Physics Basics

\[ a = \frac{f}{m} \]

energy: \( \frac{1}{2} k (l - l_0)^2 \)

force: \( k(l - l_0) \)

mass–spring system:
Physics Basics

\[ a = \frac{f}{m}; \quad a = \frac{dv}{dt}; \quad v = \frac{dp}{dt} \]

\[ \frac{dE}{dp} = f \]

\[ a = \frac{(...+f^i +...)}{m} \]

\[ a = \frac{((...+f^i_e +...)-(...+f^j_i +...))}{m} \]

\[ ma + cv + (...+f^i_j +...) = ...f^i_e \ldots \]

\[ Ma + Cv + Kp = F \]
Physics Basics

• One mass-point

\[ ma = f \]

\[ a = \frac{dv}{dt} \]

\[ v = \frac{dp}{dt} \]

\[ \frac{d^2 p}{dt^2} = f \]

\[ m \frac{d^2 p}{dt^2} + c \frac{dp}{dt} + \sum_i f_{int}^i = \sum_j f_{ext}^j \]

• Mass-spring system

\[ M \frac{d^2 p}{dt^2} + C \frac{dp}{dt} + Kp = f \]

• Numerical Simulation

\[ a^t = \frac{v^t - v^{t-\delta t}}{\delta t} \]
\[ v^t = \frac{p^t - p^{t-\delta t}}{\delta t} \]
Physics-Based Framework

- Motion equation

\[ M_q \ddot{q} + D_q \dot{q} + K_q q = f_q \]

- Equation with constraints

\[ (A^\top M_q A) \dot{p} + (A^\top D_q A) \dot{p} + (A^\top K_q A) p = (A^\top) f_q \]
Haptics Projects

- Subdivision modeling
- Trimming
- Wavelets
- Multiple patches
Other Projects

- Advanced sculpting tools
- Normal,
- Curvatures,
- Geometric constraints,
- Pressure
- Multi-patch surfaces
- Material properties
- Industrial examples and applications
Surface Discretization

- Use \( n \times n \) meshes to approximate one Bezier patch defined by \( 4 \times 4 \) control points

\[
s(u, v) = \sum_{i=0}^{3} \sum_{j=0}^{3} p_{i,j} B_i(u) B_j(v)
\]

\[
s(u, v) \approx \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} q_{i,j} N_i(u) N_j(v)
\]

where \( N_i(u) \) and \( N_j(v) \) are piecewise linear functions

- \( q_{i,j} \)'s are on this Bezier patch

- \( q_{i,j} \)'s are \textbf{NO-LONGER} independent!

\[
q_{i,j} = \sum_{k=0}^{3} \sum_{l=0}^{3} p_{i,j} B_k(a_i) B_l(b_j)
\]

- Matrix expression

\[
q = Ap
\]
- A is a \((n \times n)\) by \((4 \times 4)\) matrix
- Entries of \(A\) are computed using Bezier basis functions and parametric values
- Assume all parametric values are constants
  \[
  \dot{q} = A\dot{p}
  \]
  \[
  \ddot{q} = A\ddot{p}
  \]
- Dynamic equation
  \[
  (A^\top M_q A)\ddot{p} + (A^\top D_q A)\dot{p} + (A^\top K_q A)p = (A^\top)f_q
  \]
- Numerical integration (explicit method)
  \[
  (A^\top M_q A)\ddot{p} = (A^\top)f_q - (A^\top D_q A)\dot{p} - (A^\top K_q A)p
  \]
- What is the purpose?
  - integration of modeling and rendering
  - approximate higher-order finite element using a set of linear finite elements
  - computationally efficient
- possible to develop more advanced tools
- hierarchical models
- non-elastic models

**Parameters to be controlled:**
- orders of Bezier patches \((4, 4)\)
- sampling rate \((n, m)\)

**Generalization**
- B-spline surface
- parameters
- orders of B-spline \((k, l)\)
- control points \((n, m)\)
- sampling rate

**B-spline surface**

\[
s(u, v) = \sum_{i=0}^{n} \sum_{j=0}^{m} p_{i,j} B_{i,k}(u) B_{j,l}(v)
\]

**NURBS**

**Subdivision surfaces**