Logic and Inference: Rules

CSE 595 – Semantic Web
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http://www3.cs.stonybrook.edu/~pfodor/courses/cse595.html
Lecture Outline

• Monotonic Rules
• OWL2 RL: Description Logic Meets Rules
• Rule Interchange Format: RIF
• Semantic Web Rules Language (SWRL)
• Rules in SPARQL: SPIN
• Nonmonotonic Rules
  • Example: Brokered Trade
• Rule Markup Language (RuleML)
Introduction

• All we did until now are forms of knowledge representation (KR), like knowledge about the content of web resources, knowledge about the concepts of a domain of discourse and their relationships (ontology)

• Knowledge representation had been studied long before the emergence of the World Wide Web in the area of artificial intelligence and, before that, in philosophy
Introduction

- KR can be traced back to ancient Greece (because Aristotle is considered to be the father of logic)
- Logic is the foundation of knowledge representation, particularly in the form of predicate logic (also known as first-order logic)
Introduction

- Logic:
  - provides a high-level language in which knowledge can be expressed in a transparent way
  - has a high expressive power (maybe too high because it is intractable or undecidable in some cases)
  - has a well-understood formal semantics, which assigns an unambiguous meaning to logical statements
  - has a precise notion of *logical consequence*, which determines whether a statement follows semantically from a set of other statements (*premises*)
Introduction

- There exist proof systems that can automatically derive statements syntactically from a set of premises.
- There exist proof systems for which semantic logical consequence coincides with syntactic derivation within the proof system.
  - Proof systems should be sound (all derived statements follow semantically from the premises) and complete (all logical consequences of the premises can be derived in the proof system).
- Predicate logic is unique in the sense that sound and complete proof systems do exist - More expressive logics (higher-order logics) do not have such proof systems.
- It is possible to trace the proof that leads to a logical consequence, so logic can provide explanations for answers.
Introduction

- RDF and OWL2 profiles can be viewed as specializations of predicate logic:
  - One justification for the existence of such specialized languages is that they provide a syntax that fits well with the intended use (in our case, web languages based on tags).
  - Another justification is that they define reasonable subsets of logic where the computation is tractable (there is a trade-off between the expressive power and the computational complexity of certain logics: the more expressive the language, the less efficient the corresponding proof systems).
Introduction

- Most OWL variants correspond to a _description logic_, a subset of predicate logic for which efficient proof systems exist.
- Another subset of predicate logic with _efficient_ proof systems comprises the _Horn rule systems_ (also known as _Horn logic_ or _definite logic programs_).
  - A rule has the form:
    \[ A_1, \ldots, A_n \rightarrow B. \]
    where \( A_i \) and \( B \) are atomic formulas.
  - In Prolog notation:
    \[ B : \neg A_1, \ldots, A_n. \]
There are two intuitive ways of reading a Horn rule:

- **deductive rules:** If $A_1, \ldots, A_n$ are known to be true, then $B$ is also true.
  - There are two ways of applying deductive rules:
    - from the body ($A_1, \ldots, A_n$) to the conclusion ($B$) (**forward chaining**)
    - from the conclusion (goal) to the body (**backward reasoning**)

- **reactive rules:** If the conditions $A_1, \ldots, A_n$ are true, then carry out the action $B$. 
Introduction

• Description logics and Horn logic are orthogonal in the sense that neither of them is a subset of the other
  • For example, it is impossible to define the class of happy spouses as those who are married to their best friend in description logics, but this piece of knowledge can easily be represented using rules:

\[
\text{married}(X, Y), \text{bestFriend}(X, Y) \rightarrow \text{happySpouse}(X).
\]

• On the other hand, rules cannot (in the general case) assert:
  (a) negation/complement of classes
  (b) disjunctive/union information (for instance, that a person is either a man or a woman)
  (c) existential quantification (for instance, that all persons have a father).
Monotonic and nonmonotonic rules

• Predicate logic is *monotonic*: if a conclusion can be drawn, it remains valid even if new knowledge becomes available.

• Even if a rule uses negation,

  R1 : If birthday, then special discount.

  R2 : If *not birthday*, then not special discount.

  it works properly in cases where the birthday is known
Monotonic and nonmonotonic rules

- Imagine a customer who refuses to provide his birthday because of privacy concerns, then the preceding rules cannot be applied because their premises are not known.

R1 : If birthday, then special discount.
R2 : If not birthday, then not special discount.

R2' : If birthday is not known, then not special discount.

- R2' is not within the expressive power of predicate logic because its conclusion may become invalid if the customer’s birthday becomes known at a later stage and it happens to coincide with the purchase date.

- Adding knowledge later that invalidates some of the conclusions is called **nonmonotonic** because the addition of new information leads to a loss of a consequence.
Rules on the Semantic Web

- Rule technology has been around for decades, has found extensive use in practice, and has reached significant maturity
  - led to a broad variety of approaches
  - it is more difficult to standardize this area in the context of the (semantic) web
- A W3C working group has developed the Rule Interchange Format (RIF) standard
  - Whereas RDF and OWL are languages meant for directly representing knowledge, RIF was designed primarily for the exchange of rules across different applications
    - For example, an online store might wish to make its pricing, refund, and privacy policies, which are expressed using rules, accessible to intelligent agents
Rules on the Semantic Web

- Due to the underlying aim of serving as an interchange format among different rule systems, RIF combines many of their features, and is quite complex
- Those wishing to develop rule systems for the Semantic Web have various alternatives:
  - Rules over RDF can be expressed using SPARQL constructs
  - SPARQL is not a rule language, as basically it carries out one application of a rule.
  - SPIN is a rule system developed on top of SPARQL
  - SWRL couples OWL DL functionalities with certain types of rules
  - Model in terms of OWL but use rule technology for implementation purposes: OWL2 RL
Example Monotonic Rules: Family

- Imagine a database of facts about some family relationships which contains facts about the following base predicates:
  - mother(X, Y) X is the mother of Y
  - father(X, Y) X is the father of Y
  - male(X) X is male
  - female(X) X is female
Example Monotonic Rules: Family

- We can infer further relationships using appropriate rules:
  - a parent is either a father or a mother.

\[
\text{mother}(X, Y) \rightarrow \text{parent}(X, Y).
\]

\[
\text{father}(X, Y) \rightarrow \text{parent}(X, Y).
\]

- a brother to be a male person sharing a parent:

\[
\text{male}(X), \text{parent}(P, X), \text{parent}(P, Y), \text{notSame}(X, Y) \rightarrow \text{brother}(X, Y).
\]

- The predicate \text{notSame} denotes inequality; we assume that such facts are kept in a database

\[
\text{female}(X), \text{parent}(P, X), \text{parent}(P, Y), \text{notSame}(X, Y) \rightarrow \text{sister}(X, Y).
\]
Example Monotonic Rules: Family

- An uncle is a brother of a parent:
  \[
  \text{brother}(X, P), \text{parent}(P, Y) \rightarrow \text{uncle}(X, Y).
  \]

- A grandmother is the mother of a parent:
  \[
  \text{mother}(X, P), \text{parent}(P, Y) \rightarrow \text{grandmother}(X, Y).
  \]

- An ancestor is either a parent or an ancestor of a parent:
  \[
  \text{parent}(X, Y) \rightarrow \text{ancestor}(X, Y).
  \]
  \[
  \text{ancestor}(X, P), \text{parent}(P, Y) \rightarrow \text{ancestor}(X, Y).
  \]
Monotonic Rules: Syntax

Let us consider a simple rule stating that all loyal customers with ages over 60 are entitled to a special discount:

\[
\text{loyalCustomer}(X), \text{age}(X) > 60 \rightarrow \text{discount}(X).
\]

Rules have:

- variables, which are placeholders for values: \( X \)
- constants, which denote fixed values: \( 60 \)
- predicates, which relate objects: \( \text{loyalCustomer}, > \)
- function symbols, which denote a value, when applied to certain arguments: \( \text{age} \)

In case no function symbols are used, we discuss function-free (Horn) logic.
Rules

- A rule has the form:
  \[ \text{B}_1, \ldots, \text{B}_n \rightarrow \text{A} \]
  where \( \text{A} \) and \( \text{B}_i \) are atomic formulas
- \( \text{A} \) is the **head** of the rule
- \( \text{B}_i \) are the **premises** of the rule
- The set \( \{\text{B}_1,...,\text{B}_n\} \) is referred to as the **body** of the rule
- The commas in the rule body are read conjunctively:
  if \( \text{B}_1 \) and \( \text{B}_2 \) and \( \ldots \) and \( \text{B}_n \) are true, then \( \text{A} \) is also true
  (or equivalently, to prove \( \text{A} \) it is sufficient to prove all of \( \text{B}_1,\ldots,\text{B}_n \))
Rules

• Variables may occur in $A, B_1, \ldots, B_n$.
  • For example,
    
    $\text{loyalCustomer}(X), \text{age}(X) > 60 \rightarrow \text{discount}(X)$.

    is applied for any customer: if a customer happens to be loyal and over 60, then they gets the discount.

• The variable $X$ is implicitly universally quantified (using $\forall X$)

• In general, all variables occurring in a rule are implicitly universally quantified.
A rule $r$:

$$B_1, \ldots, B_n \rightarrow A$$

is interpreted as the following formula, denoted by $pl(r)$:

$$\forall X_1 \ldots \forall X_k ( (B_1 \land \ldots \land B_n) \rightarrow A)$$

or equivalently,

$$\forall X_1 \ldots \forall X_k (A \lor \neg B_1 \lor \ldots \lor \neg B_n)$$

where $X_1, \ldots, X_k$ are all variables occurring in $A$, $B_1, \ldots, B_n$. 
Logic Programs

• A fact is an atomic formula, such as \texttt{loyalCustomer(a345678)}.
  which says that the customer with ID \texttt{a345678} is loyal.
  • If there are variables in a fact, then they are implicitly universally quantified.

• A logic program $P$ is a finite set of facts and rules.
  • Its predicate logic translation $pl(P)$ is the set of all predicate logic interpretations of rules and facts in $P$.  

Logic Programs

- A **goal** or **query** $G$ asked to a logic program has the form

$$B_1, \ldots, B_n \rightarrow$$

- If $n = 0$, we have the empty goal $\Box$.
- The interpretation of a goal is:

$$\forall X_1 \ldots \forall X_k (\neg B_1 \lor \ldots \lor \neg B_n)$$

where $X_1, \ldots, X_k$ are all variables occurring in $B_1, \ldots, B_n$
Logic Programs

• The goal formula is equivalent to

$$\forall X_1 \ldots \forall X_k (false \lor \neg B_1 \lor \ldots \lor \neg B_n)$$

so the missing rule head can be thought of as a contradiction \textit{false}.

• An equivalent representation in predicate logic is:

$$\neg \exists X_1 \ldots \exists X_k (B_1 \land \ldots \land B_n)$$
Logic Programs

• Suppose we know the fact
  \[ p(a). \]
and we have the goal
  \[ p(x) \rightarrow \]
  • we want to know whether there is a value for which \( p \) is true
  • We expect a positive answer because of the fact \( p(a) \).
  • Thus \( p(x) \) is existentially quantified
  • Why do we negate the formula?
    • The explanation is that we use a proof technique from mathematics called *proof by contradiction*
      • This technique proves that a statement A follows from a statement B by assuming that A is false and deriving a contradiction when combined with B. Then A must follow from B.
In logic programming we prove that a goal can be answered positively by negating the goal and proving that we get a contradiction using the logic program.

For example, given the logic program

\[ p(a). \]

and the goal

\[ \neg \exists x p(x) \]

we get a logical contradiction: the second formula says that no element has the property \( p \), but the first formula says that the value of \( a \) does have the property \( p \). Thus \( \neg \exists x p(x) \) follows from \( p(a) \).
Monotonic Rules: Semantics

- Given a logic program $P$ and a query $B_1, \ldots, B_n \rightarrow$ with the variables $X_1, \ldots, X_k$, we answer positively if, and only if,
  
  $pl(P) \models \exists X_1 \ldots \exists X_k (B_1 \land \ldots \land B_n)$

  or equivalently, if

  $pl(P) \cup \{\neg \exists X_1 \ldots \exists X_k (B_1 \land \ldots \land B_n)\}$ is unsatisfiable

- We give a positive answer if the predicate logic representation of the program $P$, together with the predicate logic interpretation of the query, is unsatisfiable (a contradiction).
Monotonic Rules: Semantics

- Predicate Logic Semantics
  - A predicate logic model, $A$, consists of
    - a domain $\text{dom}(A)$, a nonempty set of objects about which the formulas make statements
    - an element from the domain for each constant
    - a concrete function on $\text{dom}(A)$ for every function symbol
    - a concrete relation on $\text{dom}(A)$ for every predicate
  - When the symbol $=$ is used to denote equality (i.e., its interpretation is fixed), we talk of Horn logic with equality
  - Logical connectives $\neg$, $\lor$, $\land$, $\rightarrow$, $\forall$, $\exists$

- A formula $\phi$ follows from a set $M$ of formulas if $\phi$ is true in all models $A$ in which $M$ is true (that is, all formulas in $M$ are true in $A$).
Monotonic Rules: Semantics

- A formula $\phi$ follows from a set $M$ of formulas if $\phi$ is true in all models $A$ in which $M$ is true (that is, all formulas in $M$ are true in $A$).

- Regardless of how we interpret the constants, predicates, and function symbols occurring in $P$ and the query, once the predicate logic interpretation of $P$ is true, $\exists x_1 \ldots \exists x_k (B_1 \land \ldots \land B_n)$ must be true: that is, there are values for the variables $x_1, \ldots, x_k$ such that all atomic formulas $B_i$ become true.
Monotonic Rules: Semantics

• Suppose $P$ is the program
  
  $p(a).$
  
  $p(X) \rightarrow q(X).$

• Consider the query
  
  $q(X) \rightarrow$

  • $q(a)$ follows from $pl(P)$
  • $\exists X q(X)$ follows from $pl(P)$
  • $pl(P) \cup \{ \neg \exists X q(X) \}$ is unsatisfiable

• If we consider the query
  
  $q(b) \rightarrow$

then we give a negative answer because $q(b)$ does not follow from $pl(P)$
Least Herbrand Model Semantics

- Instead of considering any domain $\text{dom}(A)$, we can consider only the names in the program (predicate names, constants, functors).
- Then we have **Herbrand semantics**:
  - Given an alphabet $A$, the set of all **ground terms** constructed from the constant and function symbols of $A$ is called the **Herbrand Universe** of $A$ (denoted by $U_A$).
- Consider the program:
  
  \[
  \begin{align*}
  p(\text{zero}) . \\
  p(s(s(X))) & \leftarrow p(X) .
  \end{align*}
  \]
- The Herbrand Universe of the program's alphabet is:
  
  \[U_A = \{ \text{zero}, s(\text{zero}), s(s(\text{zero})), \ldots \}\]
Least Herbrand Model Semantics

- Consider a "relations" program:

  \[
  \text{parent}(\text{pam}, \text{bob}). \quad \text{parent}(\text{bob}, \text{ann}). \\
  \text{parent}(\text{tom}, \text{bob}). \quad \text{parent}(\text{bob}, \text{pat}). \\
  \text{parent}(\text{tom}, \text{liz}). \quad \text{parent}(\text{pat}, \text{jim}). \\
  \text{grandparent}(X,Y) : - \\
  \quad \text{parent}(X,Z), \text{parent}(Z,Y).
  \]

- The Herbrand Universe of the program's alphabet is:

  \[ U_A = \{ \text{pam, bob, tom, liz, ann, pat, jim} \} \]
Least Herbrand Model Semantics

• Given an alphabet \( A \), the set of all \textbf{ground atomic formulas} over \( A \) is called the \textit{Herbrand Base} of \( A \) (denoted by \( B_A \)).

• Consider the program:

\[
\begin{align*}
p(zero) . \\
p(s(s(X))) & \leftarrow p(X) .
\end{align*}
\]

• The Herbrand Base of the program's alphabet is:

\[
B_A = \{ p(zero), p(s(zero)), p(s(s(zero))), \ldots \}
\]
Least Herbrand Model Semantics

- Consider the "relations" program:

  \[
  \begin{align*}
  \text{parent}(pam, \ bob). & \quad \text{parent}(bob, \ ann). \\
  \text{parent}(tom, \ bob). & \quad \text{parent}(bob, \ pat). \\
  \text{parent}(tom, \ liz). & \quad \text{parent}(pat, \ jim). \\
  \text{grandparent}(X,Y) :& - \\
  \quad \text{parent}(X,Z), \ \text{parent}(Z,Y).
  \end{align*}
  \]

- The Herbrand Base of the program's alphabet is:

  \[ B_A = \{ \text{parent}(pam, \ pam), \ \text{parent}(pam, \ bob), \ \\
  \text{parent}(pam, \ tom), \ldots, \ \text{parent}(bob, \ pam), \ldots, \ \\
  \text{grandparent}(pam,pam), \ldots, \text{grandparent}(bob,pam), \ldots \}. \]
Least Herbrand Model Semantics

- A **Herbrand Interpretation** of a program $P$ is an interpretation $I$ such that:
  - The domain of the interpretation: $|I| = U_P$
  - For every constant $c$: $c_I = c$
  - For every function symbol $f/n$:
    $f_I(x_1, \ldots, x_n) = f(x_1, \ldots, x_n)$
  - For every predicate symbol $p/n$: $p_I \subseteq (U_P)^n$ (i.e. some subset of $n$-tuples of ground terms)
- A **Herbrand Model** of a program $P$ is a Herbrand interpretation that is a model of $P$. 
Least Herbrand Model Semantics

- All Herbrand interpretations of a program give the same “meaning” to the constant and function symbols.
- Different Herbrand interpretations differ only in the “meaning” they give to the predicate symbols.
- We often write a Herbrand model simply by listing the subset of the Herbrand base that is true in the model.

Example: Consider our numbers program, where
\{p(zero), \ p(s(s(zero))), \ p(s(s(s(s(zero))))), \ldots\}
represents the Herbrand model that treats
\p = \{\zero, s(s(zero)), s(s(s(s(zero)))), \ldots\}
as the meaning of p.
Sufficiency of Herbrand Models

Let P be a definite program. If I' is a model of P then
\[ I = \{ A \in Bp \mid I' \models A \} \]
is a Herbrand model of P.

Proof (by contradiction):
Let I be a Herbrand interpretation.
Assume that I' is a model of P but I is not a model.

Then there is some ground instance of a clause in P:
\[ A_0 \ :- \ A_1, \ \ldots, \ A_n. \]
which is not true in I i.e., \( I \models A_1, \ldots, I \models A_n \) but I \( \not\models A_0 \)
By definition of I then, I' \( \models A_1, \ldots, I' \models A_n \) but I' \( \not\models A_0 \)
Thus, I' is not a model of P, which contradicts our earlier assumption.
Definite programs only

- Let $P$ be a definite program. If $I'$ is a model of $P$ then $I = \{ A \in Bp \mid I' \models A \}$ is a Herbrand model of $P$.
  
  - **This property holds only for definite programs!**

- Consider $P = \{ \neg p(a), \exists X.p(X) \}$
  
  - There are two Herbrand interpretations: $I_1 = \{ p(a) \}$ and $I_2 = \{ \}$
    - The first is not a model of $P$ since $I_1 \not\models \neg p(a)$.
    - The second is not a model of $P$ since $I_2 \not\models \exists X.p(X)$

- But there is a non-Herbrand model $I$
  - $| I | = N$, the set of natural numbers
  - $a_I = 0$
  - $p_I = \text{“is odd”}$
Properties of Herbrand Models

1) If $M$ is a set of Herbrand Models of a definite program $P$, then $\cap M$ is also a Herbrand Model of $P$.

2) For every definite program $P$ there is a unique least model $M_p$ such that:
   - $M_p$ is a Herbrand Model of $P$ and,
   - for every Herbrand Model $M$, $M_p \subseteq M$.

3) For any definite program, if every Herbrand Model of $P$ is also a Herbrand Model of $F$, then $P \models F$.

4) $M_p =$ the set of all ground logical consequences of $P$. 
Properties of Herbrand Models

• If $M_1$ and $M_2$ are Herbrand models of $P$, then $M = M_1 \cap M_2$ is a model of $P$.

• Assume $M$ is not a model.

• Then there is some clause $A_0 : \neg A_1, \ldots, A_n$ such that $M \models A_1, \ldots, M \models A_n$ but $M \not\models A_0$.

• Which means $A_0 \notin M_1$ or $A_0 \notin M_2$.

• But $A_1, \ldots, A_n \in M_1$ as well as $M_2$.

• Hence one of $M_1$ or $M_2$ is not a model.
Properties of Herbrand Models

• There is a unique least Herbrand model
• Let $M_1$ and $M_2$ are two incomparable minimal Herbrand models, i.e., $M = M_1 \cap M_2$ is also a Herbrand model (previous theorem), and $M \subseteq M_1$ and $M \subseteq M_2$
• Thus $M_1$ and $M_2$ are not minimal.
Least Herbrand Model

• The **least Herbrand model** $M_p$ of a definite program $P$ is the set of all ground logical consequences of the program.

$$M_p = \{ A \in B_p \mid P \models A \}$$

• First, $M_p \supseteq \{ A \in B_p \mid P \models A \}$:
  • By definition of logical consequence, $P \models A$ means that $A$ has to be in every model of $P$ and hence also in the least Herbrand model.
Least Herbrand Model

- Second, $M_P \subseteq \{ A \in B_P \mid P \models A \}$:
  - If $M_P \models A$ then $A$ is in every Herbrand model of $P$.
  - But assume there is some model $I' \not\models \neg A$.
  - By sufficiency of Herbrand models, there is some Herbrand model $I$ such that $I \models \neg A$.
  - Hence $A$ is not in some Herbrand model, and hence is not in $M_P$. 
Finding the Least Herbrand Model

- **Immediate consequence operator:**
  - Given \( I \subseteq B_p \), construct \( I' \) such that
    \[
    I' = \{ A_0 \in B_p \mid A_0 \leftarrow A_1, \ldots, A_n \text{ is a ground instance of a clause in } P \text{ and } A_1, \ldots, A_n \in I \}
    \]
  - \( I' \) is said to be the *immediate consequence of* \( I \).
  - Written as \( I' = T_p(I) \), \( T_p \) is called the *immediate consequence operator*.
  - Consider the sequence:
    \[
    \emptyset, T_p(\emptyset), T_p(T_p(\emptyset)), \ldots, T_p^i(\emptyset), \ldots
    \]
  - \( M_p \supseteq T_p^i(\emptyset) \) for all \( i \).
  - Let \( T_p \uparrow \omega = \bigcup_{i=0}^{\infty} T_p^i(\emptyset) \)
  - Then \( M_p \subseteq T_p \uparrow \omega \).
### Computing Least Herbrand Models: An Example

```
parent(pam, bob).
parent(tom, bob).
parent(tom, liz).
parent(bob, ann).
parent(bob, pat).
parent(pat, jim).

anc(X,Y) :-
  parent(X,Y).
anc(X,Y) :-
  parent(X,Z),
  anc(Z,Y).
```

<table>
<thead>
<tr>
<th>Step</th>
<th>Model</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>$\emptyset$</td>
<td>-</td>
</tr>
<tr>
<td>$M_2 = T_P(M_1)$</td>
<td>${\text{parent(pam,bob)}, \text{parent(tom,bob)}, \text{parent(tom,liz)}, \text{parent(bob,ann)}, \text{parent(bob,pat)}, \text{parent(pat,jim)}}$</td>
<td>-</td>
</tr>
<tr>
<td>$M_3 = T_P(M_2)$</td>
<td>${\text{anc(pam,bob)}, \text{anc(tom,bob)}, \text{anc(tom,liz)}, \text{anc(bob,ann)}, \text{anc(bob,pat)}, \text{anc(pat,jim)}} \cup M_2$</td>
<td>-</td>
</tr>
<tr>
<td>$M_4 = T_P(M_3)$</td>
<td>${\text{anc(pam,ann)}, \text{anc(pam,pat)}, \text{anc(tom,ann)}, \text{anc(tom,pat)}, \text{anc(bob,jim)}} \cup M_3$</td>
<td>-</td>
</tr>
<tr>
<td>$M_5 = T_P(M_4)$</td>
<td>${\text{anc(pam,jim)}, {\text{anc(tom,jim)}}} \cup M_4$</td>
<td>-</td>
</tr>
<tr>
<td>$M_6 = T_P(M_5)$</td>
<td>$M_5$</td>
<td>-</td>
</tr>
</tbody>
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Ground and Parameterized Witnesses

- Suppose we know the fact
  \[ p(a) \]
  and we have the goal
  \[ p(X) \rightarrow \]
  - Responding **true** to parametrized queries is correct, but not satisfactory
  - The appropriate answer is a substitution \( \{ X/a \} \) which gives an instantiation for \( X \), making the answer positive.
  - The constant \( a \) is called a *ground witness*
  - Given two facts: \( p(a) \) and \( p(b) \), there are two ground witnesses to the same query: \( a \) and \( b \).
Ground and Parameterized Witnesses

• Ground witnesses are not always the optimal answer
  • Consider the logic program:

\[
\text{add}(X, 0, X).
\]

\[
\text{add}(X, Y, Z) \rightarrow \text{add}(X, s(Y), s(Z)).
\]

• This program computes addition: if we read \( s \) as the “successor function,” which returns as value the value of its argument plus 1
• The \text{add} predicate computes the sum of its first two arguments into its third argument
• Consider the query:

\[
\text{add}(X, s^8(0), Z) \rightarrow
\]

• Possible ground witnesses are determined by the substitutions:
  \( \{X/0, Z/s^8(0)\} \),
  \( \{X/s(0), Z/s^9(0)\} \),
  \( \{X/s(s(0)), Z/s^{10}(0)\} \),…
The parameterized witness $Z = s^8(X)$ is the most general way to witness the existential query $\exists X \exists Z \text{ add}(X, s^8(0), Z)$ since it represents the fact that $\text{add}(X, s^8(0), Z)$ is true whenever the value of $Z$ equals the value of $X$ plus 8.

The computation of most general witnesses is the primary aim of a proof system, called *SLD resolution*.
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OWL2 RL represents the intersection of OWL and Horn logic, that is, the part of one language that can be translated in a semantics-preserving way from OWL to rules, and vice versa.

- From the modeler’s perspective, there is freedom to use either OWL or rules (and associated tools and methodologies) for modeling purposes, depending on the modeler’s experience and preferences.

- From the implementation perspective, either description logic reasoners or deductive rule systems can be used: it is possible to model using one framework, such as OWL, and to use a reasoning engine from the other framework, such as rules.
Some constructs of RDF Schema and OWL2 RL can be expressed in Horn logic, while some constructs, in general cannot be expressed.

- A triple of the form $(a, P, b)$ in RDF can be expressed as a fact:
  $P(a, b)$.
- An instance declaration of the form $\text{type}(a, C)$, stating that $a$ is an instance of class $C$, can be expressed as $C(a)$.
- The fact that $C$ is a subclass of $D$ is expressed as $C(X) \rightarrow D(X)$.
- The fact that $P$ is a subproperty of $Q$ is expressed as $P(X, Y) \rightarrow Q(X, Y)$. 
• Domain and range restrictions can also be expressed in Horn logic: \( C \) is the domain of property \( P \), while \( D \) is the range of property \( P \):

\[
\begin{align*}
P(X, Y) &\rightarrow C(X). \\
P(X, Y) &\rightarrow D(Y).
\end{align*}
\]

• \textit{equivalentClass(}\( C, D \)) can be expressed by the pair of rules:

\[
\begin{align*}
C(X) &\rightarrow D(X). \\
D(X) &\rightarrow C(X).
\end{align*}
\]

• \textit{equivalentProperty(}\( P, Q \)) can be expressed by the pair of rules:

\[
\begin{align*}
P(X, Y) &\rightarrow Q(X, Y). \\
Q(X, Y) &\rightarrow P(X, Y).
\end{align*}
\]
Transitivity of a property \( P \) is expressed as:
\[
P(X, Y), P(Y, Z) \rightarrow P(X, Z).
\]
The intersection of classes \( C_1 \) and \( C_2 \) is a subclass of \( D \):
\[
C_1(X), C_2(X) \rightarrow D(X).
\]
\( C \) is a subclass of the intersection of \( D_1 \) and \( D_2 \):
\[
C(X) \rightarrow D_1(X).
\]
\[
C(X) \rightarrow D_2(X).
\]

The union of \( C_1 \) and \( C_2 \) is a subclass of \( D \):
\[
C_1(X) \rightarrow D(X).
\]
\[
C_2(X) \rightarrow D(X).
\]

The opposite direction is outside the expressive power of Horn logic (see next slide).
C is a subclass of the union of D1 and D2 would require a disjunction in the head of the corresponding rule

\[ C(X) \rightarrow D1(X) \lor D2(X) \]

which is not available in Horn logic.
OWL2 RL: Description Logic Meets Rules

- OWL range restriction:
  \[ \text{\texttt{C rdfs:subClassOf [}} \]
  \[ \text{\texttt{rdf:type owl:Restriction ;}} \]
  \[ \text{\texttt{owl:onProperty :P ;}} \]
  \[ \text{\texttt{owl:allValuesFrom :D ]}} \]
  
  can be represented as the rule:
  \[ \text{C(X), P(X, Y) } \rightarrow \text{ D(Y).} \]

- the opposite direction cannot be expressed in Horn logic
[ rdf:type owl:Restriction ;
    owl:onProperty :P ;
    owl:allValuesFrom :D ]

    rdfs:subClassOf :C.

can be represented as the rule:

P(X, _), (Forall Y, (P(X, Y) → D(Y)))
→ C(X).

which is not available in Horn logic

• Also, cardinality constraints and complement of classes cannot be expressed in Horn logic.
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Rule Interchange Format: RIF

- Rules exhibit a broad variety (e.g., action rules, first order rules, logic programming)
  - As a consequence, the aim of the W3C Rule Interchange Format Working Group was not to develop a new rule language that would fit all purposes, but rather to focus on the interchange among the various (existing or future) rule systems on the web
- The approach taken was to develop a family of languages, from basic to existing state of the art, called dialects, that can be interchanged on the Web
- Most of the work of the RIF Working Group was dedicated to semantic aspects
  - Of course, rule interchange takes place at the syntactic level (e.g., using XML) using mappings between the various syntactic features of a logic system and RIF, but the main objective is to interchange rules in a semantics preserving way.
Rule Interchange Format: RIF

- Documents with links:
  - RIF Overview (Second Edition)
  - RIF Use Cases and Requirements (Second Edition)
  - RIF Core Dialect (Second Edition)
  - RIF Basic Logic Dialect (Second Edition)
  - RIF Production Rule Dialect (Second Edition)
  - RIF Framework for Logic Dialects (Second Edition)
  - RIF Datatypes and Built-Ins 1.0 (Second Edition)
  - RIF RDF and OWL Compatibility (Second Edition)
  - OWL 2 RL in RIF (Second Edition)
  - RIF Combination with XML data (Second Edition)
  - RIF In RDF (Second Edition)
  - RIF Test Cases (Second Edition)
  - RIF Primer (Second Edition)
Rule Interchange Format: RIF

- RIF defined two kinds of dialects:
  - **Logic-based dialects** are meant to include rule languages that are based on some form of logic; for example, first-order logic and various logic programming approaches with different interpretations of negation (answer-set programming, well-founded semantics, etc.)
  - The concrete dialects developed so far under this branch are:
    - **RIF Core** corresponding to function-free Horn logic
    - **RIF Basic Logic Dialect (BLD)** corresponding to Horn logic with equality
  - **Rules with actions** are meant to include production systems and reactive rules. The concrete dialect developed so far:
    - **Production Rule Dialect (RIF-PRD)**
Rule Interchange Format: RIF

- RIF was designed to be both uniform and extensible
  - Uniformity is achieved by expecting the syntax and semantics of all RIF dialects to share basic principles
  - Extensibility refers to the possibility of future dialects being developed and added to the RIF family
- For the logic-based side, the RIF Working Group developed the Framework for Logic Dialects (RIFFLD) which allows one to specify various rule languages by instantiating the various parameters of the approach.
The **RIF Basic Logic Dialect** corresponds to Horn logic with **equality** plus:

- **data types** (such as, integer, boolean, string, date),
- “**built-in**” **predicates** (such as, numeric-greater-than, starts-with, date-less-than), and **functions** (such as numeric-subtract, replace, hoursfrom-time), and
- **frames** (like in F-Logic) represent objects with their properties as **slots** (for example, a class professor with slots such as name, office, phone, department)

\[
\text{oid[slot1 -> value1,\ldots, slotn -> value)n]}
\]
RIF-BLD

• The syntax of RIF is straightforward, though quite verbose (of course, there is also an XML-based syntax to support interchange between rule systems)
  • Variable names begin with a question mark ?
  • The symbols =, #, and ## are used to express: equality, class membership, and subclass relationship
RIF-BLD

• Examples:
  • *A film is considered successful if it has received critical acclaim* (say, a rating higher than 8 out of 10) *or was financially successful* (produced more than $100 million in ticket sales).
  • *An actor is a movie star if he has starred in more than three successful movies, produced in a span of at least five years.*
These rules should be evaluated against the DBpedia data set:

Document(
  Prefix(func <http://www.w3.org/2007/rif-built-in-function#>)
  Prefix(pred <http://www.w3.org/2007/rif-built-in-predicate#>)
  Prefix(rdfs <http://www.w3.org/2000/01/rdf-schema#>)
  Prefix(imdbrel <http://example.com/imdbrelation#>)
  Prefix(dbpedia <http://dbpedia.org/ontology/>)
  Prefix(ibdbrel http://example.com/ibdbrelation#>)

Group(
  Forall ?Actor ?Film ?Year (  
    If And( dbpedia:starring(?Film ?Actor)  
      dbpedia:dateOfFilm(?Film ?Year)  
    )  
    Then dbpedia:starredInYear(?Film ?Actor ?Year)  
  )
)
RIF-BLD

Forall ?Film (  
  If Or (  
    External(pred:numeric-greater-than(dbpedia:criticalRating(?Film) 8)  
    External(pred:numeric-greater-than(dbpedia:boxOfficeGross(?Film) 100000000)))  
  Then dbpedia:successful(?Film)  
)  

- **External** applies built-in predicates.  
- **Group** to put together a number of rules.
RIF-BLD

Forall ?Actor (  
    If ( Exists ?Film1 ?Film2 ?Film3 ?Year1 ?Year2 ?Year3  
        And ( dbpedia:starredInYear(?Film1 ?Actor ?Year1)  
            dbpedia:starredInYear(?Film2 ?Actor ?Year2)  
            dbpedia:starredInYear(?Film3 ?Actor ?Year3)  
            External ( pred:numeric-greater-than(  
                External(func:numeric-subtract ?Year1 ?Year3) 5))  
            dbpedia:successful(?Film1)  
            dbpedia:successful(?Film2)  
            dbpedia:successful(?Film3)  
            External (pred:literal-not-identical(?Film1 ?Film2))  
            External (pred:literal-not-identical(?Film1 ?Film3))  
            External (pred:literal-not-identical(?Film2 ?Film3))  
        )  
    )  
    Then dbpedia:movieStar(?Actor)  
)
Compatibility with RDF and OWL

- A major feature of RIF is that it is compatible with the RDF and OWL standards
- Represent RDF triples using RIF frame formulas: a triple \((s \ p \ o)\) is represented as \(s[p \rightarrow o]\)
- That is, one can reason with a combination of RIF, RDF, and OWL documents
- RIF facilitates the interchange of not just rules, but also RDF graphs and/or OWL axioms
Compatibility with RDF and OWL

- The semantic definitions are such that the triple is satisfied if and only if the corresponding RIF frame formula is also satisfied.
- For example, if the RDF triple `ex:GoneWithTheWind ex:FilmYear ex:1939` is true, then so is the RIF fact `ex:GoneWithTheWind[ex:FilmYear -> ex:1939].`
Compatibility with RDF and OWL

• Given the RIF rule (which states that the Hollywood Production Code was in place between 1930 and 1968)

\[
\text{Group(}
\quad \text{Forall } ?\text{Film (}
\quad \text{If And( } ?\text{Film[ex:Year -> } ?\text{Year] }
\quad \text{External(pred:dateGreaterThan(?Year 1930))}
\quad \text{External(pred:dateGreaterThan(1968 ?Year)))}
\quad \text{Then } ?\text{Film[ex:HollywoodProductionCode -> ex:True]])}
\]

one can conclude

\[
\text{ex:GoneWithTheWind[}
\quad \text{ex:HollywoodProductionCode -> ex:True]}.
\]

as well as the corresponding RDF triple
Compatibility with RDF and OWL

- Similar techniques are used to achieve compatibility between OWL and RIF:
  - The semantics of OWL and RIF are compatible
  - One can infer conclusions from certain combinations of OWL axioms and RIF knowledge
  - OWL2 RL can be implemented in RIF
OWL2 RL in RIF

- OWL2 RL is partially described by a set of first-order rules that can form the basis for an implementation using rule technology
- To enable interoperability between rule systems and OWL2 RL ontologies, this axiomatization can be described using RIF (BLD, actually even in the simpler Core) rules
- The OWL2 RL rules can be categorized in four (non-disjoint) categories: triple pattern rules, inconsistency rules, list rules, and datatype rules
OWL2 RL in RIF

• *Triple Pattern Rules*: derive RDF triples from a conjunction of RDF triple patterns

```
Group(
  Forall ?V1 ... ?Vn(
    s[p->o] :-
    And(s1[p1->o1]... sn[pn->on]))
)
```
• **Inconsistency Rules**: indicate inconsistencies in the original RDF graph (w.r.t. the existing OWL knowledge)
  • represented in RIF as rules with the conclusion `rif:error`, a predicate symbol within the RIF namespace that can be used to express inconsistency
  • Example: an inconsistency occurs when two predicates have been declared to be **disjoint**, but connect the same entities

```riff
Group(
    rif:error :- And(
)```
• **List Rules:**

• A number of OWL2 RL rules involve processing OWL expressions that include RDF lists (for example **owl:AllDifferent**)

  ?x[rdf:type -> owl:AllDifferent]
  ?x[owl:members -> ?y]
  External(pred:list-contains(?y ?z1))
  ?iz1 = External(func:index-of(?y ?z1))
  External(pred:list-contains(?y ?z2))
  ?iz2 = External(func:index-of(?y ?z2))
  External( pred:numeric-not-equal(?iz1 ?iz2))
  ?z1[owl:sameAs->?z2] ) )
OWL2 RL in RIF

- **Datatype Rules**: provide type checking and value equality/inequality checking for typed literals in the supported datatypes
  - For example, generate an inconsistency if a literal is specified to be an instance of a data type but its value is outside the value space of that data type

```
Forall ?lt (  
    rif:error() :- And (  
        ?lt[rdf:type->xsd:decimal]  
        External(  
            pred:is-literal-not-decimal( ?lt ))) )
```
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Semantic Web Rules Language (SWRL)

- SWRL is a proposed Semantic Web language combining OWL DL with function-free Horn logic and is written in Unary/Binary Datalog RuleML.
- It allows Horn-like rules to be combined with OWL DL ontologies.
- A rule in SWRL has the form:
  \[ B_1, \ldots, B_n \rightarrow A_1, \ldots, A_m \]
  where the commas denote conjunction on both sides of the arrow.
Semantic Web Rules Language (SWRL)

- $B_1, \ldots, B_n, A_1, \ldots, A_m$ can be of the forms:
  - $C(x)$,
  - $P(x, y)$,
  - $\text{sameAs}(x, y)$, or
  - $\text{differentFrom}(x, y)$,

where

- $C$ is an OWL class,
- $P$ is an OWL property, and
- $x, y$ are Datalog variables, OWL individuals, or OWL data values.
The main complexity of the SWRL language stems from the fact that arbitrary OWL expressions, such as restrictions, can appear in the head or body of a rule. This adds significant expressive power to OWL, but at the high price of undecidability. A sublanguage is the extension of OWL DL with DL-safe rules, in which every variable must appear in a non-description logic atom in the rule body.
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Rules in SPARQL: SPIN

- Rules can be expressed in SPARQL using **CONSTRUCT**:

  \[
  \text{grandparent}(X, Z) \leftarrow
  \text{parent}(Y, Z), \text{parent}(X, Y).
  \]

  can be expressed as:

  ```sparql
  CONSTRUCT {
    ?X grandParent ?Z.
  } WHERE {
    ?Y parent ?Z.
    ?X parent ?Y.
  }
  ```
Rules in SPARQL: SPIN

- SPIN provides abstraction mechanisms for rules using *Templates*, which encapsulate parameterized SPARQL queries; and user-defined SPIN functions as a mechanism to build higher-level rules (complex SPARQL queries) on top of simpler building blocks.
Rules in SPARQL: SPIN

\[ C_2(X) \leftarrow C_1(X), \text{ equivalentClass}(C_1, C_2) \]
can be represented in SPARQL as:

```
CONSTRUCT {  
}
WHERE {  
}
```

and then instantiated as a `spin:rule` for the class `owl:Thing` to allow the rule to be applied to all possible instances.
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Nonmonotonic Rules: Motivation and Syntax

- In nonmonotonic rule systems, a rule may not be applied even if all premises are known because we have to consider contrary reasoning chains.
  - the rules we consider from now on are called **defeasible** because they can be defeated by other rules.
  - negated atomic formulas may occur in the head and the body of rules.

\[
p(X) \Rightarrow q(X).
\]
\[
r(X) \Rightarrow \neg q(X).
\]
given also the facts
\[
p(a).
\]
\[
r(a).
\]
we can conclude both \(q(a)\) and \(\neg q(a)\) (impossible)
Nonmonotonic Rules: Motivation and Syntax

- Conflicts may be resolved using priorities among rules
  - Suppose we knew somehow that the first rule is stronger than the second; then we could derive only $q(a)$.

- Priorities arise naturally in practice:
  - The source of one rule may be more reliable than the source of the second rule, or it may have higher authority
    - For example, federal law preempts state law
    - And in business administration, higher management has more authority than middle management
  - One rule may be preferred over another because it is more recent
  - One rule may be preferred over another because it is more specific
    - A typical example is a general rule with some exceptions; in such cases, the exceptions are stronger than the general rule
      - A classical example is that, in general, birds fly, however penguins are birds that do not fly
Nonmonotonic Rules: Motivation and Syntax

- Extend the rule syntax to include a unique label:
  \[ r_1: \ p(X) \ \Rightarrow \ q(X). \]
  \[ r_2: \ r(X) \ \Rightarrow \ \neg q(X). \]
  \[ r_1 > r_2. \]
  given the facts
  \[ p(a). \]
  \[ r(a). \]
  we can conclude \[ q(a). \]

- We can require the priority relation to be acyclic: it is impossible to have cycles of the form
  \[ r_1 > r_2 > \ldots > r_n > r_1 \]
Nonmonotonic Rules: Motivation and Syntax

• Priorities are meant to resolve conflicts among *competing rules*: two rules are *competing* only if the head of one rule is the negation of the head of the other

• In applications it is often the case that once a predicate $p$ is derived, some other predicates are excluded from holding
  • For example, an investment consultant may base his recommendations on three levels of risk that investors are willing to take: low, moderate, and high.
  • Only one risk level per investor is allowed to hold at any given time
  • Technically, these situations are modeled by maintaining a *conflict set* $C(L)$ for each literal $L$
  • $C(L)$ always contains the negation of $L$ but may contain more literals
Nonmonotonic Rules: Motivation and Syntax

- A defeasible rule has the form:
  \[ r : L_1, \ldots, L_n \Rightarrow L \]
  where
  - \( r \) is the label,
  - \( \{L_1, \ldots, L_n\} \) the body (or premises), and
  - \( L \) the head of the rule.
- \( L, L_1, \ldots, L_n \) are positive or negative literals (a literal is an atomic formula \( p(t_1, \ldots, t_m) \) or its negation \( \neg p(t_1, \ldots, t_m) \)).
- No function symbols may occur in the rule.
- Sometimes we denote the head of a rule as \( \text{head}(r) \), and its body as \( \text{body}(r) \).
Nonmonotonic Rules: Motivation and Syntax

- We use the label $r$ to refer to the whole rule.
- A *defeasible logic program* is a triple $(F, R, >)$ consisting of a set $F$ of facts, a finite set $R$ of defeasible rules, and an acyclic binary relation $>$ on $R$ (precisely, a set of pairs $r > r'$ where $r$ and $r'$ are labels of rules in $R$).
Example of Nonmonotonic Rules: Brokered Trade

- Electronic commerce application
- Brokered trades take place via an independent third party, the broker
- The broker matches the buyer’s requirements and the sellers’ capabilities and proposes a transaction in which both parties can be satisfied by the trade
- Concrete application: *apartment renting* (common activity that is often tedious and time-consuming)
Example of Nonmonotonic Rules: Brokered Trade

- Carlos is looking for an apartment of at least 45 sq m with at least two bedrooms.
- If it is on the third floor or higher, the house must have an elevator.
- Also, pet animals must be allowed.
- Carlos is willing to pay $300 for a centrally located 45 sq m apartment, and $250 for a similar apartment in the suburbs.
- In addition, he is willing to pay an extra $5 per square meter for a larger apartment, and $2 per square meter for a garden.
- He is unable to pay more than $400 in total.
- If given the choice, he would go for the cheapest option.
- His second priority is the presence of a garden; his lowest priority is additional space.
Formalization of Carlos’s Requirements

• Predicates that describe properties of apartments:
  • \textit{apartment}(x) stating that: \( x \) is an apartment
  • \textit{size}(x, y): \( y \) is the size of apartment \( x \) (in sq m)
  • \textit{bedrooms}(x, y): \( x \) has \( y \) bedrooms
  • \textit{price}(x, y): \( y \) is the price for \( x \)
  • \textit{floor}(x, y): \( x \) is on the \( y \)th floor
  • \textit{garden}(x, y): \( x \) has a garden of size \( y \)
  • \textit{elevator}(x): there is an elevator in the house of \( x \)
  • \textit{pets}(x): pets are allowed in \( x \)
  • \textit{central}(x): \( x \) is centrally located
Formalization of Carlos’s Requirements

- We also make use of the predicates:
  - \textbf{acceptable}(x): apartment x satisfies Carlos’s requirements
  - \textbf{offer}(x, y): Carlos is willing to pay $y for flat x
- Any apartment is a priori acceptable:
  - r1: \text{apartment}(X) \Rightarrow \text{acceptable}(X).
- However, apartment Y is unacceptable if one of Carlos’s requirements is not met:
  - r2: \text{bedrooms}(X, Y ), Y < 2 \Rightarrow \neg \text{acceptable}(X).
  - r3: \text{size}(X, Y ), Y < 45 \Rightarrow \neg \text{acceptable}(X).
  - r4: \neg \text{pets}(X) \Rightarrow \neg \text{acceptable}(X).
  - r5: \text{floor}(X, Y ), Y > 2, \neg \text{lif t}(X) \Rightarrow \neg \text{acceptable}(X).
  - r6: \text{price}(X, Y ), Y > 400 \Rightarrow \neg \text{acceptable}(X).
Formalization of Carlos’s Requirements

• Rules $\textbf{r2-r6}$ are exceptions to rule $\textbf{r1}$:

  \begin{align*}
  r2 & > r1. \\
  r3 & > r1. \\
  r4 & > r1. \\
  r5 & > r1. \\
  r6 & > r1. 
  \end{align*}
Formalization of Carlos’s Requirements

- We calculate the price Carlos is willing to pay for an apartment:
  \[ r_7 : \text{size}(X, Y), Y \geq 45, \text{garden}(X, Z), \text{central}(X) \Rightarrow \text{offer}(X, 300 + 2Z + 5(Y - 45)). \]
  \[ r_8 : \text{size}(X, Y), Y \geq 45, \text{garden}(X, Z), \neg\text{central}(X) \Rightarrow \text{offer}(X, 250 + 2Z + 5(Y - 45)). \]

- An apartment is only acceptable if the amount Carlos is willing to pay is higher than the price specified by the landlord (we assume no bargaining can take place)
  \[ r_9 : \text{offer}(X, Y), \text{price}(X, Z), Y < Z \Rightarrow \neg\text{acceptable}(X). \]
  \[ r_9 > r_1. \]
Representation of Available Apartments

- Each available apartment is given a unique name (for example \texttt{a1}), and its properties are represented as facts:
  \begin{verbatim}
  bedrooms(a1, 1).
size(a1, 50).
central(a1).
floor(a1, 1).
¬elevator(a1).
pets(a1).
garden(a1, 0).
price(a1, 300).
  \end{verbatim}

- In practice, the apartments on offer could be stored in a relational database, CSV file, or an RDF storage system.
Selecting an Apartment

<table>
<thead>
<tr>
<th>Flat</th>
<th>Bedrooms</th>
<th>Size</th>
<th>Central</th>
<th>Floor</th>
<th>Elevator</th>
<th>Pets</th>
<th>Garden</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>1</td>
<td>50</td>
<td>yes</td>
<td>1</td>
<td>no</td>
<td>yes</td>
<td>0</td>
<td>300</td>
</tr>
<tr>
<td>$a_2$</td>
<td>2</td>
<td>45</td>
<td>yes</td>
<td>0</td>
<td>no</td>
<td>yes</td>
<td>0</td>
<td>335</td>
</tr>
<tr>
<td>$a_3$</td>
<td>2</td>
<td>65</td>
<td>no</td>
<td>2</td>
<td>no</td>
<td>yes</td>
<td>0</td>
<td>350</td>
</tr>
<tr>
<td>$a_4$</td>
<td>2</td>
<td>55</td>
<td>no</td>
<td>1</td>
<td>yes</td>
<td>no</td>
<td>15</td>
<td>330</td>
</tr>
<tr>
<td>$a_5$</td>
<td>3</td>
<td>55</td>
<td>yes</td>
<td>0</td>
<td>no</td>
<td>yes</td>
<td>15</td>
<td>350</td>
</tr>
<tr>
<td>$a_6$</td>
<td>2</td>
<td>60</td>
<td>yes</td>
<td>3</td>
<td>no</td>
<td>no</td>
<td>0</td>
<td>370</td>
</tr>
<tr>
<td>$a_7$</td>
<td>3</td>
<td>65</td>
<td>yes</td>
<td>1</td>
<td>no</td>
<td>yes</td>
<td>12</td>
<td>375</td>
</tr>
</tbody>
</table>

- If we match Carlos’s requirements and the available apartments, we see:
  - flat $a_1$ is not acceptable because it has one bedroom only (rule r2)
  - flats $a_4$ and $a_6$ are unacceptable because pets are not allowed (rule r4)
  - for $a_2$, Carlos is willing to pay $300, but the price is higher (rules r7, r9)
  - flats $a_3, a_5,$ and $a_7$ are acceptable (rule r1)
Selecting an Apartment

- Carlos’s preferences are based on price, garden size, and size, in that order:
  
r10 : acceptable(X) ⇒ cheapest(X).

  r11 : acceptable(X), price(X, Z),
    acceptable(Y), price(Y, W), W<Z ⇒ ¬cheapest(X).

  r11 > r10.

- Rule r10 says that every acceptable apartment is cheapest by default.

- However, if there is an acceptable apartment cheaper than X, rule r11 (which is stronger than r10) fires and concludes that X is not cheapest.
Selecting an Apartment

• Carlos’s preferences are based on price, garden size, and size, in that order:

\[ r_{12} : \text{cheapest}(X) \Rightarrow \text{largestGarden}(X). \]

\[ r_{13} : \text{cheapest}(X), \text{gardenSize}(X, Z), \text{cheapest}(Y), \text{gardenSize}(Y,W), W > Z \Rightarrow \neg \text{largestGarden}(X). \]

\[ r_{13} > r_{12}. \]

• Rules \( r_{12} \) and \( r_{13} \) select the apartments with the largest garden among the cheapest apartments.
Selecting an Apartment

- Carlos’s preferences are based on price, garden size, and size, in that order:

  \[ r_{14} : \text{largestGarden}(X) \Rightarrow \text{rent}(X). \]

  \[ r_{15} : \text{largestGarden}(X), \text{size}(X, Z), \text{largestGarden}(Y), \text{size}(Y, W), W > Z \Rightarrow \neg \text{rent}(X). \]

  \[ r_{15} > r_{14}. \]

- Rules \( r_{14} \) and \( r_{15} \) select the proposed apartments to be rented, based on apartment size.
Selecting an Apartment

<table>
<thead>
<tr>
<th>Flat</th>
<th>Bedrooms</th>
<th>Size</th>
<th>Central</th>
<th>Floor</th>
<th>Elevator</th>
<th>Pets</th>
<th>Garden</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>1</td>
<td>50</td>
<td>yes</td>
<td>1</td>
<td>no</td>
<td>yes</td>
<td>0</td>
<td>300</td>
</tr>
<tr>
<td>$a_2$</td>
<td>2</td>
<td>45</td>
<td>yes</td>
<td>0</td>
<td>no</td>
<td>yes</td>
<td>0</td>
<td>335</td>
</tr>
<tr>
<td>$a_3$</td>
<td>2</td>
<td>65</td>
<td>no</td>
<td>2</td>
<td>no</td>
<td>yes</td>
<td>0</td>
<td>350</td>
</tr>
<tr>
<td>$a_4$</td>
<td>2</td>
<td>55</td>
<td>no</td>
<td>1</td>
<td>yes</td>
<td>no</td>
<td>15</td>
<td>330</td>
</tr>
<tr>
<td>$a_5$</td>
<td>3</td>
<td>55</td>
<td>yes</td>
<td>0</td>
<td>no</td>
<td>yes</td>
<td>15</td>
<td>350</td>
</tr>
<tr>
<td>$a_6$</td>
<td>2</td>
<td>60</td>
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<td>3</td>
<td>no</td>
<td>no</td>
<td>0</td>
<td>370</td>
</tr>
<tr>
<td>$a_7$</td>
<td>3</td>
<td>65</td>
<td>yes</td>
<td>1</td>
<td>no</td>
<td>yes</td>
<td>12</td>
<td>375</td>
</tr>
</tbody>
</table>

- Apartments $a_3$ and $a_5$ are cheapest
- $a_5$ has the largest garden and will be rented (after is inspected)
  - in this case the apartment size criterion does not play a role: $r_{14}$ fires only for $a_5$, so rule $r_{15}$ is not applicable for it
Lecture Outline

- Monotonic Rules
- OWL2 RL: Description Logic Meets Rules
- Rule Interchange Format: RIF
- Semantic Web Rules Language (SWRL)
- Rules in SPARQL: SPIN
- Nonmonotonic Rules
  - Example: Brokered Trade
- Rule Markup Language (RuleML)
Rule Markup Language (RuleML)

- RuleML is a long-running effort to develop markup of rules on the web
- It is actually not one language but a family of rule markup languages, corresponding to different kinds of rule languages: derivation rules, integrity constraints, reaction rules
- The kernel of the RuleML family is Datalog, which is function-free Horn logic
- The RuleML family provides descriptions of rule markup languages in XML
**Rule Markup Language (RuleML)**

- Vocabulary of Datalog RuleML:

<table>
<thead>
<tr>
<th>Rule Ingredient</th>
<th>RuleML</th>
</tr>
</thead>
<tbody>
<tr>
<td>fact</td>
<td>Asserted Atom</td>
</tr>
<tr>
<td>rule</td>
<td>Asserted Implies</td>
</tr>
<tr>
<td>head</td>
<td>then</td>
</tr>
<tr>
<td>body</td>
<td>if</td>
</tr>
<tr>
<td>atom</td>
<td>Atom</td>
</tr>
<tr>
<td>conjunction</td>
<td>And</td>
</tr>
<tr>
<td>predicate</td>
<td>Rel</td>
</tr>
<tr>
<td>constant</td>
<td>Ind</td>
</tr>
<tr>
<td>variable</td>
<td>Var</td>
</tr>
</tbody>
</table>
Rule Markup Language (RuleML)

• Example rule: “The discount for a customer buying a product is 7.5 percent if the customer is premium and the product is luxury”

• In RuleML 1.0:

```xml
<Implies>
  <if>
    <And>
      <Atom>
        <Rel>premium</Rel>
        <Var>customer</Var>
      </Atom>
      <Atom>
        <Rel>luxury</Rel>
        <Var>product</Var>
      </Atom>
    </And>
  </if>
  <then>
    <Atom>
      <Rel>discount</Rel>
      <Var>customer</Var>
      <Var>product</Var>
      <Ind>7.5 percent</Ind>
    </Atom>
  </then>
</Implies>
```
Rule Markup Language (RuleML)

- SWRL is an extension of RuleML and is represented in RuleML 1.0:
  \[ \text{brother}(X, Y), \text{childOf}(Z, Y) \rightarrow \text{uncle}(X, Z). \]

```
<ruleml:Implies>
  <ruleml:then>
    <swrlx:individualPropertyAtom swrlx:property="uncle">
      <ruleml:Var>X</ruleml:Var>
      <ruleml:Var>Z</ruleml:Var>
    </swrlx:individualPropertyAtom>
  </ruleml:then>
  <ruleml:if>
    <ruleml:And>
      <swrlx:individualPropertyAtom swrlx:property="brother">
        <ruleml:Var>X</ruleml:Var>
        <ruleml:Var>Y</ruleml:Var>
      </swrlx:individualPropertyAtom>
      <swrlx:individualPropertyAtom swrlx:property="childOf">
        <ruleml:Var>Z</ruleml:Var>
        <ruleml:Var>Y</ruleml:Var>
      </swrlx:individualPropertyAtom>
    </ruleml:And>
  </ruleml:if>
</ruleml:Implies>
```
Summary

• Rules on the (semantic) web form a very rich and heterogeneous landscape
• Horn logic is a subset of predicate logic that allows efficient reasoning
  • It forms a subset orthogonal to description logics
  • Horn logic is the basis of monotonic rules
• RIF is a new standard for rules on the web
  • Its logical dialect BLD is based on Horn logic
  • OWL2 RL, which is essentially the intersection of description logics and Horn logic, can be embedded in RIF
• SWRL is a much richer rule language, combining description logic features with restricted types of rules
• Nonmonotonic rules are useful in situations where the available information is incomplete
  • They are rules that may be overridden by contrary evidence (other rules)
  • Priorities are used to resolve some conflicts between nonmonotonic rules