Logic Languages

CSE 307 – Principles of Programming Languages
Stony Brook University

http://www.cs.stonybrook.edu/~cse307
Languages

- Languages:
  - Imperative = Turing machines
  - Functional Programming = lambda calculus
  - Logical Programming = first-order predicate calculus

- Prolog and its variants make up the most commonly used Logical programming languages.
  - One variant is XSB Prolog (developed here at Stony Brook)
  - Other Prolog systems: SWI Prolog, Sicstus, Yap Prolog, Ciao Prolog, GNU Prolog, etc.
    - ISO Prolog standard.
Relations/Predicates

- Predicates are building-blocks in predicate calculus: \( p(a_1, a_2, \ldots, a_k) \)
  - \( \text{parent}(X, Y) \): \( X \) is a parent of \( Y \).
    - \( \text{parent}(pam, bob) \).
    - \( \text{parent}(bob, ann) \).
    - \( \text{parent}(tom, bob) \).
    - \( \text{parent}(bob, pat) \).
    - \( \text{parent}(tom, liz) \).
    - \( \text{parent}(pat, jim) \).

- \( \text{male}(X) \): \( X \) is a male.
  - \( \text{male}(tom) \).
  - \( \text{male}(bob) \).
  - \( \text{male}(jim) \).

We attach meaning to them, but within the logical system they are simply structural building blocks, with no meaning beyond that provided by explicitly-stated interrelationships.
Relations

- **female**(X): X is a female.

  - female(pam).
  - female(pat).
  - female(ann).
  - female(liz).
Relations

parent(pam, bob).
parent(tom, bob).
parent(tom, liz).
parent(bob, ann).
parent(bob, pat).
parent(pat, jim).
female(pam).
female(pat).
female(ann).
female(liz).

male(tom).
male(bob).
male(jim).
Relations

• Rules:
  
  • \texttt{mother(X, Y)} : X is the mother of Y.
  
  - FOL:
    \[ \forall X, Y, \ parent(X, Y) \land \ female(X) \Rightarrow \ mother(X, Y) \]
  
  - In Prolog:
    \[ \texttt{mother(X,Y) :- parent(X,Y), female(X).} \]

• “,” means \textit{and} (conjunction), “:-” means \textit{if} (implication) and “;” means \textit{or} (disjunction).
Relations

• More Relations:

\[
\text{grandparent}(X,Y) :- \text{parent}(X,Z), \text{parent}(Z,Y).\]

(c) Paul Fodor (CS Stony Brook) and Elsevier
Relations

\[ \text{sibling}(X,Y) :\!-\! \text{parent}(Z,X), \]
\[ \text{parent}(Z,Y), \; X \neq Y. \]
Relations

• More Relations:

\[ \text{cousin}(X, Y) :- \ldots \]
\[ \text{greatgrandparent}(X, Y) :- \ldots \]
\[ \text{greatgreatgrandparent}(X, Y) :- \ldots \]
Recursion

\[
\text{ancestor}(X,Y) \leftarrow \\
\quad \text{parent}(X,Y).
\]

\[
\text{ancestor}(X,Y) \leftarrow \\
\quad \text{parent}(X,Z), \\
\quad \text{ancestor}(Z,Y).
\]

?- \text{ancestor}(X,\text{jim}).

?- \text{ancestor}(\text{pam},X).

?- \text{ancestor}(X,Y).
How to implement “I'm My Own Grandpa”?
https://www.youtube.com/watch?v=eYlJH81dSiw
Recursion

- What about:

\[
\text{ancestor}(X,Y) :-
\text{ancestor}(X,Z),
\text{parent}(Z,Y).
\]

\[
\text{ancestor}(X,Y) :-
\text{parent}(X,Y).
\]

? - ancestor(X,Y). INFINITE LOOP
Computations in Prolog

?- mother(M, bob).

?- parent(M, bob), female(M).

?- M=pam, female(pam).

M = pam  true

?- father(M, bob).

?- parent(M, bob), male(M)

(i) ?- M=pam, male(pam).

fail

(ii) ?- M=tom, male(tom).

M = tom  true
Prolog Execution

- Call: Call a predicate (invocation)
- Exit: Return an answer to the caller
- Fail: Return to caller with no answer
- Redo: Try next path to find an answer
The XSB Prolog System

- [http://xsb.sourceforge.net](http://xsb.sourceforge.net)
  - Developed at Stony Brook by David Warren and many contributors.

- Overview of Installation:
  - Unzip/untar; this will create a subdirectory XSB
  - Windows: you are done
  - Linux:
    - `cd XSB/build`
    - `./configure`
    - `./makexsb`
    - That’s it!
  - Cygwin under Windows: same as in Linux
Use of XSB

- Put your ruleset and data in a file with extension .P (or .pl)
  
  ```
  p(X) :- q(X,_) .
  q(1,a) .
  q(2,a) .
  q(b,c) .
  ?- p(X) .
  ```

- Don’t forget: all rules and facts end with a period (.)

- Comments: /*...*/ or %.... (% acts like // in Java/C++)

- Type
  
  ```
  .../XSB/bin/xsb  
  ...
  ```

  (Linux/Cygwin)

  ```
  ...
  ```

  (Windows)

  where ... is the path to the directory where you downloaded XSB

- You will see a prompt

  ```
  | ?-
  ```

  and are now ready to type queries
Use of XSB

• Loading your program, myprog.P

?- [myprog].

XSB will compile myprog.P (if necessary) and load it. Now you can type further queries, e.g.

?- p(X).
?- p(1).

• Some Useful Built-ins:
  • write(X) — write whatever X is bound to
  • writeln(X) — write then put newline
  • nl — output newline
  • Equality: =
  • Inequality: \= 

http://xsb.sourceforge.net/manual1/index.html (Volume 1)
Use of XSB

• Some Useful Tricks:

• XSB returns only the first answer to the query. To get the next, type ; <Return>. For instance:

```prolog
?- q(X).
X = 2;
X = 4
yes
```

• Usually, typing the ;’s is tedious. To do this programmatically, use this idiom:

```prolog
?- (q(_X), write('X='), writeln(_X), fail ; true).
```

_X here tells XSB to not print its own answers, since we are printing them by ourselves. (XSB won’t print answers for variables that are prefixed with a _.)
Syntax of Prolog Programs

- A *program* is a sequence of clauses.
- Each *clause* is of the form *head :- body*.
- Head is one *term*.
- Body is a comma-separated list of terms.
- A clause with an empty body is called a *fact*.
- A clause is also sometimes called a *rule*.
Logic Programming Concepts

- Operators:
  - conjunction, disjunction, negation, implication

- Universal and existential quantifiers

- Statements
  - sometimes true, sometimes false, sometimes unknown

- Axioms - assumed true

- Theorems - provably true

- Goals - things we'd like to prove true
Most statements can be written many ways.

That's great for people but a nuisance for computers.

It turns out that if you make certain restrictions on the format of statements you can prove theorems mechanically.

That's what logic programming systems do.
Logic Programming Concepts

- In Prolog all statements are in the form of **HORN CLAUSES** consisting of a **HEAD** and a **BODY**: HEAD :- BODY.

- The **HEAD** is a single term
- The **BODY** is a list (conjunction) of terms
- A term can be a constant, variable, or **STRUCTURE** consisting of a **FUNCTOR** and a parenthesized list of arguments
The meaning of a rule is that the conjunction of the terms in the body implies the head.

- A clause with an empty body is called a FACT:
  \[
  \text{raining(\text{ny})}.
  \]

- A clause with both sides is a RULE:
  \[
  \text{wet}(X) :- \text{raining}(X).
  \]
  Single-assignment variables: X must have the same value on both sides.

- A clause with an empty head is a QUERY, or top-level GOAL:
  \[
  ?- \text{wet}(X).
  \]

The Prolog interpreter has a collection of facts and rules in its DATABASE.

- Facts are axioms - things the interpreter assumes to be true.
- Prolog provides an automatic way to deduce true results from facts and rules.
Logic Programming Concepts

- A structure can play the role of a data structure or a predicate
- A constant is either an ATOM or a NUMBER
  - An atom is either what looks like an identifier beginning with a lowercase letter, or a single quoted character string
  - A number looks like an integer or real from some more ordinary language
- A variable looks like an identifier beginning with an uppercase letter
- There are NO declarations (vars, terms, predicates)
- All types are discovered implicitly
Logic Programming Concepts

- So, rules are theorems that allow the interpreter to infer things.
- To be interesting, rules generally contain variables:
  
  \[ \text{employed}(X) :\neg \text{employs}(Y,X). \]

  can be read:
  
  "for all X, X is employed if there exists a Y such that Y employs X"

- Note the direction of the implication.
- Also, the example does NOT say that X is employed ONLY IF there is a Y that employs X.
Logic Programming Concepts

- The scope of a variable is the clause in which it appears
  - Variables whose first appearance is on the left hand side of the clause have implicit universal quantifiers
  - Variables whose first appearance is in the body of the clause have implicit existential quantifiers
grandmother(A, C) :-
    mother(A, B),
    mother(B, C).

can be read:
"for all A, C [A is the grandmother of C if there exists a B such that A is the mother of B and B is the mother of C]"

• We probably want another rule that says:
grandmother(A, C) :-
    mother(A, B),
    father(B, C).
Recursion

• Transitive closure:
  • Example: a graph declared with facts (true statements)
    edge(1,2).
    edge(2,3).
    edge(2,4).

1) if there's an edge from X to Y, we can reach Y from X:
   reach(X,Y) :- edge(X,Y).

2) if there's an edge from X to Z, and we can reach Y from Z, then we can reach Y from X:
   reach(X,Y) :-
     edge(X,Z),
     reach(Z,Y).
?- reach(X, Y).

X = 1

Y = 2; ← Type a semi-colon repeatedly for more answers

X = 2

Y = 3;

X = 2

Y = 4;

X = 1

Y = 3;

X = 1

Y = 4;

no
Prolog Programs

• We will now explore Prolog program in more detail:

• Syntax of Prolog Programs
  • A *program* is a sequence of clauses (Horn rules).
  • Each *clause* is of the form header : - body.
  • Head is one *term*.
  • Body is a comma-separated list of terms.
  • A clause with an empty body is called a *fact*.
  • A clause is also sometimes called a *rule*. 
Terms

• Atomic data
• Variables
• Structures
Atomic Data

• Numeric constants: Integers, floating point numbers (e.g. 1024, -42, 3.1415, 6.023e23,...)

• Atoms:
  • Strings of characters enclosed in single quotes (e.g. 'Stony Brook')
  • Identifiers: sequence of letters, digits, underscore, beginning with a lower case letter (e.g. paul, r2d2, one_element).
Variables

- Variables are denoted by identifiers beginning with an *Uppercase letter or underscore* (e.g. X, Index, _param).

- **These are Single-Assignment Logical variables:**
  - Variables can be assigned only once
  - Different occurrences of the same variable in a clause denote the same data.
  - Variables are implicitly declared upon first use
  - Variables are not typed
    - All types are discovered implicitly (no declarations in LP)
  - If the variable does not start with underscore, it is assumed that it appears multiple times in the rule.
    - If it does not appear multiple times, then a warning is produced: "Singleton variable"
    - You can use variables preceded with underscore to eliminate this warning
Variables

- **Anonymous variables** (also called *Don’t care variables*): variables beginning with "_"

- Underscore, by itself (i.e., _), represents a variable
  - Each occurrence of _ corresponds to a different variable; even within a clause, _ does not stand for one and the same object.

- A variable with a name beginning with "_", but has more characters:
  - we want to give it a descriptive name
  - sometimes it is used to create relationships within a clause (and must therefore be used more than once): a warning is produced: "Singleton-marked variable appears more than once"
Variables

• Warnings are used to identify bugs (most because of copy-paste errors)
  • Instead of declarations and type checking
  • Fix all the warnings in a program, so you know that you don't miss any logical error
Variables

- Variables can be assigned only once, but that value can be further refined:

  \[ \text{- } X = f(Y), \]
  \[ \text{Y = g(Z),} \]
  \[ \text{Z = 2.} \]

  \[ X = f(g(2)) \]

- Even infinite structures:

  \[ \text{- } X = f(X). \]
Logic Programming Queries

- To run a Prolog program, one asks the interpreter a question
- This is done by asking a query which the interpreter tries to prove:
  - If it can, it says yes
  - If it can't, it says no
  - If your predicate contained variables, the interpreter prints the values it had to give them to make the predicate true.

Yes                        Yes                        No
X = ny;            X = a      X = a, Y = d
X = seattle; no

(c) Paul Fodor (CS Stony Brook) and Elsevier
Meaning of Logic Programs

- **Declarative Meaning:** What are the *logical consequences* of a program?
- **Procedural Meaning:** For what values of the variables in the query can I *prove* the query?
- The user gives the system a goal:
  - The system attempts to find axioms + inference steps to prove goal.
  - If goal contains variables, then also gives the values for those variables.
Declarative Meaning

brown(bear).
big(bear).
gray(elephant).
big(elephant).
black(cat).
small(cat).
dark(Z) :- black(Z).
dark(Z) :- brown(Z).
dangerous(X) :- dark(X), big(X).

• Logical consequence of a program L is the smallest set such that
  • All facts of the program are in L,
  • If \( H \) :- \( B_1, B_2, \ldots, B_n \)
    is an instance of a clause in the program such that \( B_1, B_2, \ldots, B_n \) are all in L, then H is also in L.

• For the above program we get dark(cat) and dark(bear) and consequently dangerous(bear).
Procedural Meaning of Prolog

brown(bear).  big(bear).
gray(elephant).  big(elephant).
black(cat).  small(cat).
dark(Z) :- black(Z).
dark(Z) :- brown(Z).
dangerous(X) :- dark(X), big(X).

• A query is, in general, a conjunction of goals: \( G_1, G_2, \ldots, G_n \)
• To prove \( G_1, G_2, \ldots, G_n \):
  • Find a clause \( H : - B_1, B_2, \ldots, B_k \) such that \( G_1 \) and \( H \) match.
  • Under that substitution for variables, prove \( B_1, B_2, \ldots, B_k, G_2, \ldots, G_n \)
If nothing is left to prove then the proof succeeds!
If there are no more clauses to match, the proof fails!
Procedural Meaning of Prolog

brown(bear). big(bear).
gray(elephant). big(elephant).
black(cat). small(cat).
dark(Z) :- black(Z).
dark(Z) :- brown(Z).
dangerous(X) :- dark(X), big(X).

• To prove: ?- dangerous(Q).

  1. Select dangerous(X) :- dark(X), big(X) and prove dark(Q), big(Q).
  2. To prove dark(Q) select the first clause of dark, i.e. dark(Z) :-
     black(Z), and prove black(Q), big(Q).
  3. Now select the fact black(cat) and prove big(cat).
  4. Go back to step 2, and select the second clause of dark, i.e. dark(Z) :-
     brown(Z), and prove brown(Q), big(Q).

This proof fails!
Procedural Meaning of Prolog

brown(bear).
big(bear).
gray(elephant).
big(elephant).
black(cat).
small(cat).
dark(Z) :- black(Z).
dark(Z) :- brown(Z).
dangerous(X) :- dark(X), big(X).

To prove: ?- dangerous(Q).

5. Now select brown(bear) and prove big(bear).
6. Select the fact big(bear).

There is nothing left to prove, so the proof succeeds
Procedural Meaning of Prolog

brown(bear).  big(bear).
gray(elephant).  big(elephant).
black(cat).  small(cat).
dark(Z) :- black(Z).
dark(Z) :- brown(Z).
dangerous(X) :- dark(X), big(X).

dangerous(Q) :-
dark(Q), big(Q).

dark(Q) :- black(Q).
dark(Q) :- brown(Q).

black(Q), big(Q)  brown(Q), big(Q)

black(cat).  big(bear).

fail  succeed
The Prolog interpreter works by what is called BACKWARD CHAINING (top-down, goal directed)

- It begins with the thing it is trying to prove and works backwards looking for things that would imply it, until it gets to facts.

It is also possible in theory to work forward from the facts trying to see if any of the things you can prove from them are what you were looking for (bottom-up resolution) - that can be very time-consuming

- Example: Answer set programming, DLV, Potassco (the Potsdam Answer Set Solving Collection), OntoBroker

- Fancier logic languages use both kinds of chaining, with special smarts or hints from the user to bound the searches
Procedural Meaning of Prolog

- The interpreter starts at the beginning of your database (this ordering is part of Prolog, NOT of logic programming in general) and looks for something with which to unify the current goal
  - If it finds a fact, great; it succeeds,
  - If it finds a rule, it attempts to satisfy the terms in the body of the rule depth first.

- This process is motivated by the RESOLUTION PRINCIPLE, due to Robinson, 1965:
  - It says that if C1 and C2 are Horn clauses, where C2 represents a true statement and the head of C2 unifies with one of the terms in the body of C1, then we can replace the term in C1 with the body of C2 to obtain another statement that is true if and only if C1 is true
Procedural Meaning of Prolog

- When it attempts resolution, the Prolog interpreter pushes the current goal onto a stack, makes the first term in the body the current goal, and goes back to the beginning of the database and starts looking again.
- If it gets through the first goal of a body successfully, the interpreter continues with the next one.
- If it gets all the way through the body, the goal is satisfied and it backs up a level and proceeds.
Procedural Meaning of Prolog

• If it fails to satisfy the terms in the body of a rule, the interpreter undoes the unification of the left hand side (this includes uninstantiating any variables that were given values as a result of the unification) and keeps looking through the database for something else with which to unify (This process is called BACKTRACKING).

• If the interpreter gets to the end of database without succeeding, it backs out a level (that's how it might fail to satisfy something in a body) and continues from there.
Procedural Meaning of Prolog

- PROLOG IS NOT PURELY DECLARATIVE:
  - The ordering of the database and the left-to-right pursuit of sub-goals gives a deterministic imperative semantics to searching and backtracking,
  - Changing the order of statements in the database can give you different results:
    - It can lead to infinite loops,
    - It can certainly result in inefficiency.
Procedural Meaning of Prolog

• Transitive closure with left recursion will run into an infinite loop:

```prolog
reach(X,Y) :-
  reach(X,Z),
  edge(Z, Y).
reach(X,Y) :-
  edge(X,Y).
?- reach(A,B).
```

infinite loop
Structures

• If $f$ is an identifier and $t_1, t_2, \ldots, t_n$ are terms, then $f(t_1, t_2, \ldots, t_n)$ is a term.

In the above, $f$ is called a **functor** and $t_i$ is an **argument**.

• Structures are used to group related data items together (in some ways similar to `struct` in C and objects in Java).

• Structures are used to construct trees (and, as a special case, lists).
Trees

- Example: expression trees:

\[
\text{plus}(\text{minus}(\text{num}(3),\text{num}(1)),\text{star}(\text{num}(4),\text{num}(2)))
\]

- Data structures may have variables. And the same variable may occur multiple times in a data structure.
Matching

• (We'll later introduce unification, a related operation that has logical semantics).

• $t_1 = t_2$: find substitutions for variables in $t_1$ and $t_2$ that make the two terms identical.

Yes, with $X = 1$, $Y = 4$. 

\[
\begin{array}{c}
\text{num} \quad \text{num} \quad \text{num} \quad \text{num} \\
3 \quad x \quad y \quad 2
\end{array}
\quad = 
\begin{array}{c}
\text{num} \quad \text{num} \quad \text{num} \quad \text{num} \\
3 \quad 1 \quad 4 \quad 2
\end{array}
\]
Matching

- Matching: given two terms, we can ask if they "match" each other. Rules:
  - A constant matches with itself: 42 unifies with 42.
  - A variable matches with anything:
    - if it matches with something other than a variable, then it instantiates,
    - if it matches with a variable, then the two variables become associated.
      - $A=35, A = B \Rightarrow B$ becomes 35.
      - $A = B, A=35 \Rightarrow B$ becomes 35.
  - Two structures match if they:
    - Have the same functor,
    - Have the same arity,
    - Match recursively.
      - $\text{foo}(g(42), 37)$ matches with $\text{foo}(A, 37), \text{foo}(g(A), B)$, etc.
Matching

- General Rule to decide whether two terms, S and T match are as follows:
  - If S and T are constants, \( S = T \) if both are same object
  - If S is a variable and T is anything, \( T = S \)
  - If T is variable and S is anything, \( S = T \)
  - If S and T are structures, \( S = T \) if
    - S and T have same functor
    - All their corresponding arguments components have to match
Matching

Yes, with $X = 1$, $Y = 4$. 
Matching

No! $X$ cannot be 1 and 4 at the same time.
Matching

- Which of these match?
  - A
  - 100
  - func(B)
  - func(100)
  - func(C, D)
  - func(+ (99, 1))
Matching

- Which of these match?
  - A
  - 100
  - func(B)
  - func(100)
  - func(C, D)
  - func(+ (99, 1))

A matches with 100, func(B), func(100), func(C,D), func(+ (99, 1)).
100 matches with A.
func(B) matches with A, func(100), func(+ (99, 1))
func(C, D) matches with A.
func(+ (99, 1)) matches with A, func(B).
Accessing arguments of a structure

- Matching is the predominant means for accessing a structures arguments.

- Let date('Sep', 1, 2015) be a structure used to represent dates, with the month, day and year as the three arguments (*in that order!*).

  Then $\text{date}(\text{M, D, Y}) = \text{date('Sep', 1, 2015)}$ makes
  $\text{M} = \text{'Sep'}, \quad \text{D} = 1, \quad \text{Y} = 2015$.

- If we want to get only the day, we can write
  $\text{date( _, D, _) = date('Sep', 1, 2015)}$.

  Then we only get: $\text{D} = 1$. 
Lists

- Prolog uses a special syntax to represent and manipulate lists (syntactic sugar = internally, it uses structures):
  - \([1,2,3,4]\): represents a list with 1, 2, 3 and 4, respectively.
  - This can also be written as \([1 \mid [2,3,4]]\): a list with 1 as the *head* (first element) and \([2,3,4]\) as its *tail* (the list of remaining elements).
  - If \(X = 1\) and \(Y = [2,3,4]\) then \([X \mid Y]\) is same as \([1,2,3,4]\).
  - The empty list is represented by \([\ ]\) or *nil*.
  - The symbol "|" (*pipe*) and is used to separate the beginning elements of a list from its tail.
    - For example: \([1,2,3,4] = [1 \mid [2,3,4]] = [1 \mid [2 \mid [3,4]]] = [1,2 \mid [3,4]]\)
Lists

• Lists are special cases of trees (i.e., (syntactic sugar = internally, it uses structures).

• For instance, the list \([1,2,3,4]\) is represented by the following structure:

[Diagram of a tree structure representing \([1,2,3,4]\)]

• where the function symbol \(./2\) is the list constructor. \([1,2,3,4]\) is same as \(.(1, .(2, .(3, .(4, []))))\)
Lists

• *Strings*: A sequence of characters surrounded by quotes is equivalent to a list of (numeric) character codes: “abc”, “to be, or not to be”.

First example: \texttt{member/2}, to find if a given element occurs in a list:

The program:

\begin{verbatim}
member(X, [X|_]).
member(X, [_|Ys]) :-
    member(X,Ys).
\end{verbatim}

Example queries:

?- member(2, [1,2,3]).
?- member(X, [l,i,s,t]).
?- member(f(X), [f(1),g(2),f(3),h(4)]).
Programming with Lists

- **append/3**: concatenate two lists to form the third list:

  - **The program**:
    - Empty list append A is A.
      
      ```
      append([], L, L).
      ```
    - Otherwise, break the first list up into a head X, tail L: if L append M is N, then X | N append M is X | N:
      
      ```
      append([X|L], M, [X|N]) :-
      append(L,M,N).
      ```

  - **Example queries**:
    
    ```
    ?- append([1,2],[3,4],X).
    ?- append(X, Y, [1,2,3,4]).
    ?- append(X, [3,4], [1,2,3,4]).
    ```
Programming with Lists

• Is the predicate a function?

• No. We are not applying arguments to get a result. Instead, we are proving that a theorem holds. Therefore, we can leave other variables unbound.

?- append(L, [2, 3], [1, 2, 3]).
   L = [ 1 ]
?- append([ 1 ], L, [1, 2, 3]).
   L = [2, 3]
?- append(L1, L2, [1, 2, 3]).
   L1 = [], L2 = [1, 2, 3];
   L1 = [1], L2 = [2, 3];
   L1 = [1, 2], L2 = [3];
   L1 = [1, 2, 3], L2 = [];
no
Append example trace

`append([],L,L).`

`append([X|L], M, [X|N]) :- append(L,M,N).`

`append([1,2],[3,4],X)?`
Append example trace

append([],L,L).
append([X|L],M,[X|N]) :- append(L,M,N).

append([1,2],[3,4],A)?  X=1, L=[2], M=[3,4], A=[X|N]
Append example trace

\[
\text{append}([],L,L).
\]
\[
\text{append}([X|L],M,[X|N]) :- \text{append}(L,M,N).
\]

\[
\begin{array}{l}
\text{append}([2],[3,4],N) ? \\
\text{append}([1,2],[3,4],A) ?
\end{array}
\]

\[
\begin{array}{l}
X=1,L=[2],M=[3,4],A=[X|N]
\end{array}
\]
Append example trace

```prolog
append([], L, L).
append([X|L], M, [X|N']) :- append(L, M, N').
```

Append example trace:

- `append([2], [3,4], N)`?
  - `X=2, L= [], M=[3,4], N=[2|N']`
- `append([1,2], [3,4], A)`?
  - `X=1, L=[2], M=[3,4], A=[1|N]`
Append example trace

```
append([],L,L).
append([X|L],M,[X|N’]) :- append(L,M,N’).
```

<table>
<thead>
<tr>
<th>append([],[3,4],N’) ?</th>
</tr>
</thead>
<tbody>
<tr>
<td>append([2],[3,4],N) ?</td>
</tr>
<tr>
<td>append([1,2],[3,4],A) ?</td>
</tr>
</tbody>
</table>

- append([],[3,4],N’)?
  - X=2, L=[], M=[3,4], N=[2|N’]
- append([2],[3,4],N)?
  - X=1, L=[2], M=[3,4], A=[1|N]
- append([1,2],[3,4],A)?
  - X=1, L=[2], M=[3,4], A=[1|N]
Append example trace

\texttt{append([],L,L).
append([X|L],M,[X|N']) :- append(L,M,N').}

\begin{tabular}{|l|l|}
\hline
\texttt{append([], [3,4], N')?} & \texttt{L = [3,4], N' = L} \\
\hline
\texttt{append([2], [3,4], N)?} & \texttt{X=2, L=[], M=[3,4], N=[2|N']} \\
\hline
\texttt{append([1,2], [3,4], A)?} & \texttt{X=1, L=[2], M=[3,4], A=[1|N]} \\
\hline
\end{tabular}
Append example trace

\begin{align*}
\text{append}([], L, L). \\
\text{append}([X|L], M, [X|N']) & : \text{- append}(L, M, N'). \\
\end{align*}

\begin{align*}
A & = [1|N] \\
N & = [2|N'] \\
N' & = L \\
L & = [3,4] \\
\text{Answer: } A = [1,2,3,4]
\end{align*}

\begin{tabular}{|c|}
\hline
append([], [3,4], N')? & \quad L = [3,4], N' = L \\
append([2], [3,4], N)? & \quad X=2, L=[], M=[3,4], N=[2|N'] \\
append([1,2], [3,4], A)? & \quad X=1, L=[2], M=[3,4], A=[1|N] \\
\hline
\end{tabular}
Programming with Lists

- **len/2** finds the length of a list (first argument):
  - The program:
    ```prolog
    len([], 0).
    len([_|Xs], N+1) :-
      len(Xs, N).
    ```
  - Example queries:
    ```prolog
    ?- len([], X).
      X = 0
    ?- len([l,i,s,t], 4).
      false
    ?- len([l,i,s,t], X).
      X = 0+1+1+1+1
    ```
Arithmetic

?- 1+2 = 3.

false

• In Predicate logic, the basis for Prolog, the only symbols that have a meaning are the predicates themselves.

• In particular, function symbols are uninterpreted: have no special meaning and can only be used to construct data structures.
Arithmetic

- Meaning for arithmetic expressions is given by the built-in predicate "is":

\[ ?- X \text{ is } 1 + 2. \]

\textit{succeeds, binding } X \text{ = 3.}

\[ ?- 3 \text{ is } 1 + 2. \]

\textit{succeeds.}

- General form: \( R \text{ is } E \) where \( E \) is an expression to be evaluated and \( R \) is matched with the expression's value.

- \( Y \text{ is } X + 1 \), where \( X \) is a free variable, will give an error because \( X \) does not (yet) have a value, so, \( X + 1 \) cannot be evaluated.
The list length example revisited

- **length/2** finds the length of a list (first argument):
  - The program:
    
    ```prolog
    length([], 0).
    length([_|Xs], M):-
        length(Xs, N),
        M is N+1.
    ```
  - Example queries:
    
    ```prolog
    ?- length([], X).
    ?- length([l,i,s,t], 4).
    ?- length([l,i,s,t], X).
    X = 4
    ?- length(List, 4).
    List = [_1, _2, _3, _4].
    ```
Conditional Evaluation

- Conditional operator: the if-then-else construct in Prolog:
  - if A then B else C is written as ( A -> B ; C)
  - To Prolog this means: try A. If you can prove it, go on to prove B and ignore C. If A fails, however, go on to prove C ignoring B.

```
max(X,Y,Z) :-
    ( X =< Y -> Z = Y ; Z = X )
```

?- max(1,2,X).
X = 2.
Conditional Evaluation

- Consider the computation of $n!$ (i.e. the factorial of $n$)

```
factorial(N, F) :- ... 
```

- $N$ is the input parameter; and $F$ is the output parameter!
- The body of the rule species how the output is related to the input.
  - For factorial, there are two cases: $N \leq 0$ and $N > 0$.
    - if $N \leq 0$, then $F = 1$
    - if $N > 0$, then $F = N \times \text{factorial}(N - 1)$

```
factorial(N, F) :-  
    (N > 0 
     -> N1 is N-1,  
        factorial(N1, F1),  
        F = N*F1 
      ; F = 1 
    ).  
```

?- factorial(12,X).
X = 479001600
Imperative features

- Other imperative features: we can think of prolog rules as imperative programs w/ backtracking.

```prolog
program :-
    member(X, [1, 2, 3, 4]),
    write(X),
    nl,
    fail.

program.
?- program. % prints all solutions
```

- **fail**: always fails, causes backtracking.
- **!** is the cut operator: prevents other rules from matching (we will see it later).
Therefore, Prolog Syntax:

- Assignments with arithmetic expressions is done using the keyword "is".
- If-then-else is written as
  
  \[
  ( \text{cond} \rightarrow \text{then-part} ; \text{else-part} )
  \]
- Arithmetic expressions are not directly used as arguments when calling a predicate; they are first evaluated, and then passed to the called predicate.
- If more than one action needs to be performed in a rule, they are written one after another, separated by a comma.
Arithmetic Operators

- Integer/Floating Point operators: +, -, *, /
- Automatic detection of Integer/Floating Point
- Integer operators: mod, // (integer division)
- Comparison operators: <, >, =<, >=,

$Expr1 =:= Expr2$ (succeeds if expression $Expr1$ evaluates to a number equal to $Expr2$),

$Expr1 =\not= Expr2$ (succeeds if expression $Expr1$ evaluates to a number non-equal to $Expr2$)
Programming with Lists

- We want to define `delete/3`, to remove a given element from a list (called `select/3` in XSB's basics library)

- Examples:
  - `delete([1,2,3], 2, X)` should succeed with `X = [1,3]`.
  - `delete([1,2,3], X, [1,3])` should succeed with `X = 2`.
  - `delete(X, 2, [1,3])` should succeed with 
    - `X=[2,1,3]`
    - `X=[1,2,3]`
    - `X=[1,3,2]`; fail
Programming with Lists

• Algorithm:
  • When $X$ is selected from $[X \mid Ys]$, $Ys$ results.
  • When $X$ is selected from the tail of $[H \mid Ys]$, $[H \mid Zs]$ results, where $Zs$ is the result of taking $X$ out of $Ys$. 
Programming with Lists

- The program:

  \[
  \text{delete}([], X, _) :- \text{fail}.
  \]

  \[
  \text{delete}([X|Ys], X, Ys).
  \]

  \[
  \text{delete}([Y|Ys], X, [Y|Zs]) :-
    \text{delete}(Ys, X, Zs).
  \]

- Example queries:

  \[
  ?- \text{delete}([l,i,s,t], s, X).
  \]

  \[
  X = [l, i, t]
  \]

  \[
  ?- \text{delete}([l,i,s,t], X, Y).
  \]

  \[
  ?- \text{delete}(X, s, [l,i,t]).
  \]

  \[
  ?- \text{delete}(X, Y, [l,i,s,t]).
  \]
Permutations

- Define `permute/2`, to find a permutation of a given list.
  - E.g. `permute([1,2,3], X)` should return $X=[1,2,3]$ and upon backtracking, $X=[1,3,2]$, $X=[2,1,3]$, $X=[2,3,1]$, $X=[3,1,2]$, and $X=[3,2,1]$.
  - Hint: What is the relationship between the permutations of [1,2,3] and the permutations of [2,3]?

<table>
<thead>
<tr>
<th>permute([2,3], Y)</th>
<th>permute([1,2,3], Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[2,3]</td>
<td>[1,2,3]</td>
</tr>
<tr>
<td></td>
<td>[2,1,3]</td>
</tr>
<tr>
<td></td>
<td>[2,3,1]</td>
</tr>
<tr>
<td>[3,2]</td>
<td>[1,3,2]</td>
</tr>
<tr>
<td></td>
<td>[3,1,2]</td>
</tr>
<tr>
<td></td>
<td>[3,2,1]</td>
</tr>
</tbody>
</table>
The program:

\[
\text{permute}([], []) .
\]
\[
\text{permute}([X|Xs], Ys) :-
    \text{permute}(Xs, Zs),
    \text{delete}(Ys, X, Zs).
\]

Example query:

\[
?- \text{permute}([1,2,3], X).
\]
The Issue of Efficiency

• Define a predicate, \texttt{rev/2} that finds the reverse of a given list.
  • E.g. \texttt{rev([1,2,3], X)} should succeed with \(X = [3,2,1]\).
  • Hint: what is the relationship between the reverse of \([1,2,3]\) and the reverse of \([2,3]\)?

\begin{verbatim}
rev([], []).
rev([X|Xs], Ys) :- rev(Xs, Zs),
                  append(Zs, [X], Ys).
\end{verbatim}

• How long does it take to evaluate \texttt{rev([1, 2, \ldots, n], X)}?
  • \(T(n) = T(n - 1) + \text{ time to add 1 element to the end of an } n - 1 \text{ element list} = T(n - 1) + n - 1 = T(n - 2) + n - 2 + n - 1 = \ldots\)
  • \(\Rightarrow T(n) = O(n^2)\)
Making rev/2 faster

- Keep an accumulator: a stack all elements seen so far.
  - i.e. a list, with elements seen so far in reverse order.

- The program:

\[
\text{rev}(L1, L2) :- \text{rev}(L1, [], L2).
\]
\[
\text{rev}([X|Xs], \text{AccBefore}, \text{AccAfter}) :- \\
\quad \text{rev}(Xs, [X|\text{AccBefore}], \text{AccAfter}).
\]
\[
\text{rev}([], \text{Acc}, \text{Acc}). \text{ % Base case}
\]

- Example query:

\[
?- \text{rev}([1,2,3], [], X).
\]

which calls \text{rev}([2,3], [1], X)

which calls \text{rev}([3], [2,1], X)

which calls \text{rev}([], [3,2,1], X)
Tree Traversal

Assume you have a binary tree, represented by

node/3 facts: for internal nodes: node(a,b,c) means that a has b and c as children.

leaf/1 facts: for leaves: leaf(a) means that a is a leaf.

Example:

node(5, 3, 6). node(3, 1, 4). leaf(1). leaf(4). leaf(6).

Write a predicate preorder/2 that traverses the tree (starting from a given node) and returns the list of nodes in pre-order.
Tree Traversal

preorder(Root, [Root]) :-
    leaf(Root).

preorder(Root, [Root|L]) :-
    node(Root, Child1, Child2),
    preorder(Child1, L1),
    preorder(Child2, L2),
    append(L1, L2, L).

- The program takes $O(n^2)$ time to traverse a tree with $n$ nodes.
Difference Lists

- The lists in Prolog are singly-linked; hence we can access the first element in constant time, but need to scan the entire list to get the last element.

- However, unlike functional languages like Lisp or SML, we can use variables in data structures:
  - We can exploit this to make lists “open tailed”
Difference Lists

- When \( X = [1,2,3|Y] \), \( X \) is a list with 1, 2, 3 as its first three elements, followed by \( Y \).
- Now if \( Y = [4|Z] \) then \( X = [1,2,3,4|Z] \).
- We can think of \( Z \) as “pointing to” the end of \( X \).
- We can now add an element to the end of \( X \) in constant time!!
  - (e.g. \( Z = [5 | W] \))
- Open-tailed lists are also called *difference lists* in Prolog.
Tree Traversal, Revisited

preorder1(Node, List, Tail) :-
   node(Node, Child1, Child2),
   List = [Node|List1],
   preorder1(Child1, List1, Tail1),
   preorder1(Child2, Tail1, Tail).
preorder1(Node, [Node|Tail], Tail) :-
   leaf(Node).
preorder(Node, List) :-
   preorder1(Node, List, []).

• The program takes $O(n)$ time to traverse a tree with $n$ nodes.
Difference Lists: Conventions

- An difference list is represented by two variables: one referring to the entire list, and another to its (uninstantiate) tail.
  - e.g. \( X = [1,2,3 | Z] \).
- Most Prolog programmers use the notation List - Tail to denote a list List with tail Tail.
- Note that “-” is used as a data structure symbol (not used here for arithmetic).
Difference Lists: Conventions

- The preorder traversal program may be written as:

```prolog
preorder1(Node, [Node|L]-T) :-
    node(Node, Child1, Child2),
    preorder1(Child1, L-T1),
    preorder1(Child2, T1-T),
    preorder1(Node, [Node|T]-T).
```
Graphs in Prolog

- There are several ways to represent graphs in Prolog:
  - represent each edge separately as one clause (fact):
    
    \[
    \text{edge}(a,b). \quad \text{edge}(b,c).
    \]
  
  - isolated nodes cannot be represented, unless we have also \text{node}/1 facts
  
  - the whole graph as one data object: as a pair of two sets (nodes and edges):
    
    \[
    \text{graph}([a,b,c,d,f,g], \quad [\text{e}(a,b), \text{e}(b,c), \text{e}(b,f)])
    \]
    
  - list of arcs: \([a\rightarrow b, b\rightarrow c, b\rightarrow f]\)

  - adjacency-list: \([\text{n}(a,[b]), \text{n}(b,[c,f]), \text{n}(d,[])]\)
Graphs in Prolog

- Path from one node to another one:
  - a predicate `path(G,A,B,P)` to find an acyclic path P from node A to node B in the graph G.
  - The predicate should return all paths via backtracking.
  - We will solve it using the graph as a data object, like in `graph([a,b,c,d,f,g],[e(a,b), e(b,c), e(b,f)])`
Graphs in Prolog

Path from one node to another one:

\[
\text{path}(G,A,B,P) :- \\
\quad \text{path1}(G,A,[B],P).
\]

\[
\text{path1}(_,A,[A|P1],[A|P1]). \% \text{Base}
\]

\[
\text{path1}(G,A,[Y|P1],P) :- \\
\quad \text{adjacent}(X,Y,G), \\
\quad \neg \text{member}(X,[Y|P1]), \\
\quad \text{path1}(G,A,[X,Y|P1],P).
\]
Graphs in Prolog

- Acyclic graph path:

\[
\text{adjacent}(X,Y,\text{graph}(\_\_\_\_\_\_\_, Es)) \leftarrow \\
\quad \text{member}(\text{e}(X,Y), Es) . \\
\text{adjacent}(X,Y,\text{graph}(\_\_\_\_\_\_\_, Es)) \leftarrow \\
\quad \text{member}(\text{e}(Y,X), Es) .
\]
Graphs in Prolog

- Cycle from a given node:
  - a predicate \texttt{cycle(G,A,P)} to find a closed path (cycle) \texttt{P} starting at a given node \texttt{A} in the graph \texttt{G}.
  - The predicate should return all cycles via backtracking.

\[
\text{cycle}(G,A,P) \leftarrow \\
\text{adjacent}(B,A,G), \\
\text{path}(G,A,B,P1), \\
\text{length}(P1,L), \\
L > 2, \\
\text{append}(P1,\texttt{[A]},P).
\]
Aggregates in XSB

- `setof(?Template, +Goal, ?Set)`: ?Set is the set of all instances of Template such that Goal is provable.
- `bagof(?Template, +Goal, ?Bag)`: has the same semantics as `setof/3` except that the third argument returns an unsorted list that may contain duplicates.
- `findall(?Template, +Goal, ?List)`: is similar to predicate `bagof/3`, except that variables in Goal that do not occur in Template are treated as existential, and alternative lists are not returned for different bindings of such variables.
- `tfindall(?Template, +Goal, ?List)`: is similar to predicate `findall/3`, but the Goal must be a call to a single tabled predicate.
XSB Prolog

- Negation: \textit{not} (\textbackslash+/): negation-as-failure
- Another negation called \textit{tnot} (\texttt{TABLENING = memoization})
  - Use: \ldots: = \ldots, \textit{tnot}(\texttt{foobar}(X))
  - All variables under the scope of \textit{tnot} must also occur to the left of that scope in the body of the rule in other positive relations:
  - Ok: \ldots: = p(X,Y), \textit{tnot}(\texttt{foobar}(X,Y)), \ldots
  - Not ok: \ldots: = p(X,Z), \textit{tnot}(\texttt{foobar}(X,Y)), \ldots
- XSB also supports Datalog:
  \begin{verbatim}
  :- auto_table.
  \end{verbatim}
  at the top of the program file
XSB Prolog

- **Read/write from and to files:**
- **Edinburgh style:**

```prolog
?- see('a.txt'), read(X), seen.
?- tell('a.txt'),
    write('Hello, World!'), told.
```
• Read/write from and to files:
  • ISO style:

```prolog
?- open('a.txt', write, X),
   write(X, 'Hello, World!'),
   close(X).
```

Cut (logic programming)

- Cut (! in Prolog) is a goal which always succeeds, but cannot be backtracked past.

- **Green cut**
  
  ```prolog
  gamble(X) :- gotmoney(X), !.
  gamble(X) :- gotcredit(X), \ + gotmoney(X).
  ```

  *cut says “stop looking for alternatives”*

- by explicitly writing \ + gotmoney(X), it guarantees that the second rule will always work even if the first one is removed by accident or changed

- **Red cut**
  
  ```prolog
  gamble(X) :- gotmoney(X), !.
  gamble(X) :- gotcredit(X).
  ```

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Cut (logic programming)

• Consider:

\[ p(a).\ p(b). \]
\[ q(a).\ q(b).\ q(c). \]
\[ \textit{?- p(X),!}. \]
\[ X=a ; \]
\[ \text{no} \]
\[ \textit{?- p(X),!,q(Y)}. \]
\[ X=a, Y=a ; \]
\[ X=a, Y=b ; \]
\[ X=a, Y=c ; \]
\[ \text{no} \]
Testing types

- \texttt{atom(X)}
  Tests whether \( X \) is bound to a symbolic atom.
  
  \texttt{?- atom(a).}
  
  yes
  
  \texttt{?- atom(3).}
  
  no

- \texttt{integer(X)}
  Tests whether \( X \) is bound to an integer.

- \texttt{real(X)}
  Tests whether \( X \) is bound to a real number.
Testing for variables

• **is_list**(L)
  Tests whether L is bound to a list.

• **ground**(G)
  Tests whether G has unbound logical variables.

• **var**(X)
  Tests whether X is bound to a Prolog variable.
Control / Meta-predicates

- **call(P)**
  Force P to be a goal; succeed if P does, else fail.
Assert and retract

- **asserta(C)**
  Assert clause C into database above other clauses with the same key predicate. The key predicate of a clause is the first predicate encountered when the clause is read from left to right.

- **assertz(C), assert(C)**
  Assert clause C into database below other clauses with the same key predicate.

- **retract(C)**
  Retract C from the database. C must be sufficiently instantiated to determine the predicate key.
Prolog terms and clauses

- **clause(H,B)**
  Retrieves clauses in memory whose head matches H and body matches B. H must be sufficiently instantiated to determine the main predicate of the head.

- **functor(E,F,N)**
  E must be bound to a functor expression of the form 'f(...)'.
  F will be bound to 'f', and N will be bound to the number of arguments that f has.

- **arg(N,E,A)**
  E must be bound to a functor expression, N is a whole number, and A will be bound to the Nth argument of E.
Prolog terms and clauses

- `=..` converts between term and list. For example,

```prolog
?- parent(a,X) = .. L.
L = [parent, a, _X001]
```
Conclusion for this course

- That is all!
- My guess on where languages will be going: languages that combine:
  - Multiparadigm
  - High-level data structures
  - With: speed, simplicity (dynamic weakly typed)
- JavaScript frameworks, node.js, Google Go, Swift, and what else?
- More and more languages every day!!! What should we learn? All!
  - Youtube is implemented with Python,
  - IBM Watson uses Prolog,
  - Wikipedia is implemented with PHP,
  - Microsoft F# is a functional programming language, etc.
- More Scripting Languages: Writing programs by coordinating pre-existing components, rather than writing components from scratch.
Scripting-style:
- Speed: Trade almost everything for developer productivity
- Economy of Expression
- Lack of Declarations
- Simple rules
- Flexible dynamic typing
- Access to the OS
- Sophisticated string processing
- High-level data structures: Maps, Lists, Tuples, Sets.
- Batch and Interactive
- Open and Portable
- Single Canonical Implementation
- Interpreted: Fast to start
- Easily extended
- Easily embedded
Future is here
Future is here

- Coroutines
- Reification
- Java on GPUs (Sumatra)
- Self-Tuning JVM
- Meta-Object Protocol
- Multi-Tenancy
- JNI 2.0
- Java 9 ... and beyond
- Modular Platform (Jigsaw)
- More and More Ports
- Memory-Efficient Data Structures
- Tail Calls

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I'm hoping that this course prepared you for the change the future will bring in programming languages.

Thank you!