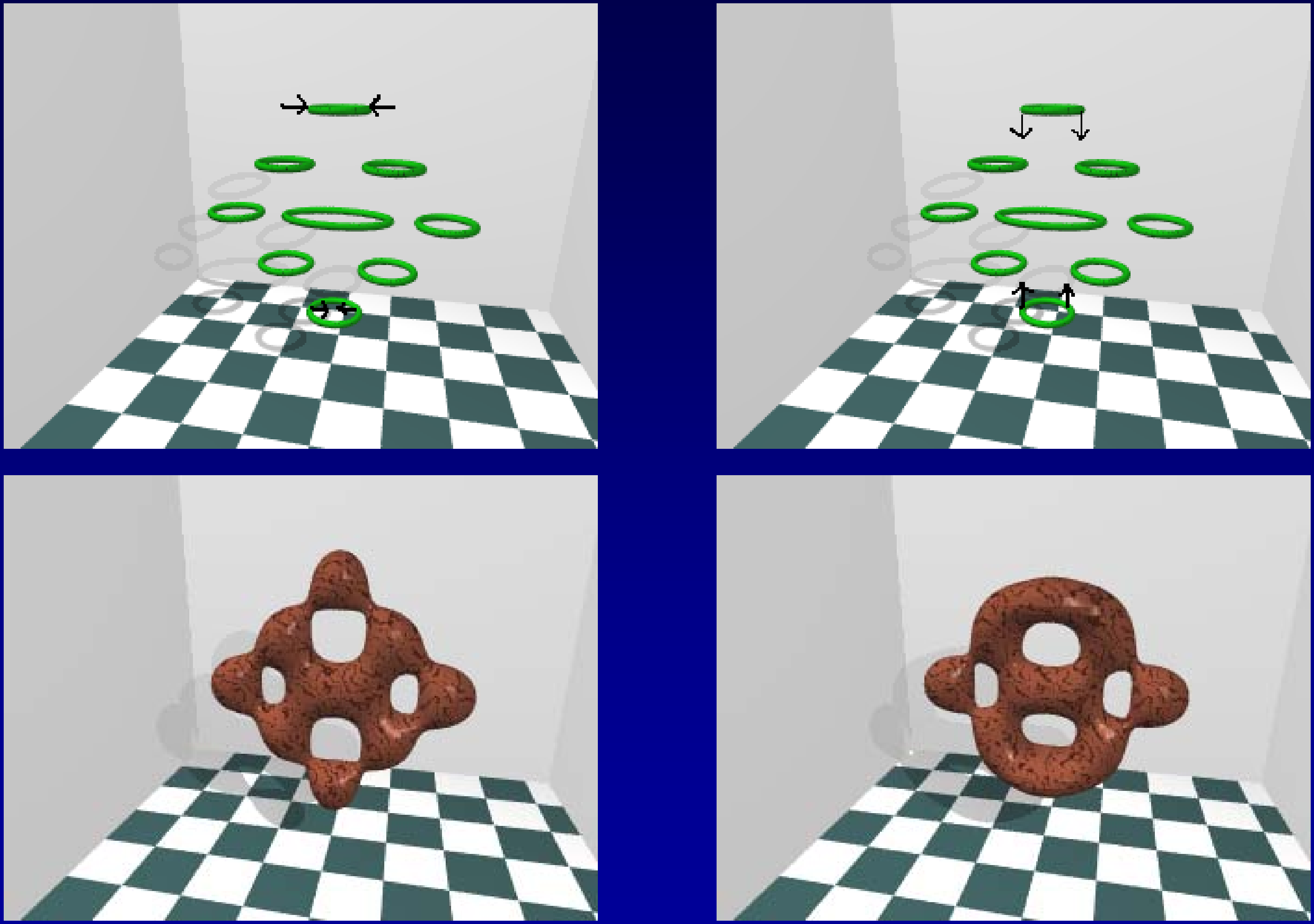
# Changing Gradient Directions



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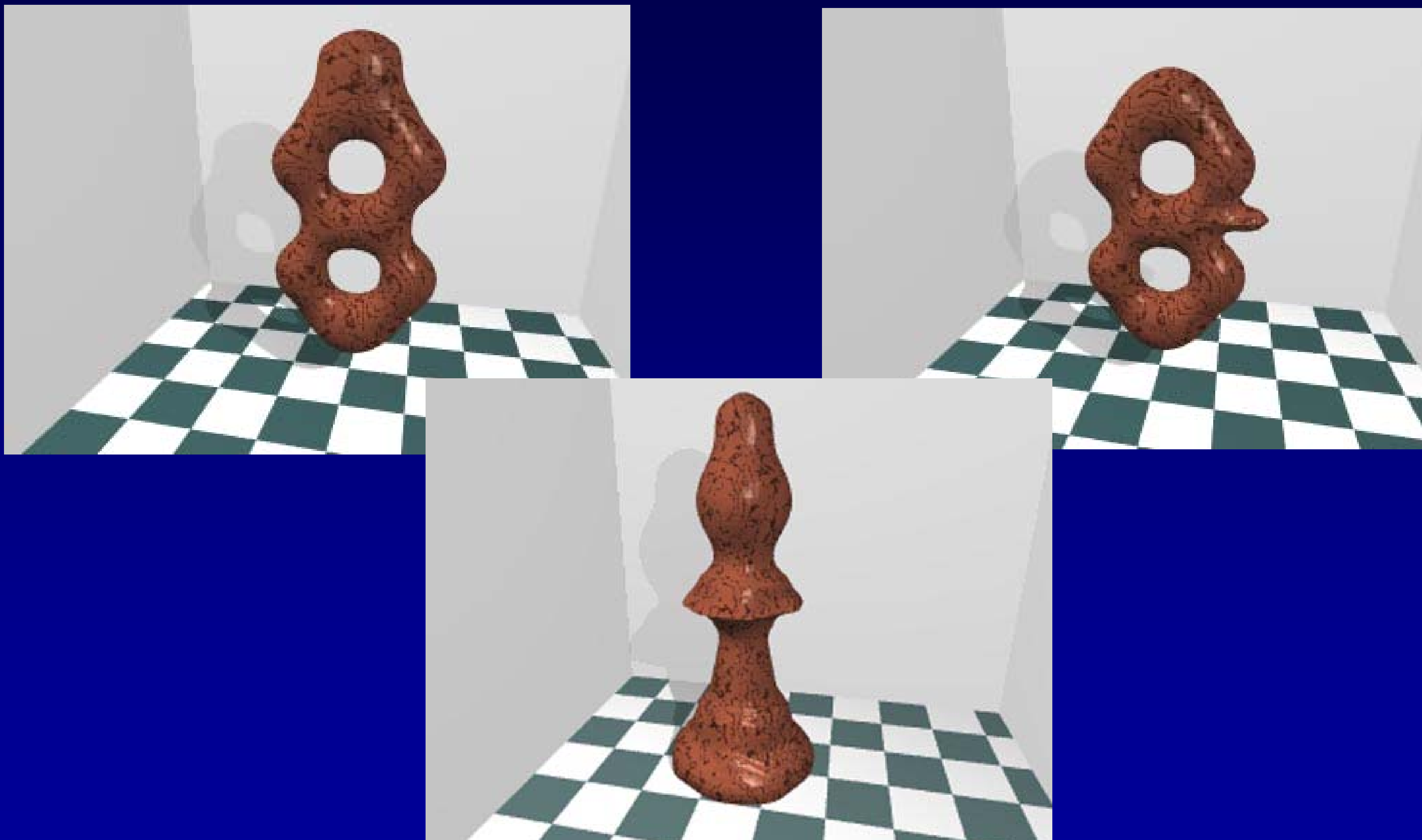
# Direct Intensity Manipulations



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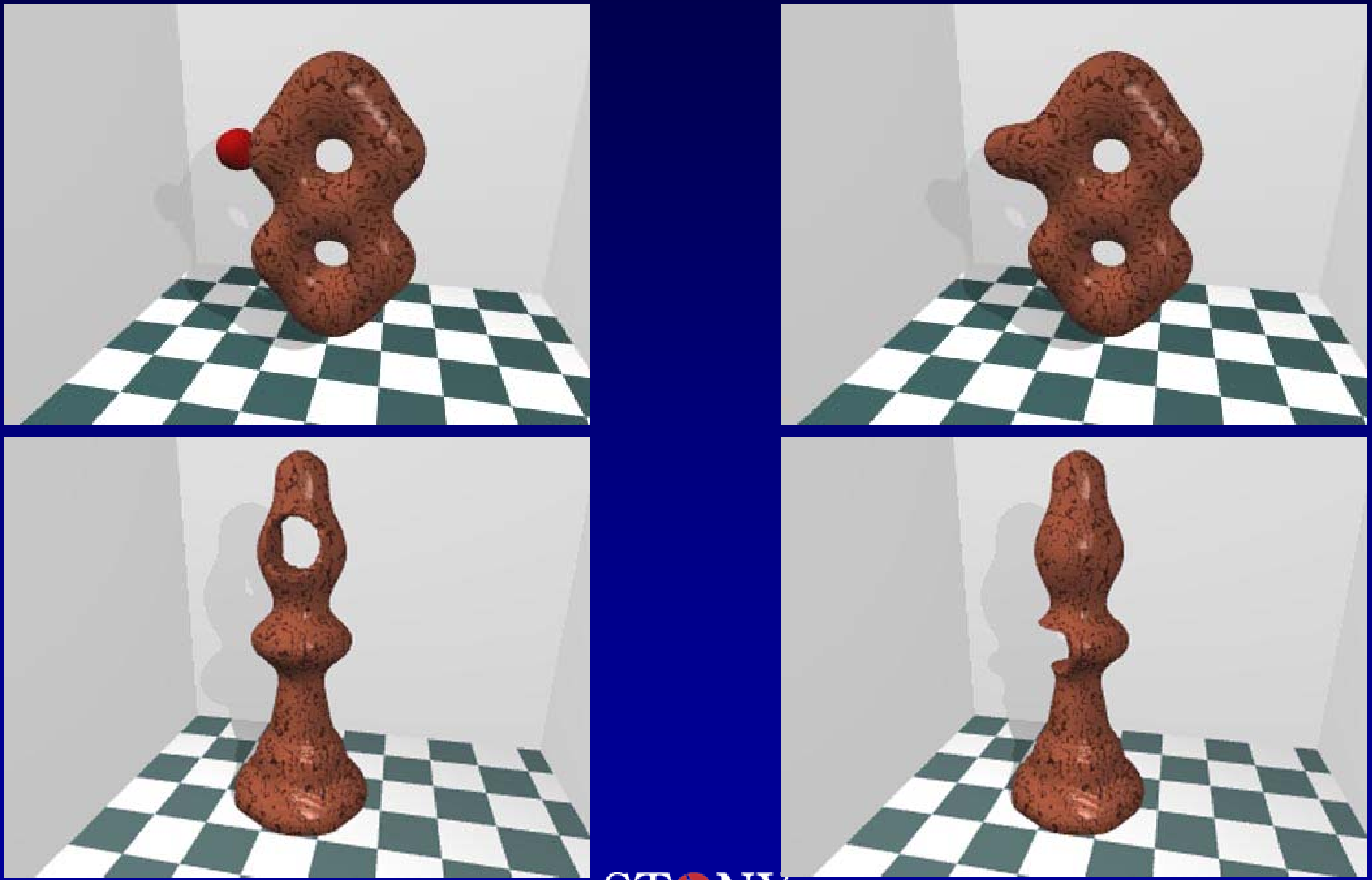
# Direct CSG Manipulations



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# Gradient and Curvature Approximation

- Gradient approximation

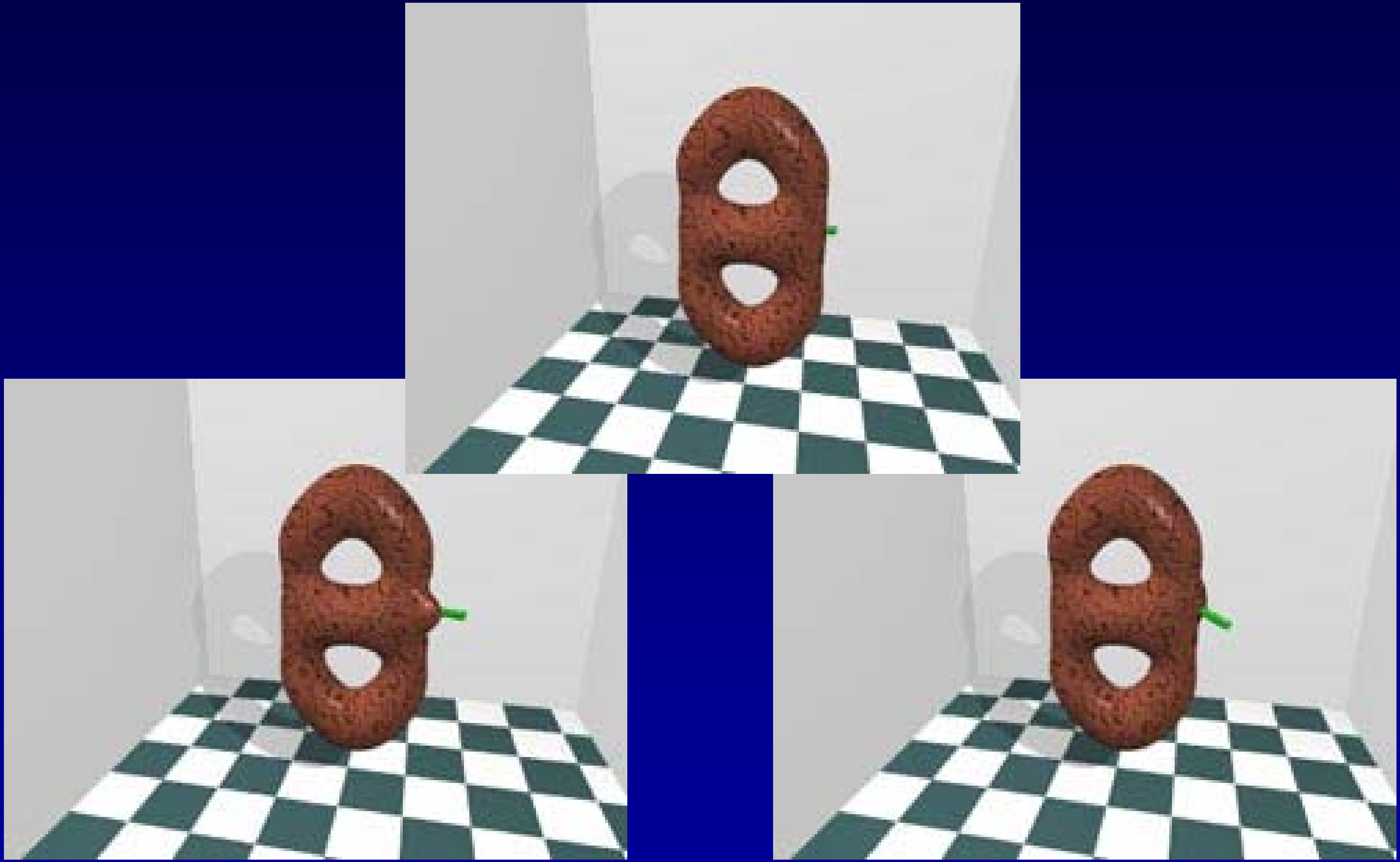
$$\nabla d(x,y,z) \approx \left(\frac{d_{i+1,j,k}-d_{i-1,j,k}}{2\Delta x}, \frac{d_{i,j+1,k}-d_{i,j-1,k}}{2\Delta y}, \frac{d_{i,j,k+1}-d_{i,j,k-1}}{2\Delta z}\right)$$

- Curvature approximation

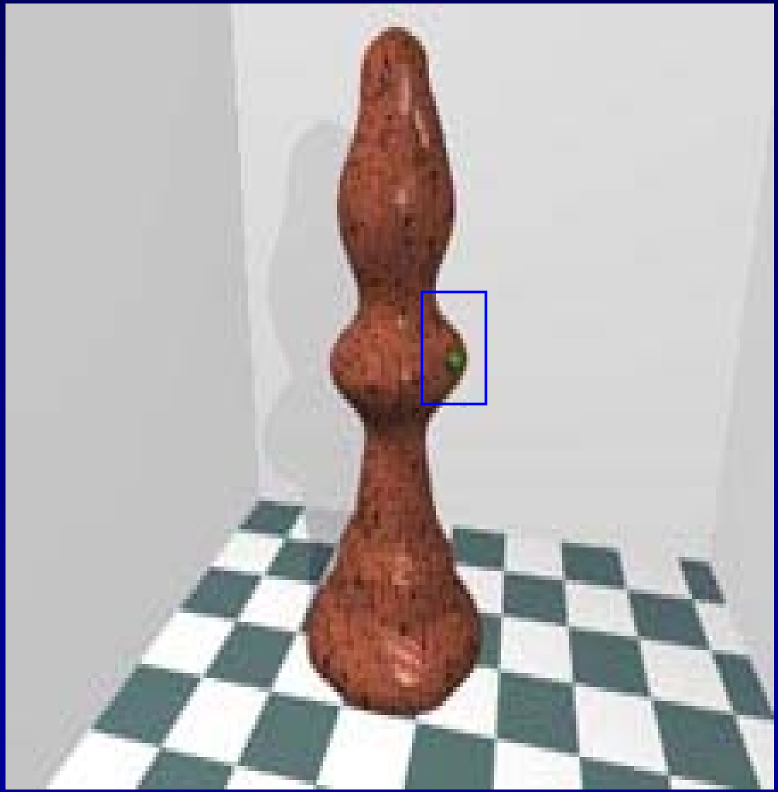
  Mean curvature:

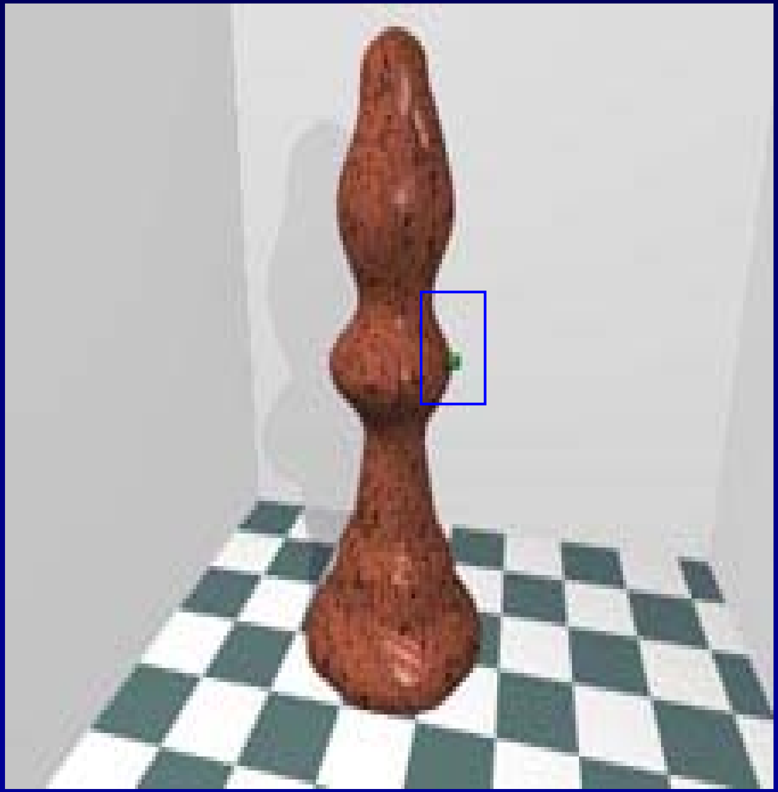
$$\nabla \cdot \nabla d(x,y,z)$$

# Gradient Manipulations



# Curvature Manipulations





# Summary of Implicit PDE Modeling

- General boundary constraints for shape design, reconstruction, blending, and recovery
- RBF method or distance field approximation for initial guess with generalized constraints
- Manipulation of implicit PDE objects
  - Sketch curve sculpting
  - Blending coefficient manipulation
  - Direct manipulation of implicit objects

# Outline

- Motivation and contributions
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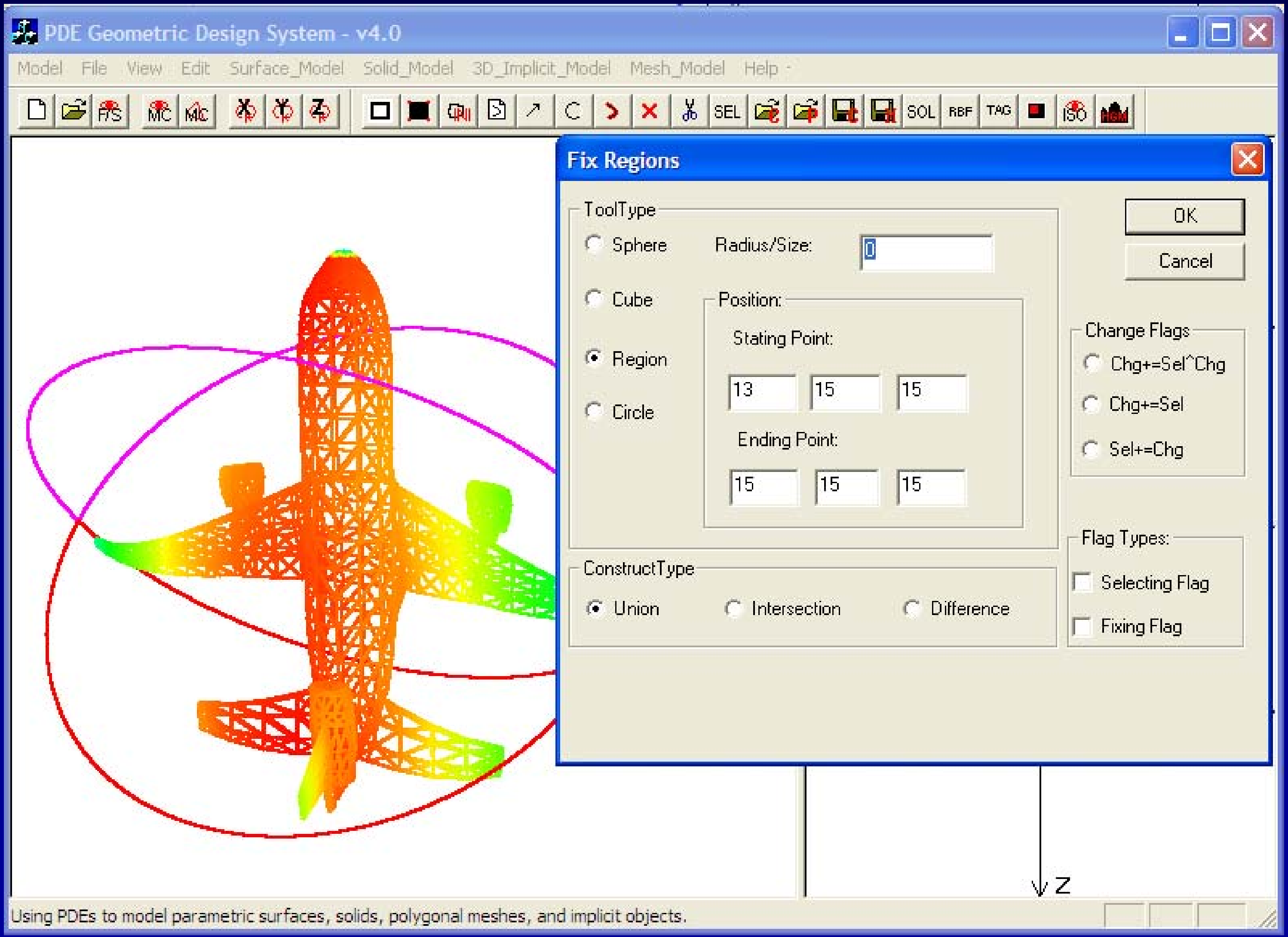
# Free-Form PDE Modeling and Deformation

- Formulations
  - Geometry
  - Intensity Integration
- Boundary constraints and manipulations
- Direct sculpting of PDE solid geometry
- Intensity-based free-form modeling and deformation

# Interface of PDE Solids

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# Numerical Techniques

- Spectral approximation
- Finite-element method (FEM)
- Finite-difference method (FDM)
- Solving linear equation system
  - Iterative method
  - Multi-grid improvement

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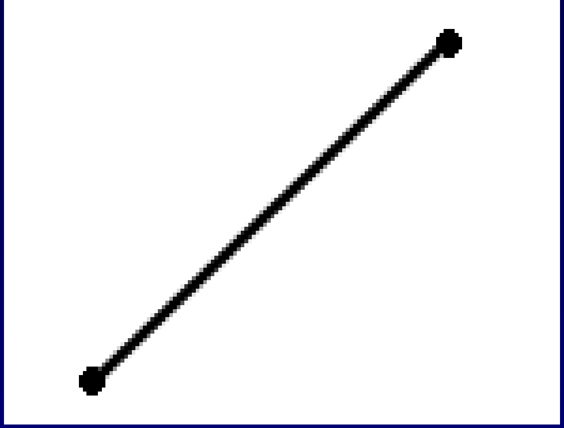
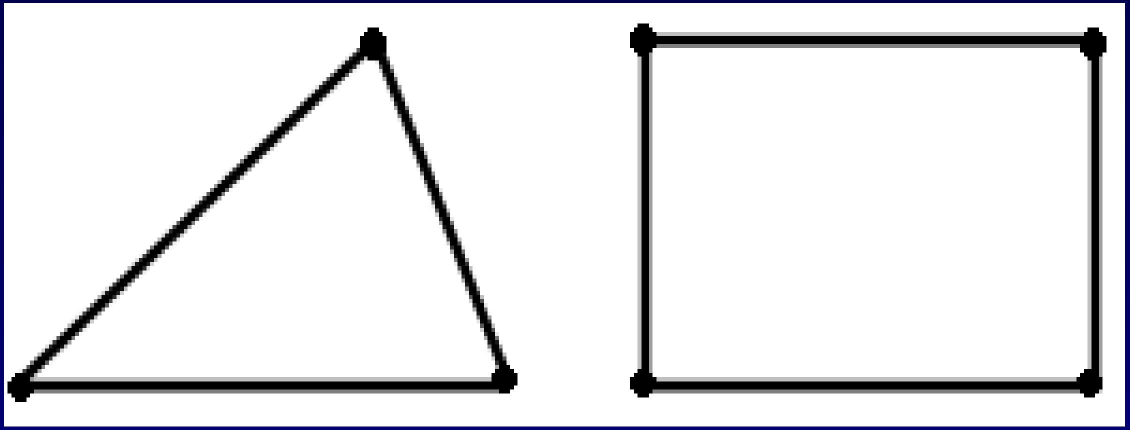
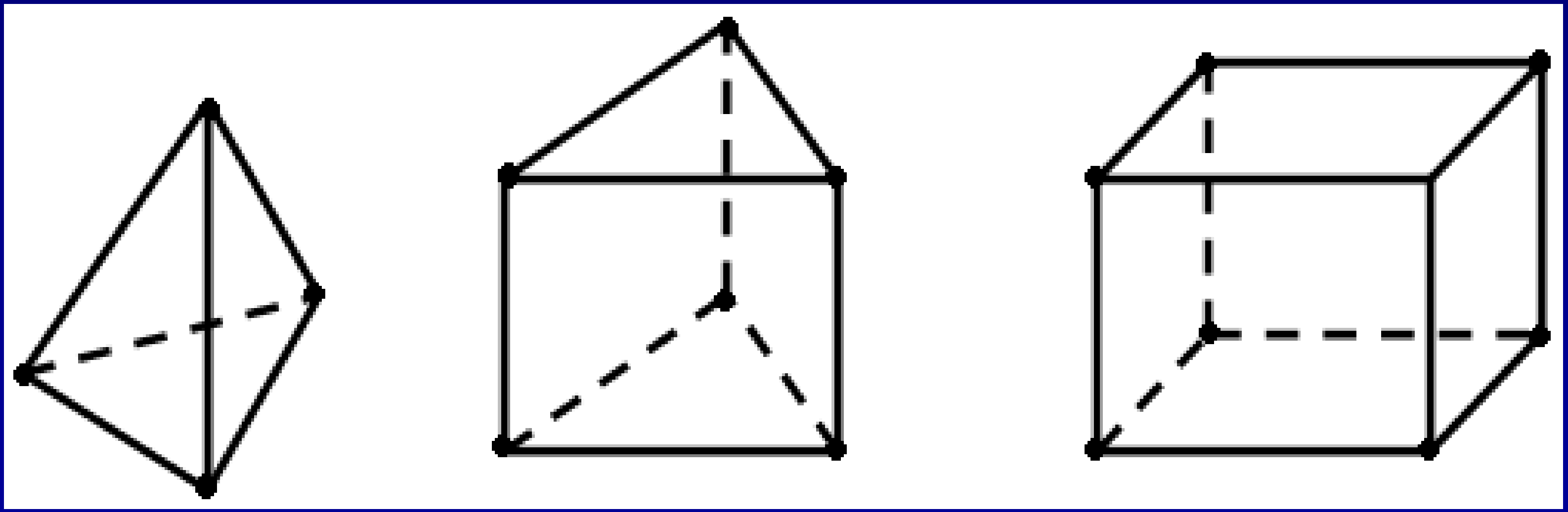
# Finite Element Method

- Approximate the infinite problem by interpolation functions over sub-domains
  - Discretize the domain into sub-domains
  - Select the interpolation functions
  - Formulate the system of equations
  - Solve the equations for coefficients of the interpolation to approximate the solution

# Typical Finite Elements







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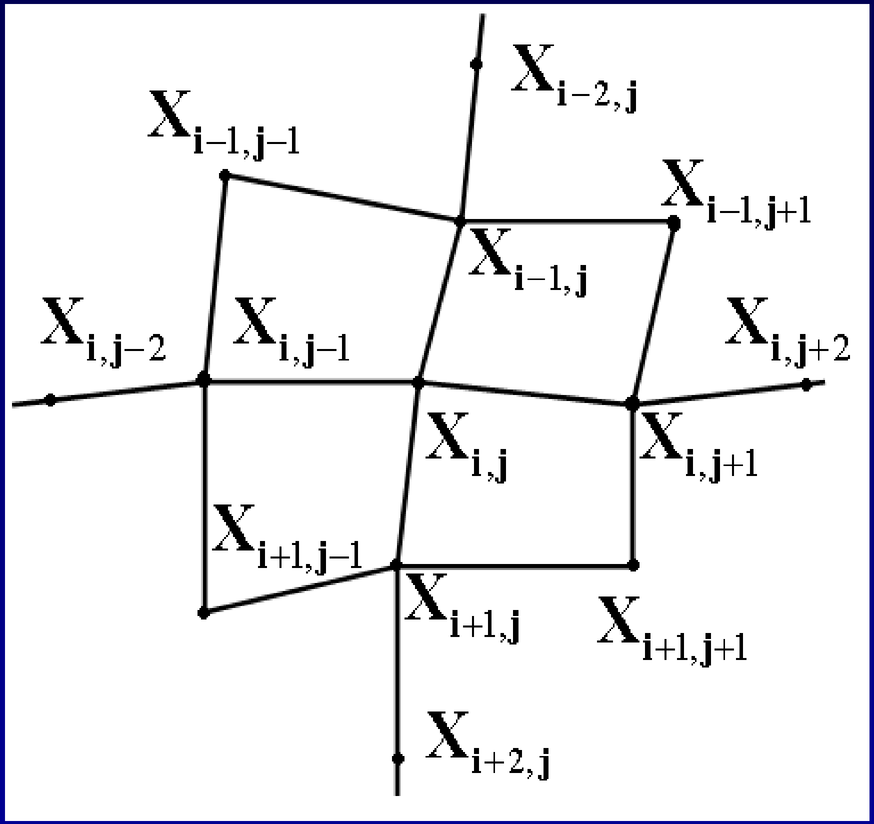
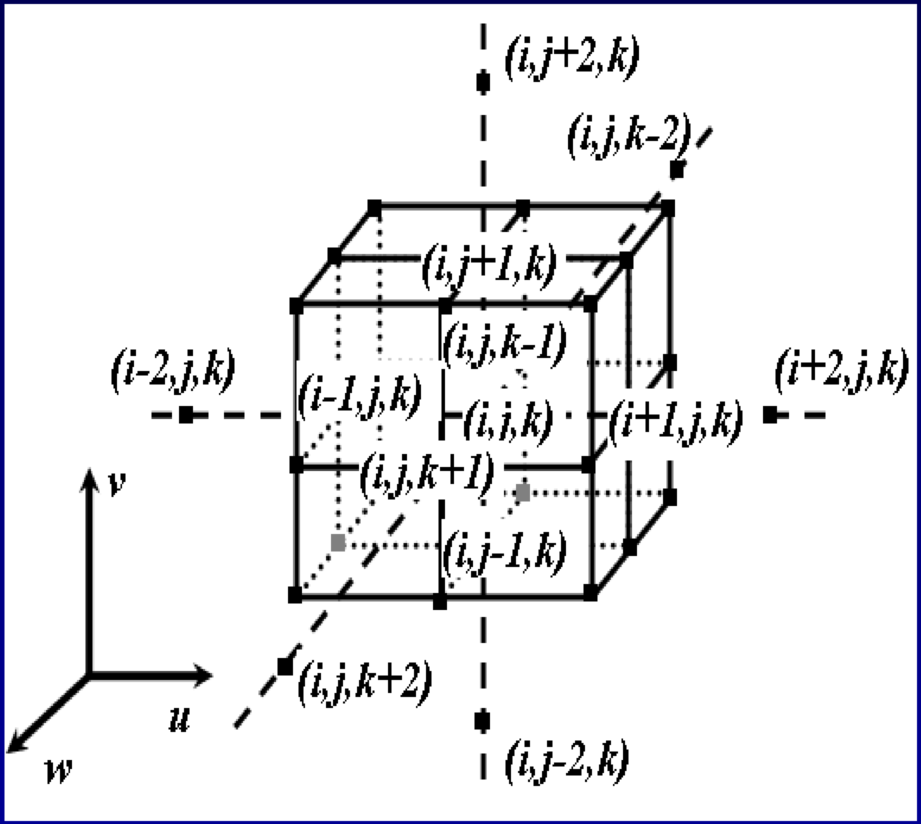
# Finite Difference Method

- Divides the working space into discrete grids
- Samples the PDE at grid points with discretized approximations
- Forms a set of algebraic equations
- Uses iterative techniques and multi-grid algorithm to improve the performance

# Working Space Discretization



# Difference Equation Approximation

$$\frac{\partial^4 f_{i,j}}{\partial u^4} = \frac{f_{i-2,j} + f_{i+2,j} - 4f_{i-1,j} - 4f_{i+1,j} + 6f_{i,j}}{\Delta u^4},$$

$$\frac{\partial^4 f_{i,j}}{\partial v^4} = \frac{f_{i,j-2} + f_{i,j+2} - 4f_{i,j-1} - 4f_{i,j+1} + 6f_{i,j}}{\Delta v^4},$$

$$\frac{\partial^4 f_{i,j}}{\partial u^2 v^2} = \frac{f_{i-1,j-1} + f_{i+1,j-1} + f_{i-1,j+1} + f_{i+1,j+1} - 2f_{i-1,j} - 2f_{i+1,j} - 2f_{i,j-1} - 2f_{i,j+1} + 4f_{i,j}}{\Delta u^2 \Delta v^2}$$

# Finite Difference Method

- Simple and easy for implementation
- Allows flexible and generalized boundary conditions and additional constraints
- Enables local control and direct manipulation
- Guarantees an approximate solution
- Time performance depends on resolution of discretization of working space

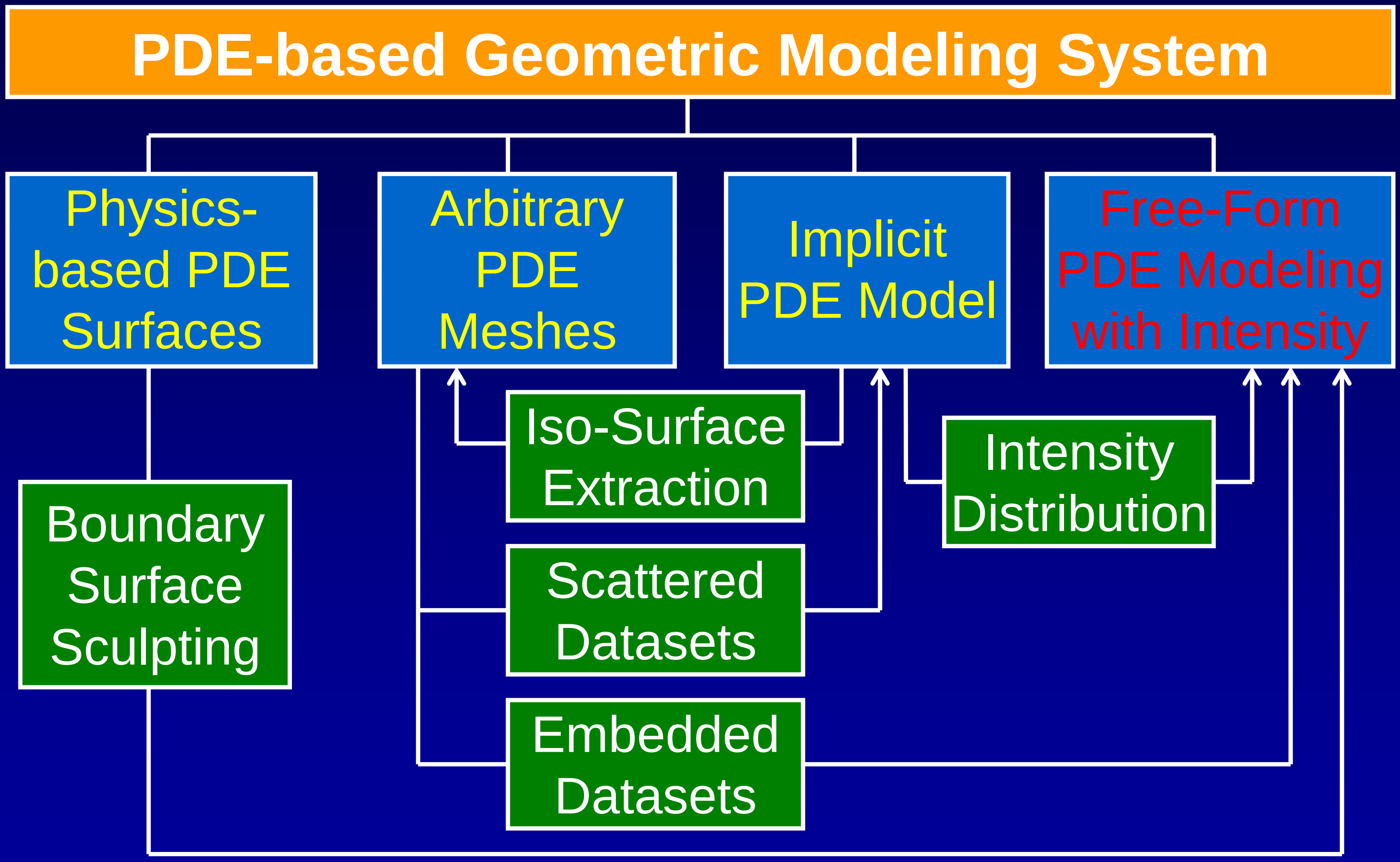
# Solving Linear Equations

- Iterative methods
  - Gauss-Seidel iteration

    $$\mathbf{AX} = \mathbf{b}, \mathbf{A} = \mathbf{A}_d - \mathbf{A}_r, \mathbf{A}_d\mathbf{X} = \mathbf{A}_r\mathbf{X} + \mathbf{b}$$

    $$\mathbf{A}_d\mathbf{X}^{(n)} = \mathbf{A}_r\mathbf{X}^{(n-1)} + \mathbf{b}$$
  - SOR iteration
    - Difference between approximation and the real solution
- Multi-grid method improvement
  - Starting from coarsest grids, linear interpolating the coarse solution to get initial guess of finer resolution

# System Outline

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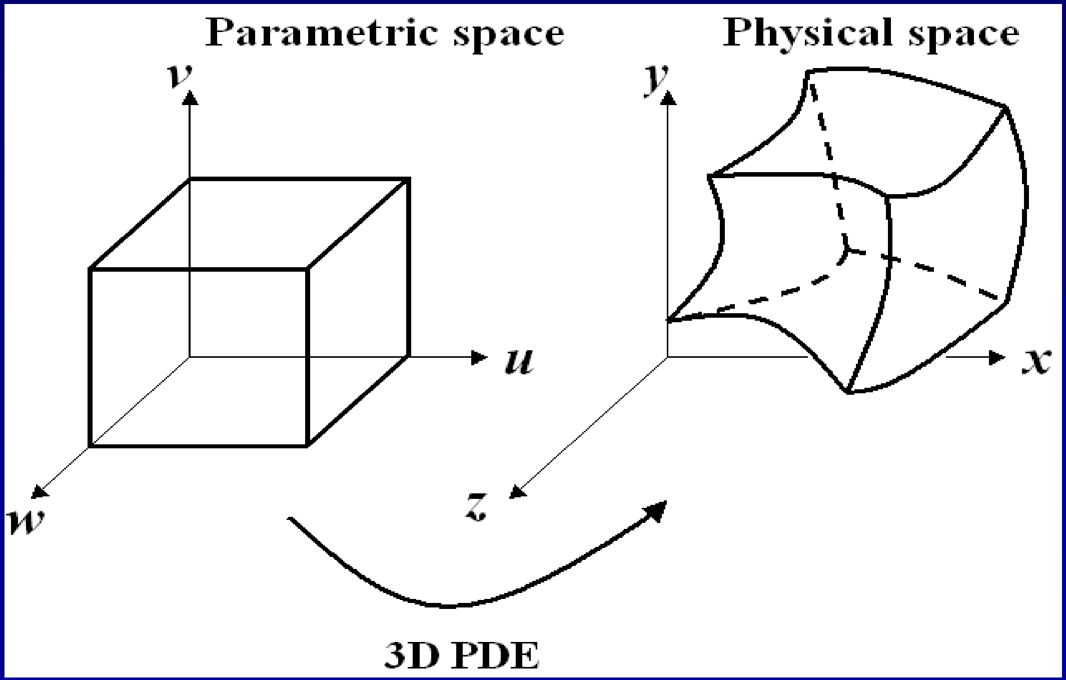
# Free-Form PDE Solid Geometry

- PDE formulation:

$$\left(a^2(u,v,w)\frac{\partial^2}{\partial u^2}+b^2(u,v,w)\frac{\partial^2}{\partial v^2}+c^2(u,v,w)\frac{\partial^2}{\partial w^2}\right)^2\mathbf{X}(u,v,w)=\mathbf{0}$$

- Free-form deformation for explicit model:

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# Free-Form PDE Solid Geometry

- Boundary conditions:
  - Surfaces:

$$\mathbf{X}(0,v,w)=\mathbf{U}_0(v,w),\ \mathbf{X}(1,v,w)=\mathbf{U}_1(v,w),$$
$$\mathbf{X}(u,0,w)=\mathbf{V}_0(u,w),\ \mathbf{X}(u,1,w)=\mathbf{V}_1(u,w),$$
$$\mathbf{X}(u,v,0)=\mathbf{W}_0(u,v),\ \mathbf{X}(u,v,1)=\mathbf{W}_1(u,v)$$

  - Curve network:

$$\mathbf{X}(u,v_i,w_j)=\mathbf{U}_{ij}(u),\ v_i\in\{0,1\}\text{ or }w_j\in\{0,1\};$$
$$\mathbf{X}(u_k,v,w_l)=\mathbf{V}_{kl}(v),\ u_k\in\{0,1\}\text{ or }w_l\in\{0,1\};$$
$$\mathbf{X}(u_r,v_s,w)=\mathbf{W}_{rs}(w),\ u_r\in\{0,1\}\text{ or }v_s\in\{0,1\}$$

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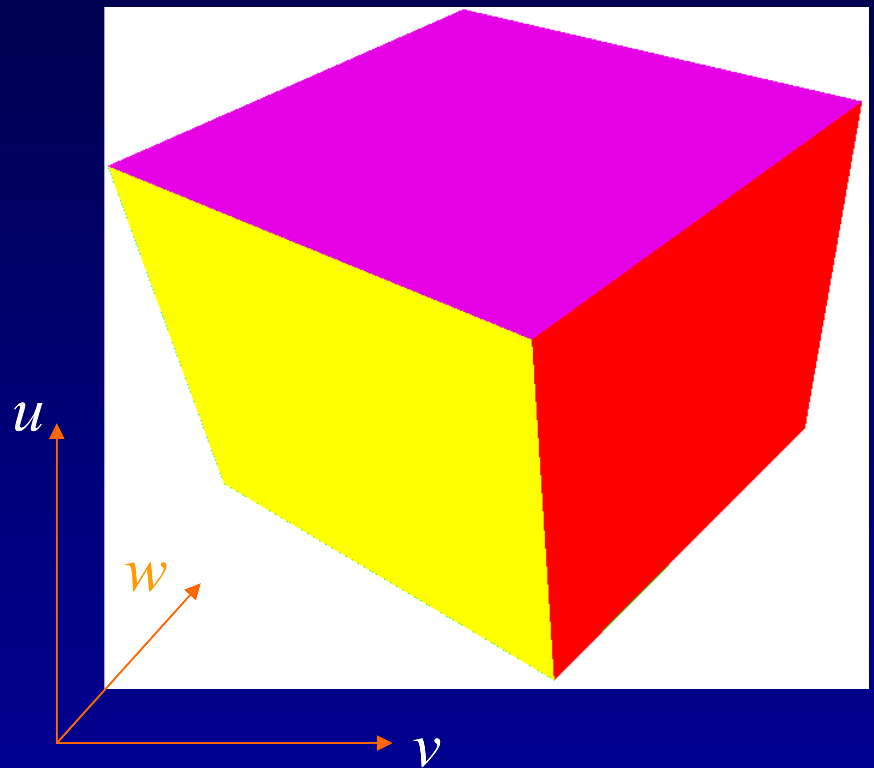
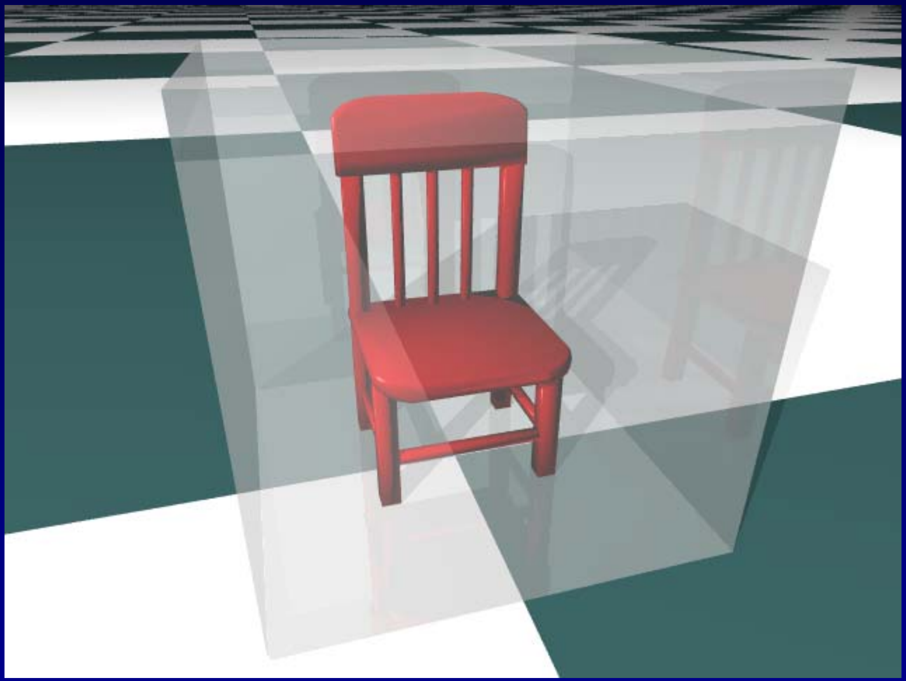
# PDE Solid from Boundary Surfaces



Boundary surfaces

Corresponding PDE solid

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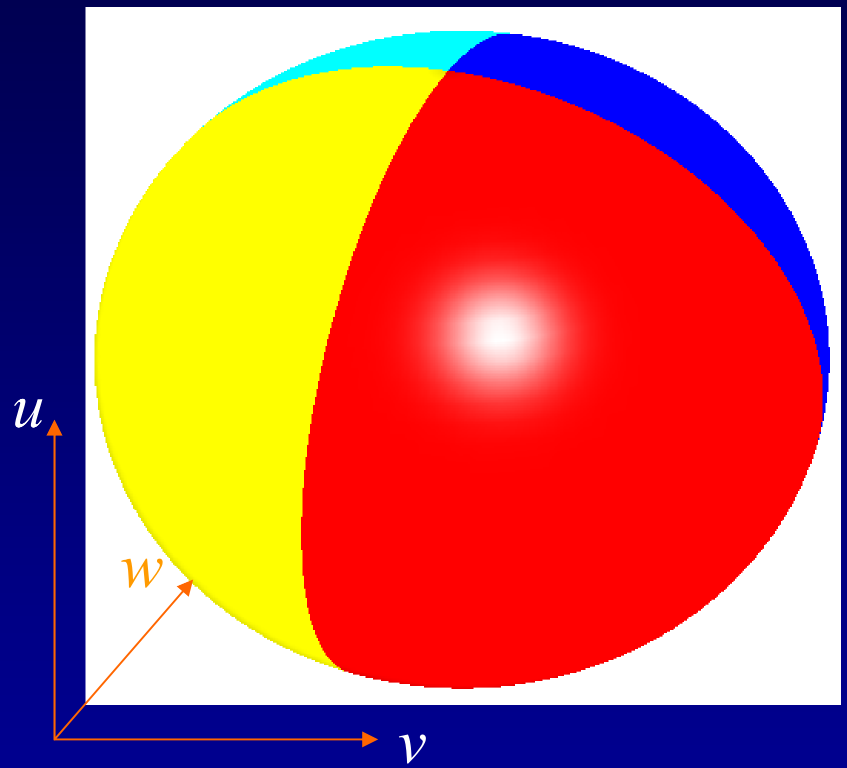
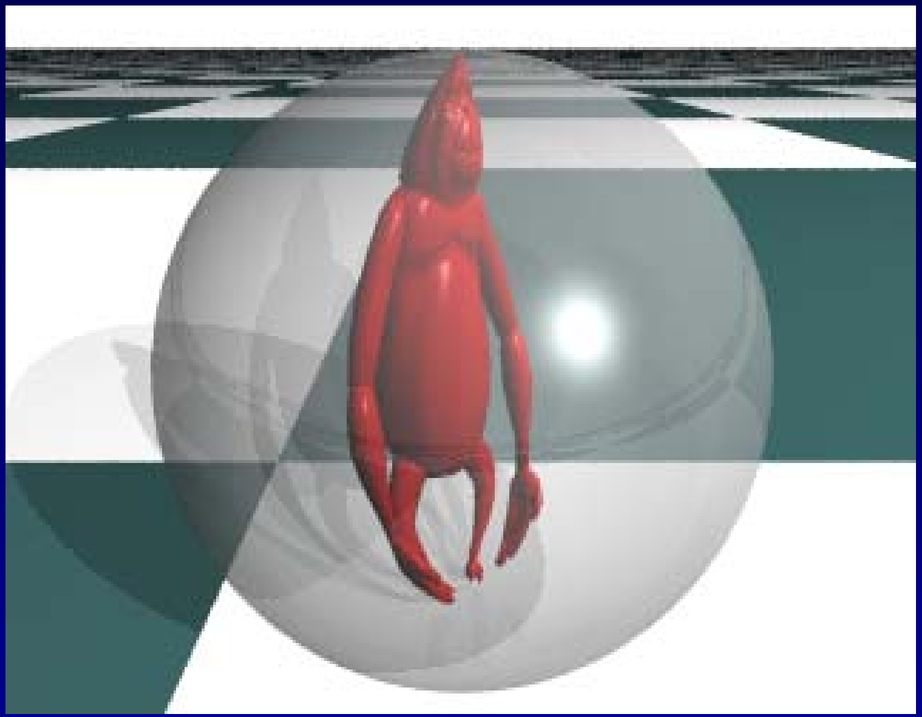
# PDE Solid from Boundary Surfaces

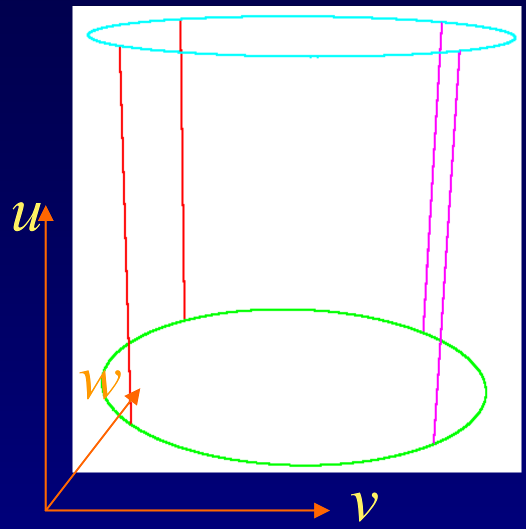


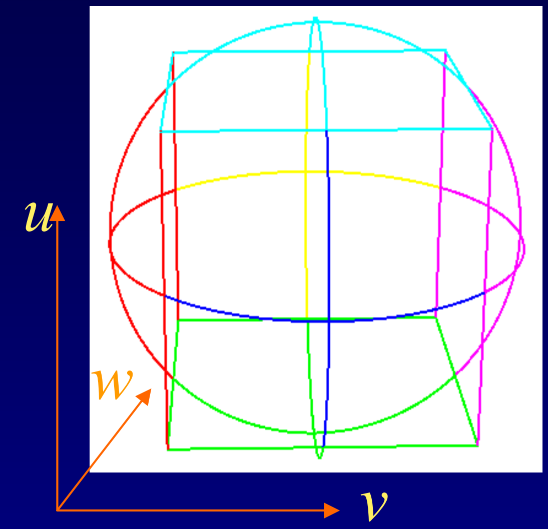

Boundary surfaces

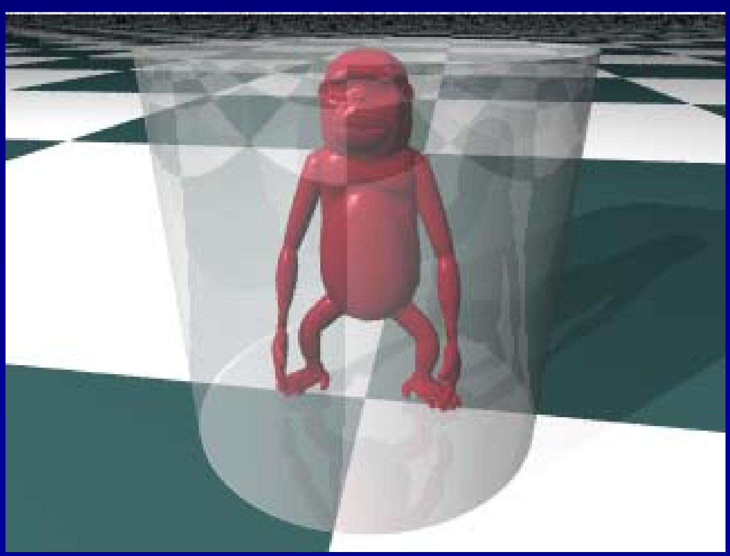


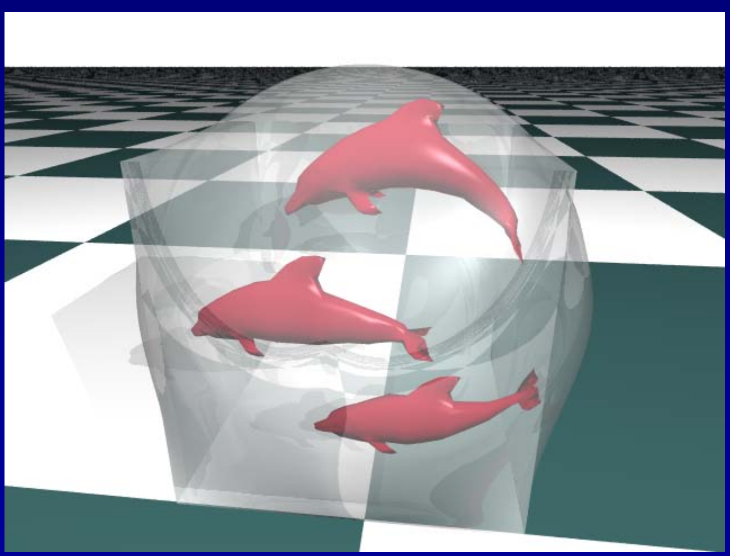
Corresponding PDE solid

# PDE Solids from Boundary Curves





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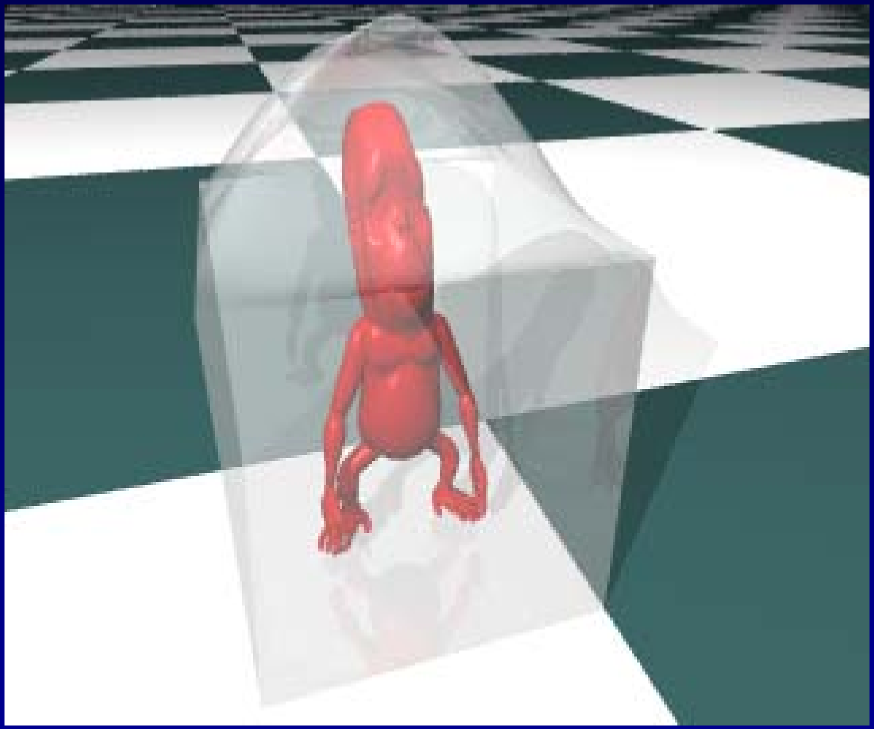
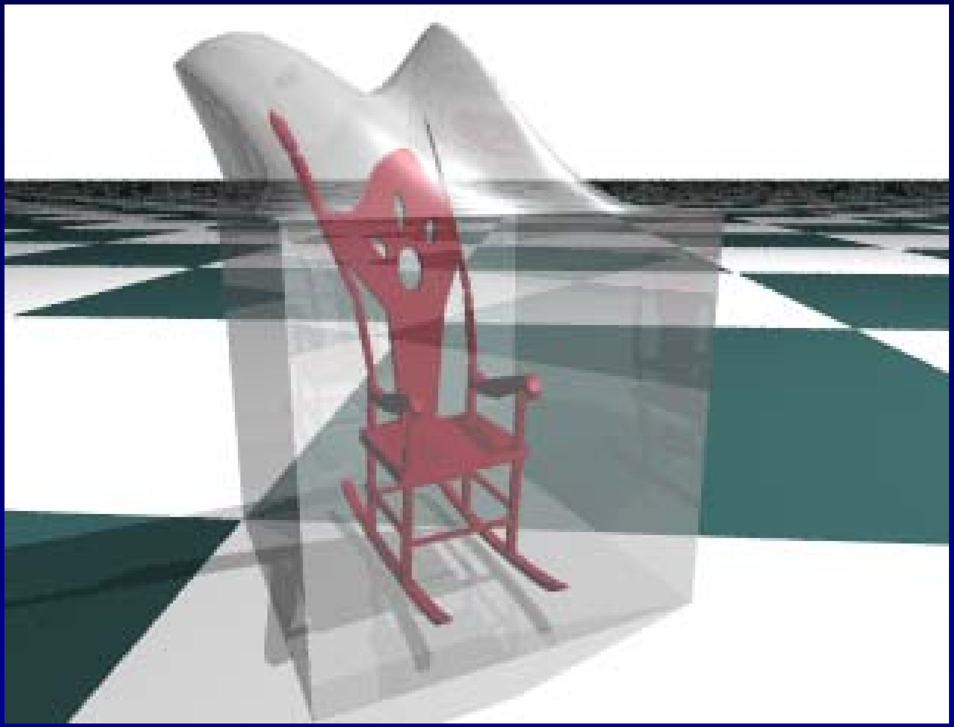
# Boundary Surface Sculpting





Curve editing on the boundary surface of $u$=1

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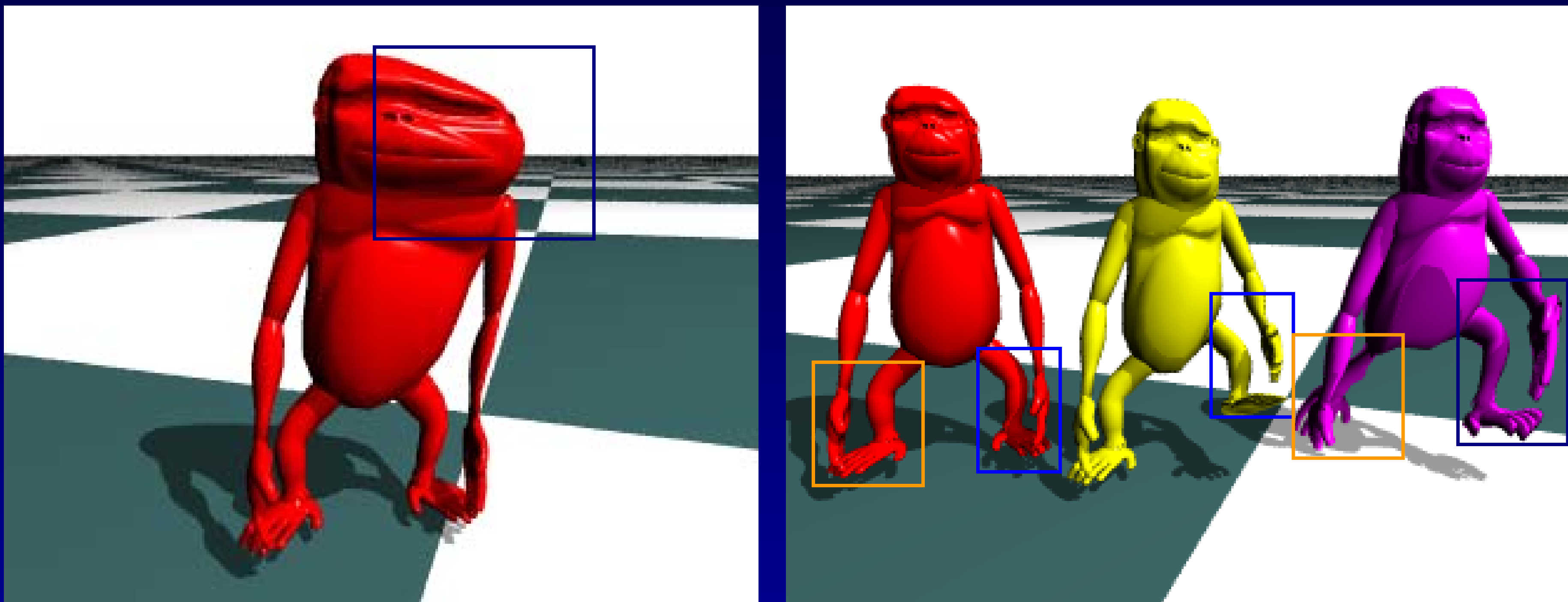
# Direct Manipulation of PDE Solids



Modifying selected regions on an embedded dataset

# Direct Manipulation of PDE Solids

Directly moving a point on an embedded dataset

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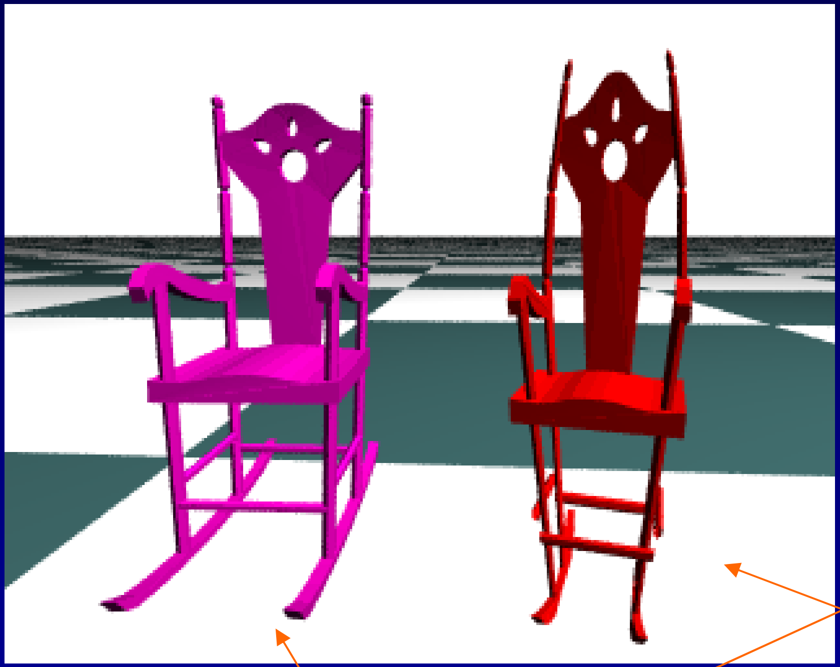
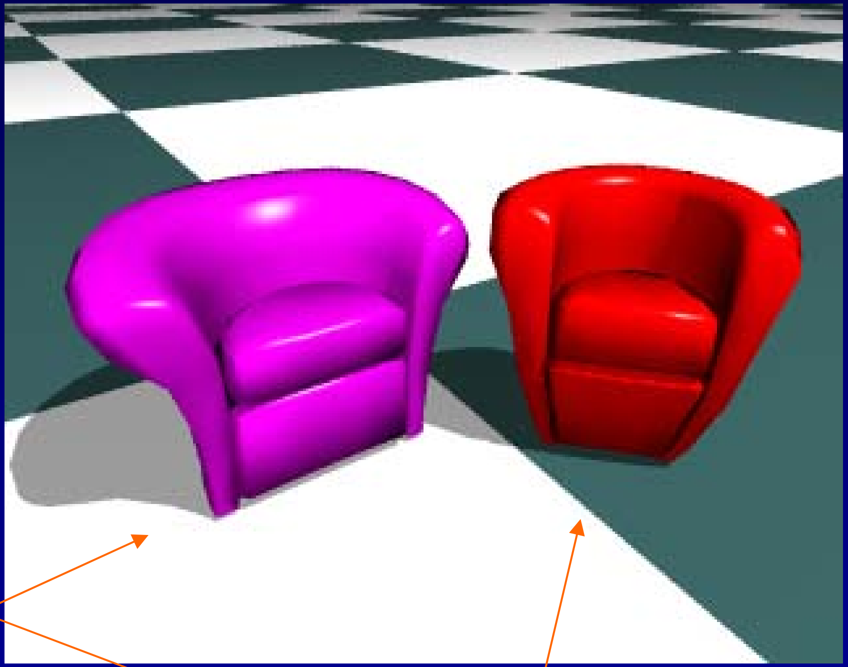
# Geometric Free-Form Deformation





From a PDE solid cube

From a PDE solid sphere

# Integrating Intensity Attributes

- Formulation:

$$\left(\mathbf{a}^2(u,v,w)\frac{\partial^2}{\partial u^2}+\mathbf{b}^2(u,v,w)\frac{\partial^2}{\partial v^2}+\mathbf{c}^2(u,v,w)\frac{\partial^2}{\partial w^2}\right)^2\mathbf{P}(u,v,w)=\mathbf{0},$$

$$\mathbf{P}(u,v,w)=\begin{pmatrix}\mathbf{X}(u,v,w)\\ d(u,v,w)\end{pmatrix},$$

$$\mathbf{a}(u,v,w)=\begin{bmatrix}a_{\mathrm{x}}(u,v,w) & 0\\ 0 & a_d(u,v,w)\end{bmatrix}$$

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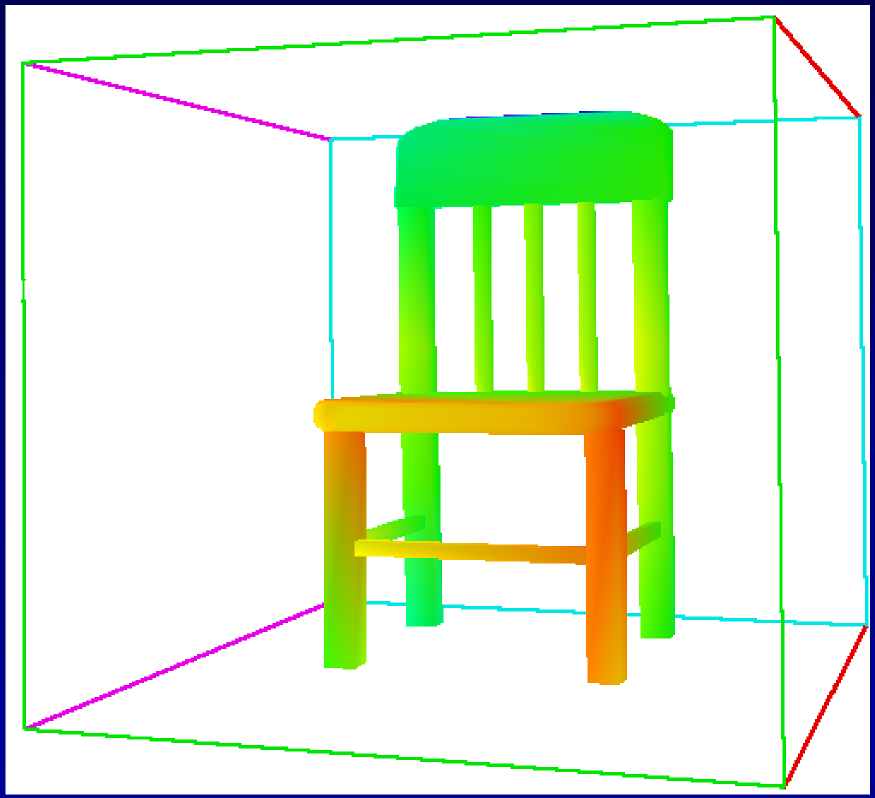
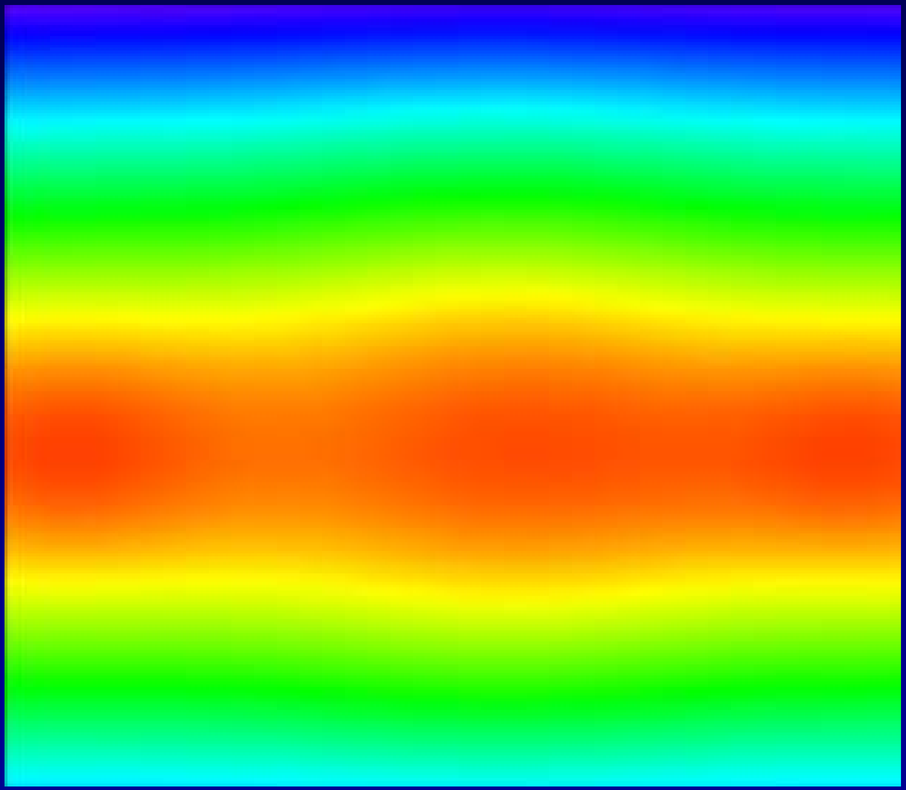
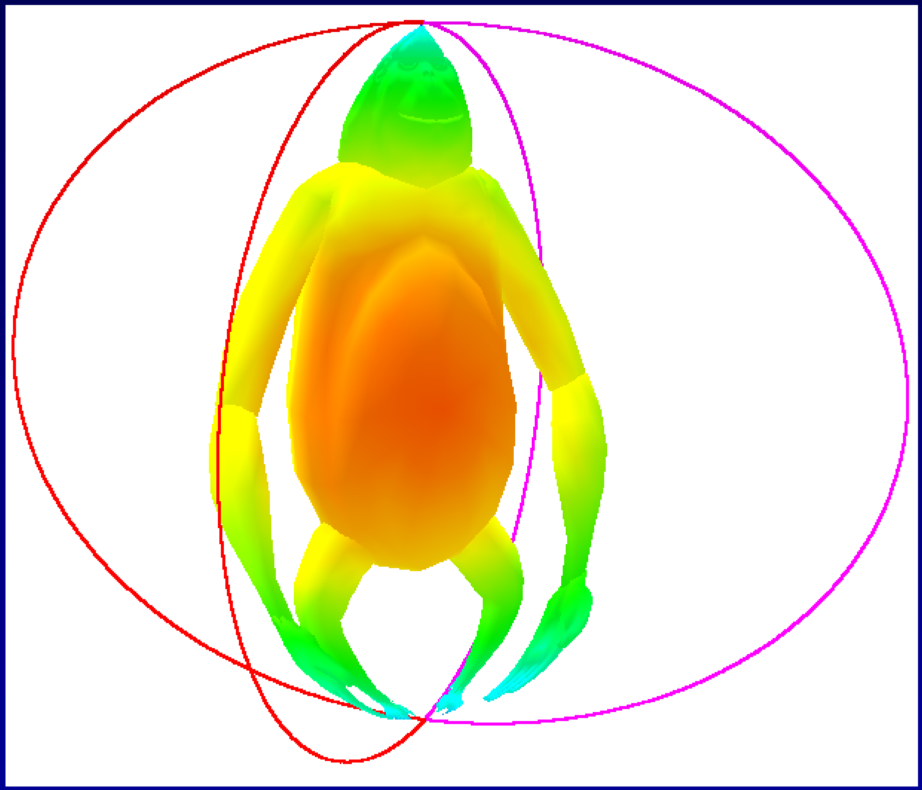
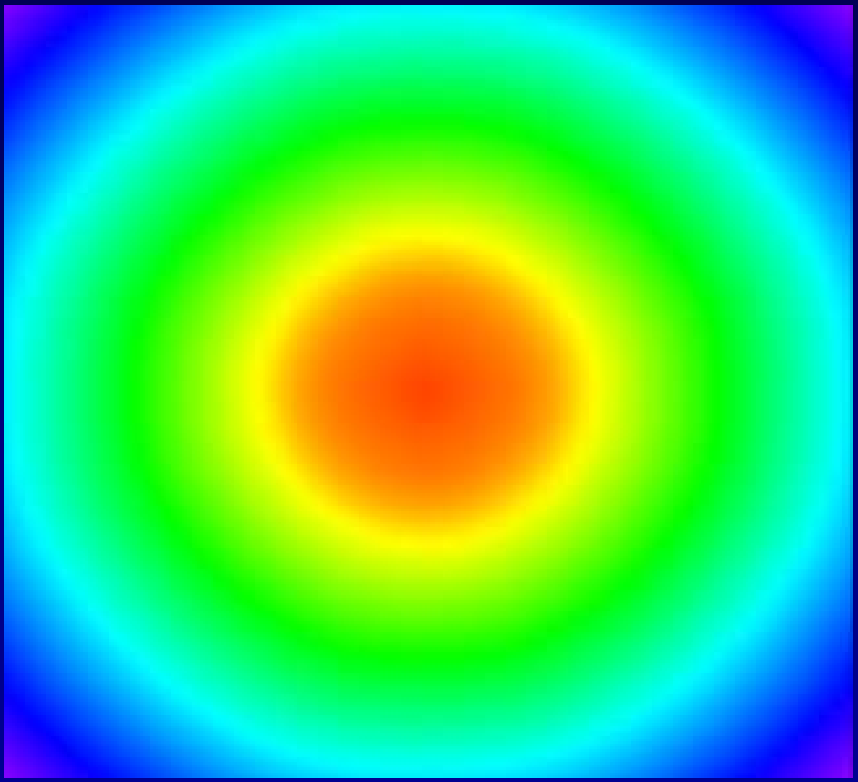
# Intensity Initialization





# Intensity Initialization





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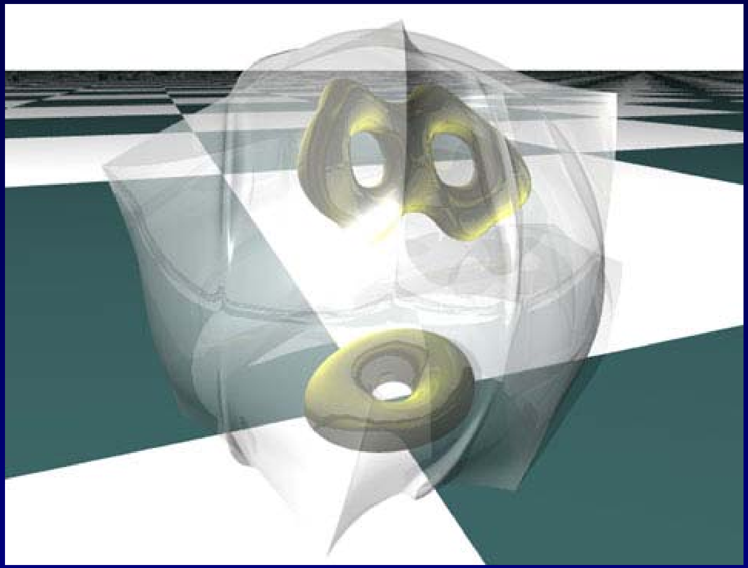
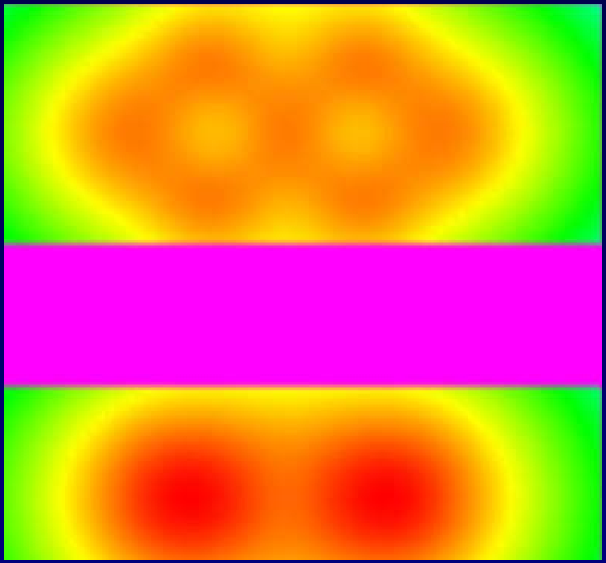
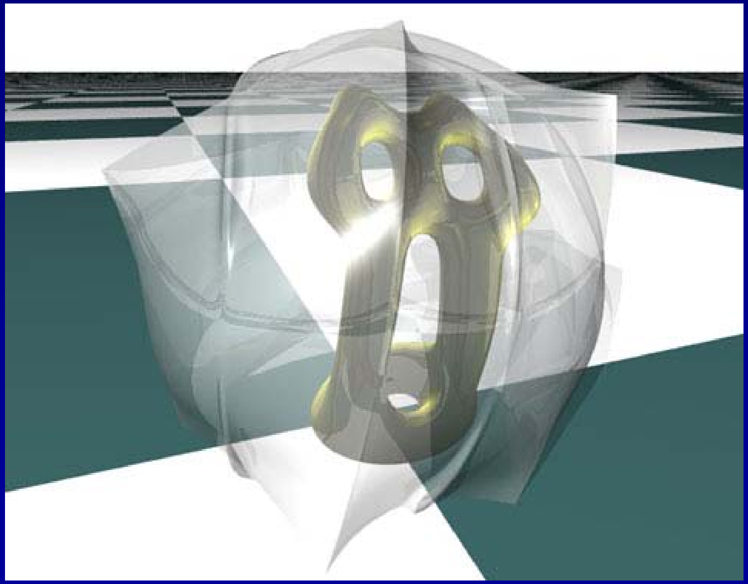
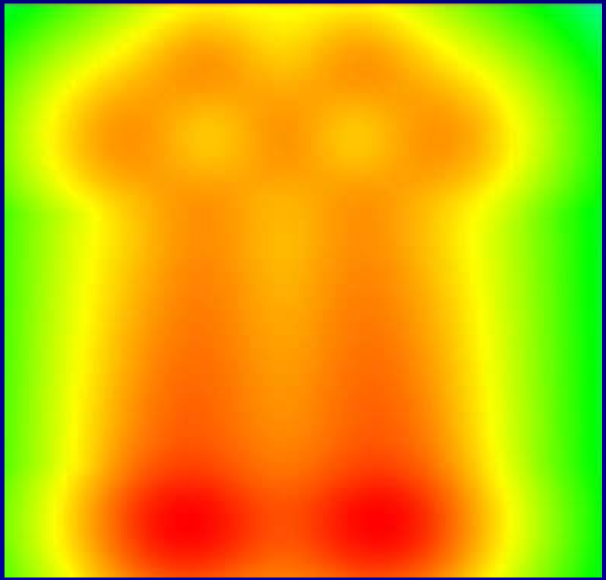
# Arbitrary Shape Blending









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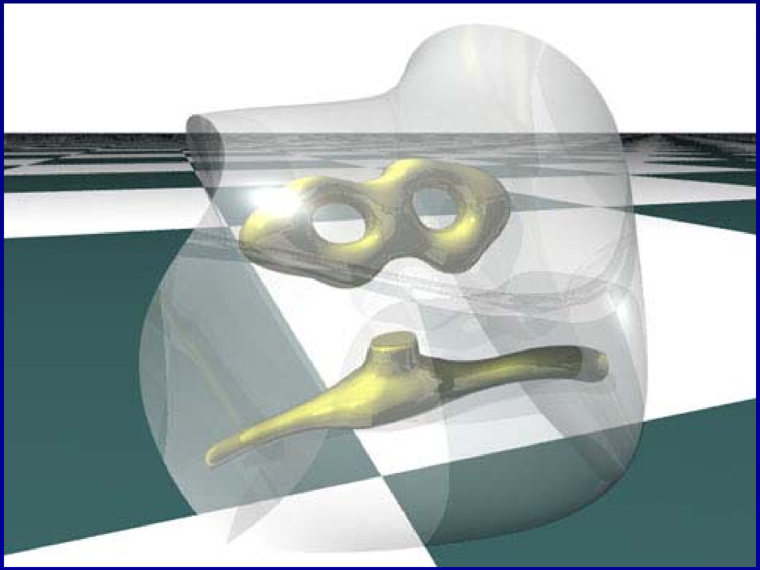
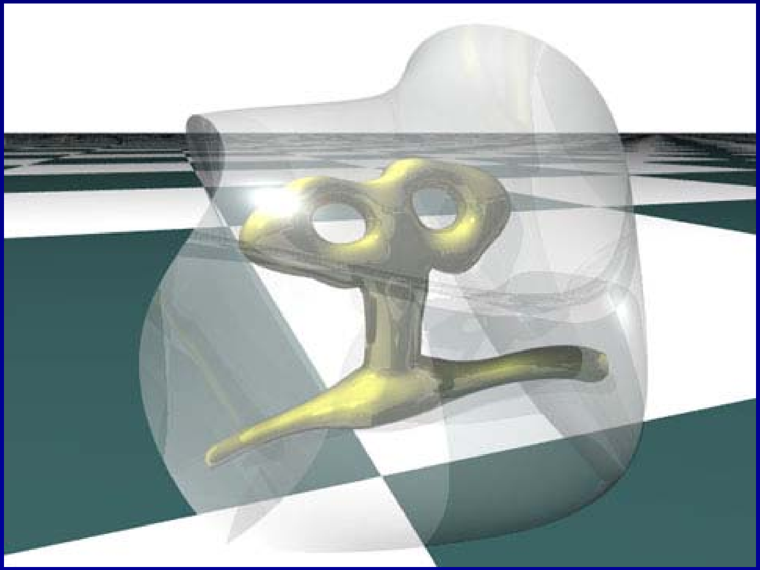
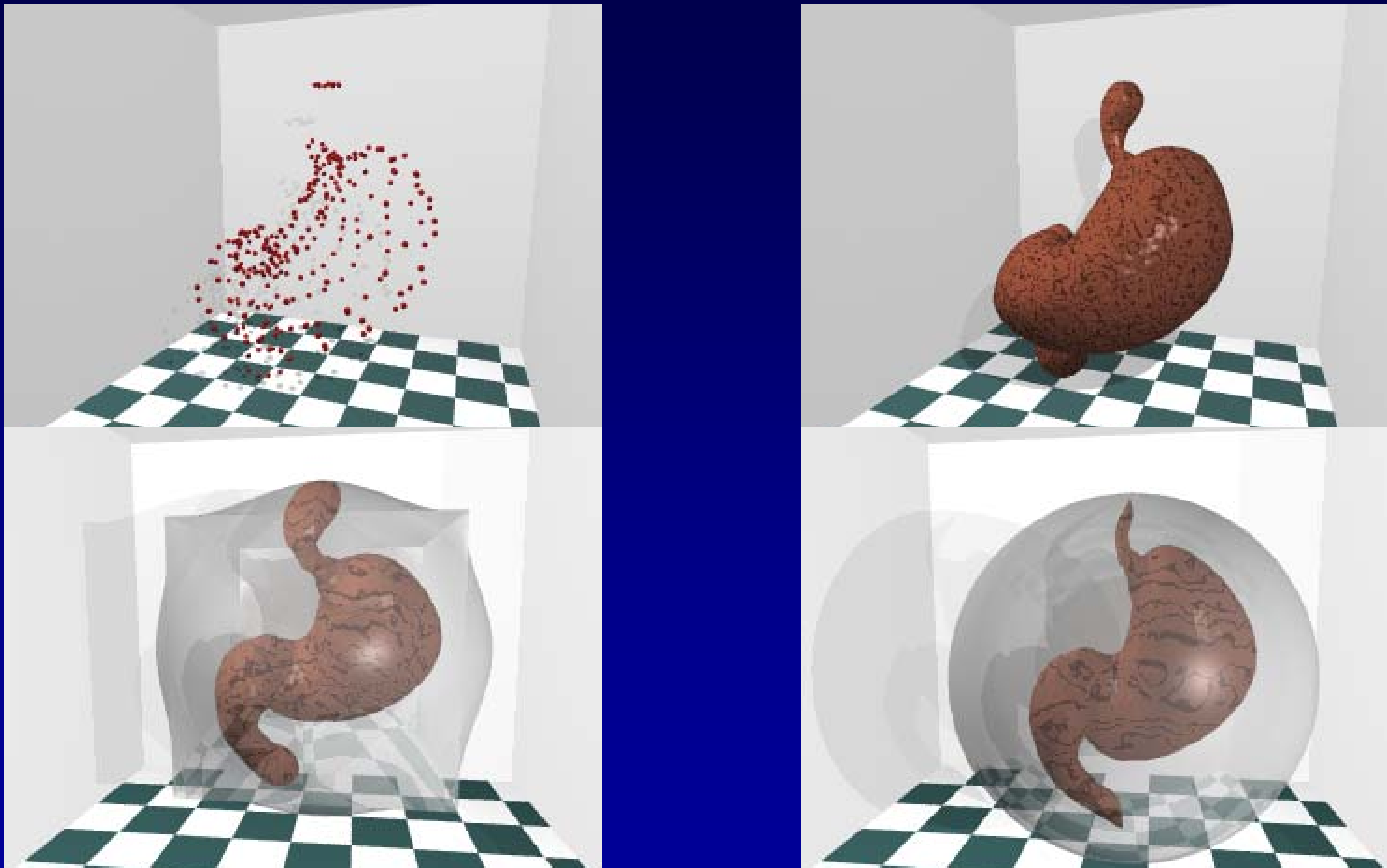
# Arbitrary Shape Blending





# Iso-surface Deformation



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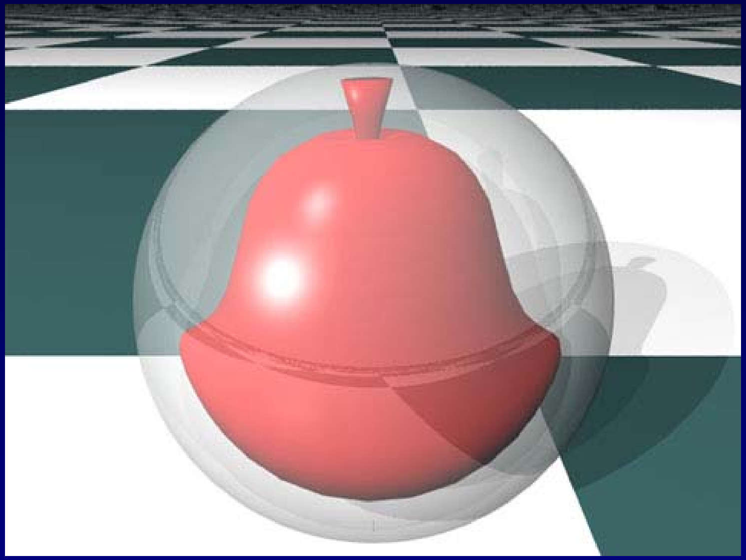
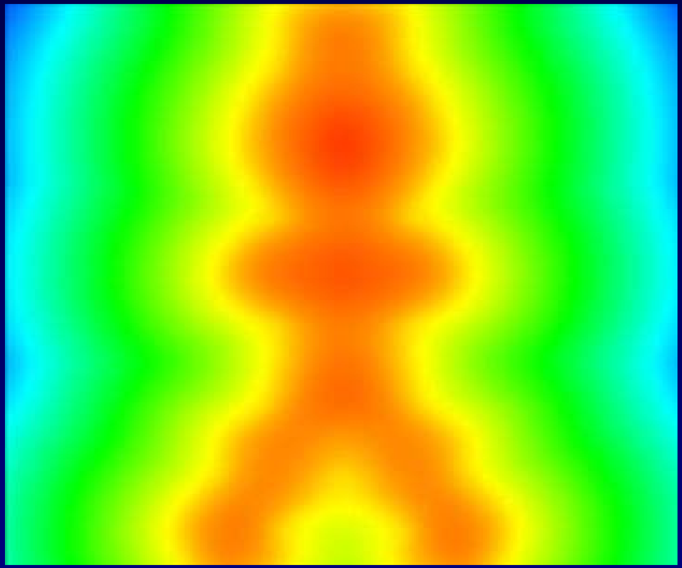
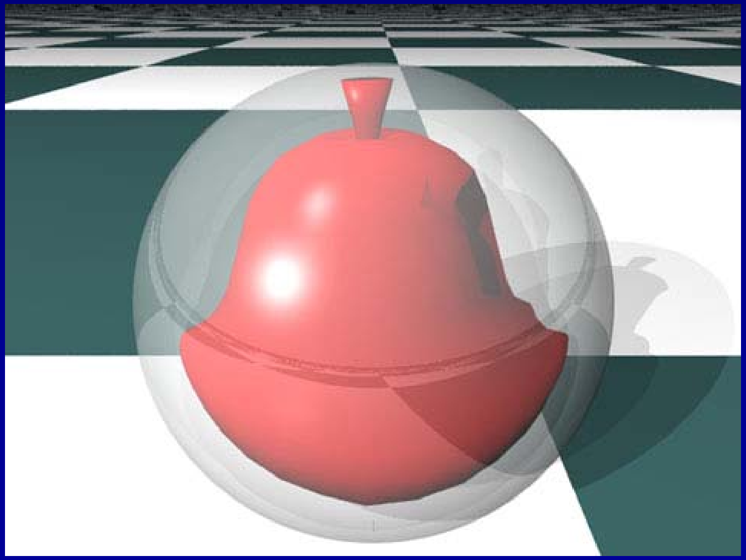
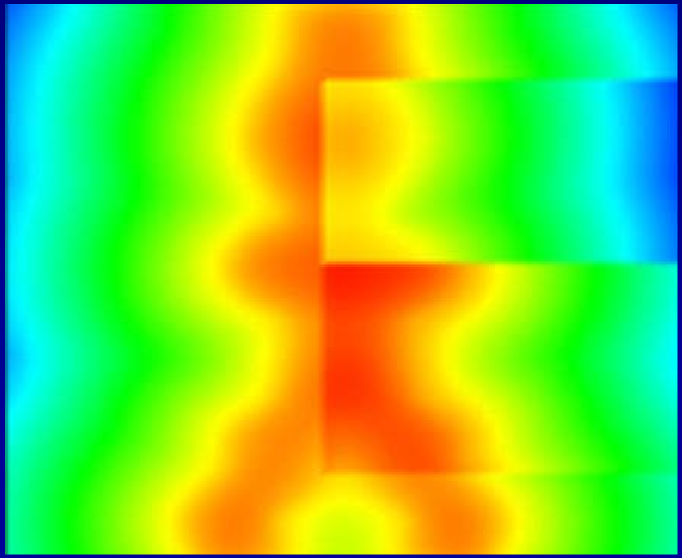
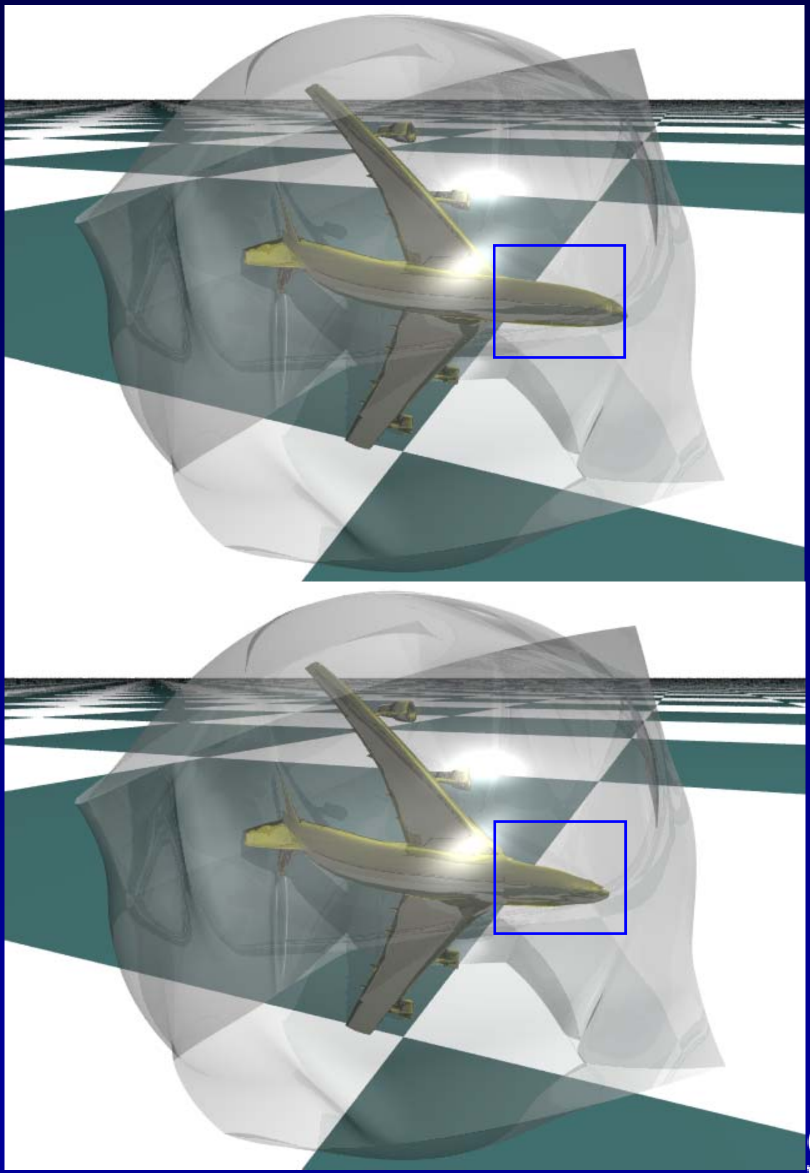


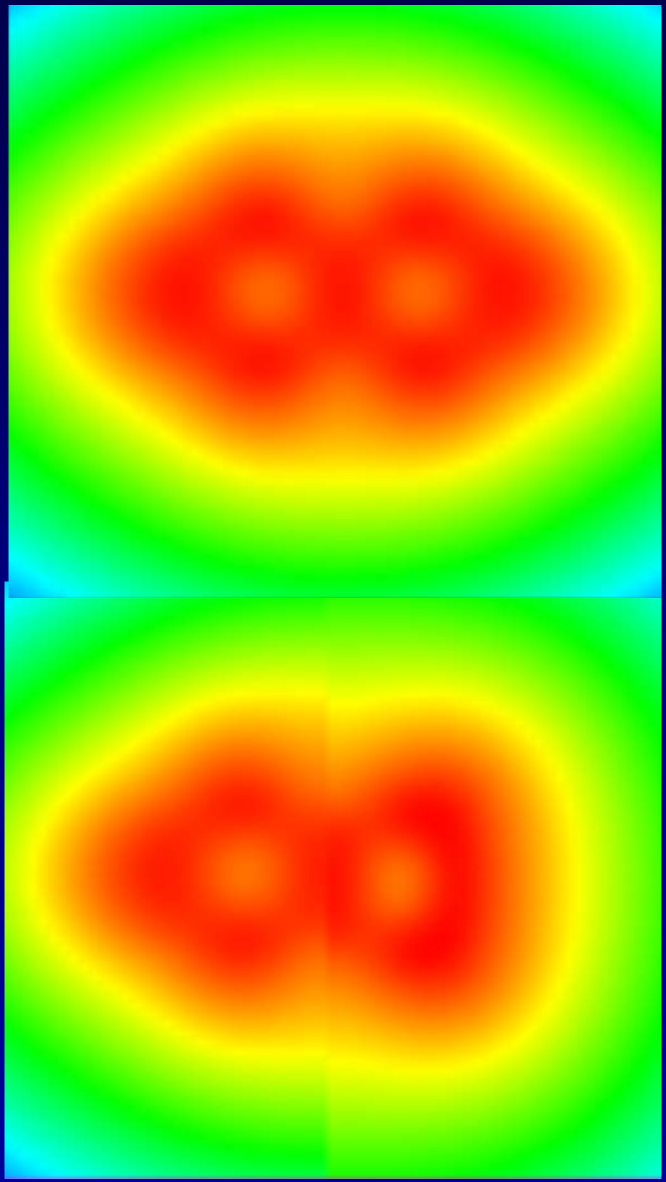






# Intensity Field Modification





# Free-Form PDE Modeling Summary

- Boundary surfaces or curve network as boundary constraints
- Boundary surfaces manipulation for solid deformation
- Free-form deformation for embedded datasets
- Sculpting toolkits for direction manipulation
- Integrating with implicit PDE for more general modeling
  - Arbitrary shape blending based on intensity
  - Intensity-based shape manipulation and deformation

# Outline

- Motivation and contributions
- Related work
- PDE-based geometric modeling system
  - Physics-based PDE surfaces/displacements
  - PDE-based arbitrary mesh modeling
  - Implicit elliptic PDE model
  - PDE-based free-form modeling and deformation
- Conclusion

# Conclusion

- Integrated PDE modeling system for parametric objects, arbitrary meshes, and implicit models
- Incorporation of popular geometric modeling techniques and representations
- Information recovery from partial input
- Physical properties for dynamic behavior
- Various modeling toolkits for direct manipulation and interactive sculpting
- Shape design, recovery, abstraction, and modification in a single framework

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# Future Work

- Geometric modeling
  - Shape design, morphing, reconstruction
- Image processing and medical imaging
  - Enhancement, denoising, medical data reconstruction
- Simulation and animation
  - Natural phenomena simulation, medical simulation

# Related Publications

- Haixia Du and Hong Qin. Dynamic PDE-Based Surface Design Using Geometric and Physical Constraints. Accepted by *Graphical Models*, 2003.
- Haixia Du and Hong Qin. A Shape Design System Using Volumetric Implicit PDEs. Accepted by *CAD Special Issue of the ACM Symposium on Solid Modeling and Applications, 2003*.
- Haixia Du and Hong Qin. PDE-based Free-Form Deformation of Solid Objects. In preparation for journal submission, 2004.
- Haixia Du and Hong Qin. PDE-based Skeletonization and Propagation for Abitrary Topological Shapes. In preparation for journal submission, 2004.
- Haixia Du and Hong Qin. Medial Axis Extraction and Shape Manipulation of Solid Objects Using Parabolic PDEs. Accepted by *The Nineth ACM Symposium on Solid Modeling and Applications 2004*.
- Haixia Du and Hong Qin. Interactive Shape Design Using Volumetric Implicit PDEs. In Proceedings of *The Eighth ACM Symposium on Solid Modeling and Applications 2003*, p235-246.
- Haixia Du and Hong Qin. Integrating Physics-based Modeling with PDE Solids for Geometric Design. In Proceedings of *Pacific Graphics 2001*, p198-207.
- Haixia Du and Hong Qin. Dynamic PDE Surfaces with Flexible and General Constraints. In Proceedings of *Pacific Graphics 2000*, p213-222,
- Haixia Du and Hong Qin. Direct Manipulation and Interactive Sculpting of PDE Surfaces. In Proceedings of *EuroGraphics 2000*, pC261-C270.

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- Committee members
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