Logic Languages

CSE 307 – Principles of Programming Languages
Stony Brook University
http://www.cs.stonybrook.edu/~cse307
Languages

- Paradigms of Programming Languages:
  - Imperative = Turing machines
  - Functional Programming = lambda calculus
  - Logical Programming = first-order predicate calculus

- Prolog and its variants make up the most commonly used Logical programming languages.
  - One variant is XSB Prolog (developed here at Stony Brook)
  - Other Prolog systems: SWI Prolog, Sicstus, Yap Prolog, Ciao Prolog, GNU Prolog, etc.
    - ISO Prolog standard.
Relations/Predicates

- Predicates/relations are building-blocks in predicate calculus: \( p(a_1,a_2,\ldots,a_k) \)
- \( \text{parent}(X, Y) : X \) is a parent of \( Y \).

\[
\begin{align*}
\text{parent}(\text{pam}, \text{bob}). & \quad \text{parent}(\text{bob}, \text{ann}). \\
\text{parent}(\text{tom}, \text{bob}). & \quad \text{parent}(\text{bob}, \text{pat}). \\
\text{parent}(\text{tom}, \text{liz}). & \quad \text{parent}(\text{pat}, \text{jim}).
\end{align*}
\]

- We attach meaning to them, but within the logical system they are simply structural building blocks, with no meaning beyond that provided by explicitly-stated interrelationships.
Relations

- **female**(X): X is a female.
  
  - female(pam).
  - female(pat).
  - female(ann).
  - female(liz).

- **male**(X): X is a male.
  
  - male(tom).
  - male(bob).
  - male(jim).
Relations

parent(pam, bob).
parent(tom, bob).
parent(tom, liz).
parent(bob, ann).
parent(bob, pat).
parent(pat, jim).
female(pam).
female(pat).
female(ann).
female(liz).
male(tom).
male(bob).
male(jim).
Relations

- Rules:
  - `mother(X, Y)`: X is the mother of Y.

    - In FOL:
      \[
      \forall X, Y \, (parent(X, Y) \land \text{female}(X) \implies \text{mother}(X, Y))
      \]

    - In Prolog:
      \[
      \text{mother}(X,Y) :- \\
      \quad \text{parent}(X,Y), \\
      \quad \text{female}(X).
      \]

- “,” means and (conjunction), “:-” means if (implication) and “;” means or (disjunction).
More Relations:

grandparent(X,Y) :- parent(X,Z), parent(Z,Y).
Relations

\[ \text{sibling}(X,Y) : \neg \text{parent}(Z,X), \text{parent}(Z,Y), X \neq Y. \]

?- sibling(ann,Y).
Relations

• More Relations:

  $\text{cousin}(X,Y) :- \ldots$

  $\text{greatgrandparent}(X,Y) :- \ldots$

  $\text{greatgreatgrandparent}(X,Y) :- \ldots$
Recursion

\[
\text{ancestor}(X,Y) :- \\
\quad \text{parent}(X,Y).
\]

\[
\text{ancestor}(X,Y) :- \\
\quad \text{parent}(X,Z), \\
\quad \text{ancestor}(Z,Y).
\]

?- \text{ancestor}(X,jim).

?- \text{ancestor}(pam,X).

?- \text{ancestor}(X,Y).
• How to implement “I'm My Own Grandpa”?
https://www.youtube.com/watch?v=eYlJH81dSiw
Recursion

What about:

\[
\text{ancestor}(X, Y) \ :- \\
\quad \text{ancestor}(X, Z), \\
\quad \text{parent}(Z, Y).
\]

\[
\text{ancestor}(X, Y) \ :- \\
\quad \text{parent}(X, Y).
\]

\[- \quad \text{ancestor}(X, Y). \quad \text{INFINITE LOOP}\]
Computations in Prolog

?- mother(M, bob).

?- parent(M, bob), female(M).

?- M=pam, female(pam).

M = pam true

?- father(M, bob).

?- parent(M, bob), male(M)

(i) ?- M=pam, male(pam).

fail

(ii) ?- M=tom, male(tom).

M = tom true
Prolog Execution

- **Call**: Call a predicate (invocation)
- **Exit**: Return an answer to the caller
- **Fail**: Return to caller with no answer
- **Redo**: Try next path to find an answer
The XSB Prolog System

- [http://xsb.sourceforge.net](http://xsb.sourceforge.net)
  - Developed at Stony Brook by David Warren and many contributors.
- Overview of Installation:
  - Unzip/untar; this will create a subdirectory XSB
  - Windows: you are done
  - Linux:
    ```
    cd XSB/build
    ./configure
    ./makexsb
    ```
    That’s it!
- Cygwin under Windows: same as in Linux
Use of XSB

• Put your ruleset and data in a file with extension .P (or .pl)
  
  \[
p(X) :- q(X, _).
  
  q(1,a).
  
  q(2,a).
  
  q(b,c).
  
  ?- p(X).
  \]

• Don’t forget: all rules and facts end with a period (.)

• Comments: /*...*/ or %.... (% acts like // in Java/C++)

• Type
  
  \[.../XSB/bin/xsb \quad \text{(Linux/Cygwin)}\]
  
  \[...\XSB/config\x86-pc-windows/bin/xsb \quad \text{(Windows)}\]

  where ... is the path to the directory where you downloaded XSB

• You will see a prompt
  
  | ?-

  and are now ready to type queries
Use of XSB

• Loading your program, myprog.P

  ?- [myprog].

  XSB will compile myprog.P (if necessary) and load it. Now you can type further queries, e.g.

  ?- p(X).
  ?- p(1).

• Some Useful Built-ins:
  - write(X) – write whatever X is bound to
  - writeln(X) – write then put newline
  - nl – output newline
  - Equality: =
  - Inequality: \=

http://xsb.sourceforge.net/manual1/index.html (Volume 1)
Use of XSB

- Some Useful Tricks:
  - XSB returns only the first answer to the query. To get the next, type ; <Return>. For instance:

    | ?- q(X).
    X = 2;
    X = 4
    yes

  - Usually, typing the ;’s is tedious. To do this programmatically, use this idiom:

    | ?- q(_X), write('X='), writeln(_X), fail ; true.

    _X here tells XSB to not print its own answers, since we are printing them by ourselves. (XSB won’t print answers for variables that are prefixed with a _.)
Syntax of Prolog Programs

- A program is a sequence of clauses.
- Each clause is of the form head :- body.
- Head is one term.
- Body is a comma-separated list of terms.
- A clause with an empty body is called a fact.
- A clause is also sometimes called a rule.
Logic Programming Concepts

- Operators:
  - conjunction, disjunction, negation, implication

- Universal and existential quantifiers

- Statements
  - sometimes true, sometimes false, sometimes unknown
  - axioms - assumed true
  - theorems - provably true
  - goals - things we'd like to prove true
Most statements can be written many ways

That's great for people but a nuisance for computers

It turns out that if you make certain restrictions on the format of statements you can prove theorems mechanically

That's what logic programming systems do
Logic Programming Concepts

• In Prolog all statements are in the form of **HORN CLAUSES** consisting of a **HEAD** and a **BODY**:

  HEAD :- BODY.

• The **HEAD** is a single term

• The **BODY** is a list (conjunction) of terms

• A term can be a constant, variable, or **STRUCTURE** consisting of a **FUNCTOR** and a parenthesized list of arguments
Logic Programming Concepts

- A structure can play the role of a data structure or a predicate.
- A constant is either an ATOM or a NUMBER.
  - An atom is either what looks like an identifier beginning with a lowercase letter, or a single quoted character string.
  - A number looks like an integer or real from some more ordinary language.
- A variable looks like an identifier beginning with an uppercase letter.
- There are NO declarations (vars, terms, predicates).
- All types are discovered implicitly.
Logic Programming Concepts

- The meaning of a rule is that the conjunction of the structures in the body implies the head.
- A clause with an empty body is called a FACT:
  \[ \text{raining(\text{ny})}. \]
- A clause with both sides is a RULE:
  \[ \text{wet(X)} :\text{- raining(X)}. \]
  
  Single-assignment variables: X must have the same value on both sides
- A clause with an empty head is a QUERY, or top-level GOAL:
  \[ ?- \text{wet(X)}. \]
- The Prolog interpreter has a collection of facts and rules in its DATABASE.
  
  - Facts are axioms - things the interpreter assumes to be true.
  - Prolog provides an automatic way to deduce true results from facts and rules.
So, rules are theorems that allow the interpreter to infer things

To be interesting, rules generally contain variables

```
employed(X) :- employs(Y,X).
```

can be read:

"for all X, X is employed if there exists a Y such that Y employs X"

Note the direction of the implication

Also, the example does NOT say that X is employed ONLY IF there is a Y that employs X
The scope of a variable is the clause in which it appears:

- Variables whose first appearance is on the left hand side of the clause (the head) have implicit universal quantifiers.
- Variables whose first appearance is in the body of the clause have implicit existential quantifiers in that body.
grandmother(A, C) :-
mother(A, B),
mother(B, C).

can be read:
"for all A, C [A is the grandmother of C if there exists a B such that A is the mother of B and B is the mother of C]"

- We probably want another rule that says:

grandmother(A, C) :-
mother(A, B),
father(B, C).
Recursion

- Transitive closure:
  - Example: a graph declared with facts (true statements)
    \[
    \text{edge}(1, 2). \\
    \text{edge}(2, 3). \\
    \text{edge}(2, 4). \\
    \]
  1) if there's an edge from \(X\) to \(Y\), we can reach \(Y\) from \(X\):
     \[
     \text{reach}(X, Y) :- \text{edge}(X, Y). \\
     \]
  2) if there's an edge from \(X\) to \(Z\), and we can reach \(Y\) from \(Z\), then we can reach \(Y\) from \(X\):
     \[
     \text{reach}(X, Y) :- \\
     \text{edge}(X, Z), \\
     \text{reach}(Z, Y). \\
     \]
?- reach(X,Y).

X = 1
Y = 2 ;  ← Type a semi-colon repeatedly for
X = 2     more answers
Y = 3 ;
X = 2
Y = 4 ;
X = 1
Y = 3 ;
X = 1
Y = 4 ;
no

reach(X,Y) :- edge(X,Y).
reach(X,Y) :-
    edge(X,Z),
    reach(Z, Y).
Prolog Programs

• We will now explore Prolog programs in more detail:

• Syntax of Prolog Programs
  • A *program* is a sequence of clauses (Horn rules).
  • Each *clause* is of the form *head* : - *body*.
  • Head is one *term*.
  • Body is a comma-separated list of terms.
  • A clause with an empty body is called a *fact*.
  • A clause is also sometimes called a *rule*.
Terms

- Atomic data
- Variables
- Structures
Atomic Data

- **Numeric constants**: Integers, floating point numbers (e.g. 1024, -42, 3.1415, 6.023e23, …)
- **Atoms**:
  - Identifiers: sequence of letters, digits, underscore, beginning with a lower case letter (e.g. paul, r2d2, one_element).
  - Strings of characters enclosed in single quotes (e.g. 'Stony Brook')
Variables

- Variables are denoted by identifiers beginning with an **Uppercase letter or underscore** (e.g. \(X\), \(Index\), \(_param\)).

- **These are Single-Assignment Logical variables:**
  - Variables can be assigned only once
  - Different occurrences of the same variable in a clause denote the same data.
  - Variables are implicitly declared upon first use
  - Variables are not typed
    - All types are discovered implicitly (no declarations in LP)
  - If the variable does not start with underscore, it is assumed that it appears multiple times in the rule.
    - If it does not appear multiple times, then a warning is produced: "Singleton variable"
  - You can use variables preceded with underscore to eliminate this warning
Variables

- **Anonymous variables** (also called Don’t care variables): variables beginning with "/_
- Underscore, by itself (i.e., _), represents a variable
  - Each occurrence of _ corresponds to a different variable; even within a clause, _ does not stand for one and the same object.
- A variable with a name beginning with "_", but has more characters. E.g.: _radius, _Size
  - we want to give it a descriptive name
  - sometimes it is used to create relationships within a clause (and must therefore be used more than once): a warning is produced: "Singleton-marked variable appears more than once"
Variables

- Warnings are used to identify bugs (most because of copy-paste errors)
  - Instead of declarations and type checking
  - Fix all the warnings in a program, so you know that you don't miss any logical error
Variables

- Variables can be assigned only once, but that value can be further refined:

  \[ ?- \ X = f(Y), \]
  \[ Y = g(Z), \]
  \[ Z = 2. \]

  Therefore, \( X = f(g(2)) \)

- The order also does not matter:

  \[ ?- \ Z = 2, \]
  \[ Y = g(Z), \]
  \[ X = f(Y). \]

  Therefore, \( X = f(g(2)) \)

- Even infinite structures:

  \[ ?- \ X = f(X). \]

  \( X = f(f(f(f(f(f(f(f(f(\ldots)))))))) \)

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Logic Programming Queries

• To run a Prolog program, one asks the interpreter a question
  • This is done by asking a query which the interpreter tries to prove:
    • If it can, it says yes
    • If it can't, it says no
    • If your predicate contained variables, the interpreter prints the values it had to give them to make the predicate true.

```prolog
?- wet(ny).  \?- reach(a, d).  \?- reach(d, a).
Yes   Yes   No
?- wet(X).  \?- reach(X, d).  \?- reach(X, Y).
X = ny ;   X=a               X=a, Y=d
X = seattle ;
no
```

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Meaning of Logic Programs

- **Declarative Meaning:** What are the *logical consequences* of a program?
- **Procedural Meaning:** For what values of the variables in the query can I prove the query?

The user gives the system a goal:

- The system attempts to find axioms + inference steps to prove goal.
- If goal contains variables, then also gives the values for those variables.
Declarative Meaning

brown(bear). big(bear).
gray(elephant). big(elephant).
black(cat). small(cat).
dark(Z) :- black(Z).
dark(Z) :- brown(Z).
dangerous(X) :- dark(X), big(X).

• **Logical consequence of a program** $L$ is the *smallest* set such that
  • All facts of the program are in $L$,
  • If $H :- B_1, B_2, \ldots, B_n$ is an instance of a clause in the program such that $B_1, B_2, \ldots, B_n$ are all in $L$, then $H$ is also in $L$.

• For the above program we get $\text{dark(cat)}$ and $\text{dark(bear)}$ and consequently $\text{dangerous(bear)}$. 

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Procedural Meaning of Prolog

brown(bear).  big(bear).
gray(elephant).  big(elephant).
black(cat).  small(cat).
dark(Z) :- black(Z).
dark(Z) :- brown(Z).
dangerous(X) :- dark(X), big(X).

- A query is, in general, a conjunction of goals: $G_1, G_2, \ldots, G_n$
- To prove $G_1, G_2, \ldots, G_n$:
  - Find a clause $H : - B_1, B_2, \ldots, B_k$ such that $G_1$ and $H$ match.
  - Under the substitution for variables, prove $B_1, B_2, \ldots, B_k, G_2, \ldots, G_n$

If nothing is left to prove then the proof succeeds!
If there are no more clauses to match, the proof fails!
To prove:  \(?-\) **dangerous**(Q).

1. Select **dangerous**(X) :- **dark**(X), **big**(X) and prove **dark**(Q), **big**(Q).
2. To prove **dark**(Q) select the first clause of dark, i.e. **dark**(Z) :- **black**(Z), and prove **black**(Q), **big**(Q).
3. Now select the fact **black**(cat) and prove **big**(cat).
4. Go back to step 2, and select the second clause of dark, i.e. **dark**(Z) :- **brown**(Z), and prove **brown**(Q), **big**(Q).

This proof fails!
Procedural Meaning of Prolog

brown(bear). big(bear).
gray(elephant). big(elephant).
black(cat). small(cat).
dark(Z) :- black(Z).
dark(Z) :- brown(Z).
dangerous(X) :- dark(X), big(X).

• To prove: ?- dangerous(Q).

5. Now select brown(bear) and prove big(bear).
6. Select the fact big(bear).

There is nothing left to prove, so the proof succeeds
Procedural Meaning of Prolog

brown(bear).  big(bear).
gray(elephant).  big(elephant).
black(cat).  small(cat).
dark(Z) :- black(Z).
dark(Z) :- brown(Z).
dangerous(X) :- dark(X), big(X).

dangerous(Q)
dangerous(Q) :-
dark(Q), big(Q).
dark(Q), big(Q)
dark(Q) :- black(Q).
dark(Q) :- brown(Q).
black(Q), big(Q) brown(Q), big(Q)
black(cat).
brown(bear).
big(cat) big(bear)
fail succeed
The Prolog interpreter works by what is called BACKWARD CHAINING (top-down, goal directed)

- It begins with the thing it is trying to prove and works backwards looking for things that would imply it, until it gets to facts.

- It is also possible in theory to work forward from the facts trying to see if any of the things you can prove from them are what you were looking for (bottom-up resolution) - that can be very time-consuming

- Example: Answer Set Programming (ASP), (such as DLV, Potassco (the Potsdam Answer Set Solving Collection)), OntoBroker (uses LP well founded semantics)

- Fancier logic languages use both kinds of chaining, with special hints from the user to bound the searches
The interpreter starts at the beginning of your database (this ordering is part of Prolog, NOT of logic programming in general) and looks for something with which to unify the current goal:

- If it finds a fact, great; it succeeds,
- If it finds a rule, it attempts to satisfy the terms in the body of the rule depth first.

This process is motivated by the RESOLUTION PRINCIPLE, due to Robinson, 1965:

- It says that if C1 and C2 are Horn clauses, where C2 represents a true statement and the head of C2 unifies with one of the terms in the body of C1, then we can replace the term in C1 with the body of C2 to obtain another statement that is true if and only if C1 is true.
Procedural Meaning of Prolog

- When it attempts resolution, the Prolog interpreter pushes the current goal onto a stack, makes the first term in the body the current goal, and goes back to the beginning of the database and starts looking again.
- If it gets through the first goal of a body successfully, the interpreter continues with the next one.
- If it gets all the way through the body, the goal is satisfied and it backs up a level and proceeds.
Procedural Meaning of Prolog

- If it fails to satisfy the terms in the body of a rule, the interpreter undoes the unification of the left hand side (this includes uninstantiating any variables that were given values as a result of the unification) and keeps looking through the database for something else with which to unify (BACKTRACKING).

- If the interpreter gets to the end of database without succeeding, it backs out a level (that's how it might fail to satisfy something in a body) and continues from there.
Procedural Meaning of Prolog

- **PROLOG IS NOT PURELY DECLARATIVE:**
  - The ordering of the database and the left-to-right pursuit of sub-goals gives a deterministic imperative semantics to searching and backtracking.
  - Changing the order of statements in the database can give you different results:
    - It can lead to infinite loops,
    - It can certainly result in inefficiency.
Transitive closure with left recursion in Prolog will run into an infinite loop:

```prolog
reach(X,Y) :-
    reach(X,Z),
    edge(Z, Y).
reach(X,Y) :-
    edge(X,Y).

?- reach(A,B).
Infinite loop
```
Structures

• If $f$ is an identifier and $t_1, t_2, \ldots, t_n$ are terms, then $f(t_1, t_2, \ldots, t_n)$ is a term.

• In the above, $f$ is called a functor and $t_i$ is an argument.

• Structures are used to group related data items together (in some ways similar to `struct` in C and objects in Java).

• Structures are used to construct trees (and, as a special case, lists).
Trees

- Example: expression trees:

\[
\text{plus}(\text{minus}(\text{num}(3), \text{num}(1)), \text{star}(\text{num}(4), \text{num}(2)))
\]

- Data structures may have variables. And the same variable may occur multiple times in a data structure.
Matching

- (We'll later introduce unification, a related operation that has logical semantics).
- $t_1 = t_2$: find substitutions for variables in $t_1$ and $t_2$ that make the two terms identical.

Yes, with $X = 1$, $Y = 4$. 
Matching

- Matching: given two terms, we can ask if they "match" each other.
- Rules:
  - A constant matches with itself: $42$ unifies with $42$.
  - A variable matches with anything:
    - if it matches with something other than a variable, then it instantiates,
    - if it matches with a variable, then the two variables become associated.
    - $A=35$, $A=B \Rightarrow B$ becomes $35$.
    - $A=B$, $A=35 \Rightarrow B$ becomes $35$.
  - Two structures match if they:
    - Have the same functor,
    - Have the same arity,
    - Match recursively.
    - $\text{foo}(g(42),37)$ matches with $\text{foo}(A,37)$, $\text{foo}(g(A),B)$, etc.
Matching

• General Rules to decide whether two terms, $S$ and $T$ match are as follows:
  • If $S$ and $T$ are constants, $S=T$ if both are same object
  • If $S$ is a variable and $T$ is anything, $T=S$
  • If $T$ is variable and $S$ is anything, $S=T$
  • If $S$ and $T$ are structures, $S=T$ if
    • $S$ and $T$ have same functor and same arity
    • All their corresponding arguments components have to match
Matching

Yes, with $X = 1, Y = 4$. 
No! $X$ cannot be 1 and 4 at the same time.
Matching

- Which of these match?
  - A
  - 100
  - func(B)
  - func(100)
  - func(C, D)
  - func(+\(99, 1\))
Which of these match?

- A
- 100
- \texttt{func(B)}
- \texttt{func(100)}
- \texttt{func(C, D)}
- \texttt{func(+99\, 1)}

- A matches with 100, \texttt{func(B)}, \texttt{func(100)}, \texttt{func(C,D)}, \texttt{func(+99\, 1)}.
- 100 matches only with A.
- \texttt{func(B)} matches with A, \texttt{func(100)}, \texttt{func(+99\, 1)}.
- \texttt{func(C, D)} matches with A.
- \texttt{func(+99\, 1)} matches with A and \texttt{func(B)}. 
Accessing arguments of a structure

- Matching is the predominant means for accessing a structures' arguments.
- Let \( \text{date('Sep', 1, 2015)} \) be a structure used to represent dates, with the month, day and year as the three arguments (\textbf{in that order!}).

Then \( \text{date(M,D,Y)} = \text{date('Sep',1,2020)} \) makes

\[
M = 'Sep', \quad D = 1, \quad Y = 2015.
\]

- If we want to get only the day, we can write
\( \text{date(_, D, _)} = \text{date('Sep', 1, 2015)} \).
Then we only get: \( D = 1 \).
Lists

- Prolog uses a special syntax to represent and manipulate lists (syntactic sugar = internally, it uses structures):
  - \([1, 2, 3, 4]\): represents a list with 1, 2, 3 and 4, respectively.
  - This can also be written as \([1| [2, 3, 4]]\): a list with 1 as the head (first element) and [2, 3, 4] as its tail (the list of remaining elements).
    - If \(X = 1\) and \(Y = [2, 3, 4]\) then \([X|Y]\) is same as \([1, 2, 3, 4]\).
  - The empty list is represented by \([\ ]\) or \(nil\).
  - The symbol "|" (pipe) and is used to separate the beginning elements of a list from its tail.
    - For example: \([1, 2, 3, 4] = [1| [2, 3, 4]] = [1| [2| [3, 4]]] = [1, 2| [3, 4]] = [1, 2, 3| [4]] = [1| [2| [3| [4| [\ ]]]]]\).
Lists

- Lists are special cases of trees (i.e., (syntactic sugar = internally, it uses structures).
- For instance, the list \([1,2,3,4]\) is represented by the following structure:

\[
[1,2,3,4] \text{ is same as } .(1,.(2,.(3,.(4,[]))))
\]

- where the function symbol \(./2\) is the list constructor.
Lists

• *Strings*: A sequence of characters surrounded by double quotes is equivalent to a list of (numeric) character codes: "abc", "John Smith", "to be, or not to be".

• ?- X="abc".

  X = [97,98,99]
• First example: `member/2`, to find if a given element occurs in a list:

  • The program:

    ```prolog
    member(X, [X|_]).
    member(X, [_|Ys]) :-
      member(X,Ys).
    ```

  • Example queries:

    ```prolog
    ?- member(2,[1,2,3]).
    ?- member(X, [l,i,s,t]).
    ?- member(f(X),[f(1),g(2),f(3),h(4)]).
    ```
Programming with Lists

- **append/3**: concatenate two lists to form the third list:
  
  - The program:
    
    - Empty list append L is L.
      
      \[
      \text{append}([], L, L).
      \]
    
    - Otherwise, break the first list up into a head X, tail L: if L append M is N, then X | N append M is X | N:
      
      \[
      \text{append}([X|L], M, [X|N]) :- \quad \text{append}(L,M,N).
      \]
  
  - Example queries:
    
    ?- \ append([1,2],[3,4],X).
    
    ?- \ append(X, Y, [1,2,3,4]).
    
    ?- \ append(X, [3,4], [1,2,3,4]).
Programming with Lists

- Is the predicate a function?
  - No. We are not applying arguments to get a result.
  - Instead, we are proving that a theorem holds.
  - Therefore, we can leave any variables unbound.

```prolog
?- append(L, [2, 3], [1, 2, 3]).
   L = [1 ]
?- append([1], L, [1, 2, 3]).
   L = [2, 3]
?- append(L1, L2, [1, 2, 3]).
   L1 = [], L2 = [1, 2, 3];
   L1 = [1], L2 = [2, 3];
   L1 = [1, 2], L2 = [3] ;
   L1 = [1, 2, 3], L2 = [];
no
```
Append example trace

\[\text{append}([], L, L).\]
\[\text{append}([X|L], M, [X|N]) :- \text{append}(L, M, N).\]
Append example trace

```prolog
append([], L, L).
append([X|L], M, [X|N]) :- append(L, M, N).
```

```prolog
append([1,2],[3,4],A)?
```

Answer:

X=1, L=[2], M=[3,4], A=[X|N]
Append example trace

append([],L,L).
append([X|L],M,[X|N]) :- append(L,M,N).

append([2],[3,4],N)?

append([1,2],[3,4],A)? X=1, L=[2], M=[3,4], A=[X|N]
Append example trace

append([],L,L).
append([X|L],M,[X|N']) :- append(L,M,N').

append([2],[3,4],N)?  X=2, L=[], M=[3,4], N=[2|N']
append([1,2],[3,4],A)?  X=1, L=[2], M=[3,4], A=[1|N]
Append example trace

```prolog
append([], L, L).
append([X|L], M, [X|N']) :- append(L, M, N').
```

| append([1,2], [3,4], A)? | X=1, L=[2], M=[3,4], A=[1|N] |
|--------------------------|----------------------------------|
| append([2], [3,4], N)?   | X=2, L=[], M=[3,4], N=[2|N']    |
| append([], [3,4], N')?   |                                  |
Append example trace

```
append([],L,L).
append([X|L],M,[X|N']) :- append(L,M,N').
```

<table>
<thead>
<tr>
<th>Query</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>append([], [3,4], N')?</code></td>
<td>( L = [3,4], N' = L )</td>
</tr>
<tr>
<td><code>append([2], [3,4], N)?</code></td>
<td>( X=2, L=[], M=[3,4], N=[2</td>
</tr>
<tr>
<td><code>append([1,2], [3,4], A)?</code></td>
<td>( X=1, L=[2], M=[3,4], A=[1</td>
</tr>
</tbody>
</table>
Append example trace

```prolog
append([], L, L).
append([X|L], M, [X|N']) :- append(L, M, N').
```

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>append([], [3,4], N')</code></td>
<td><code>L = [3,4], N' = L</code></td>
</tr>
<tr>
<td><code>append([2], [3,4], N)</code></td>
<td>`X=2, L=[], M=[3,4], N=[2</td>
</tr>
<tr>
<td><code>append([1,2], [3,4], A)</code></td>
<td>`X=1, L=[2], M=[3,4], A=[1</td>
</tr>
</tbody>
</table>

Answer: \( A = [1,2,3,4] \)
Programming with Lists

- **len**/2 finds the length of a list (first argument):
  - The program:
    
    ```prolog
    len([], 0).
    len([_|Xs], N+1) :-
        len(Xs, N).
    ```
  - Example queries:
    ```prolog
    ?- len([], X).
    X = 0
    ?- len([l,i,s,t], 4).
    false
    ?- len([l,i,s,t], X).
    X = 0+1+1+1+1
    ```
Arithmetic

?- 1+2 = 3.

false

- In Predicate logic, the basis for Prolog, the only symbols that have a meaning are the predicates themselves.
- In particular, function symbols are uninterpreted: have no special meaning and can only be used to construct data structures.
Arithmetic

- Meaning for arithmetic expressions is given by the built-in predicate "is":

  ?- x is 1 + 2.

  succeeds, binding x = 3.

  ?- 3 is 1 + 2.

  succeeds.

- General form: \( R \text{ is } E \) where E is an expression to be evaluated and R is matched with the expression's value.

- \( Y \text{ is } X + 1 \), where X is a free variable, will give an error because X does not (yet) have a value, so, \( X + 1 \) cannot be evaluated.
The list length example revisited

- **length/2** finds the length of a list (first argument).

  - The program:

    ```prolog
    length([], 0).
    length([_|Xs], M):-
        length(Xs, N),
        M is N+1.
    ```

  - Example queries:

    ```prolog
    ?- length([], X).
    ?- length([1,i,s,t], 4).
    ?- length([1,i,s,t], X).
        X = 4
    ?- length(List, 4).
        List = [_1, _2, _3, _4]
    ```
Conditional Evaluation

- Conditional operator: the if-then-else construct in Prolog:
  - \textit{if} \( A \) \textit{then} \( B \) \textit{else} \( C \) is written as \(( A \rightarrow B ; C)\)
  - To Prolog this means: try \( A \). If you can prove it, go on to prove \( B \) and ignore \( C \). If \( A \) fails, however, go on to prove \( C \) ignoring \( B \).

\[
\text{max}(X,Y,Z) : - \\
( X =< Y \\
\rightarrow Z = Y \\
; Z = X \\
).
\]

?- \text{max}(1,2,X).
\text{X} = 2.
Conditional Evaluation

- Consider the computation of \( n! \) (i.e. the factorial of \( n \))

\[
\text{factorial}(N, F) :- \ldots
\]

- \( N \) is the input parameter; and \( F \) is the output parameter!
- The body of the rule species how the output is related to the input.
  - For factorial, there are two cases: \( N \leq 0 \) and \( N > 0 \).
    - if \( N \leq 0 \), then \( F = 1 \)
    - if \( N > 0 \), then \( F = N \times \text{factorial}(N - 1) \)

\[
\text{factorial}(N, F) :-
\quad (N > 0)
\quad -> N1 \text{ is } N-1,
\quad \text{factorial}(N1, F1),
\quad F \text{ is } N\times F1
\quad ; F = 1
\).
\]

?- factorial(12, X).
X = 479001600
Imperative features

• Other imperative features: we can think of Prolog rules as imperative programs with backtracking.

```prolog
program :-
    member(X, [1, 2, 3, 4]),
    write(X),
    nl,
    fail;
    true.

?- program. % prints all solutions
```

• `fail`: always fails, causes backtracking.
• `!` is the cut operator: prevents other rules from matching (we will see it later).
Arithmetic Operators

• Integer/Floating Point operators: +, -, *, /
• Automatic detection of Integer/Floating Point

• Integer operators: mod, // (integer division)

• Comparison operators: <, >, =<, >=,

Expr1 =:= Expr2 (succeeds if expression Expr1 evaluates to a number equal to Expr2),

Expr1 =\= Expr2 (succeeds if expression Expr1 evaluates to a number non-equal to Expr2)
Programming with Lists

- Quicksort:

  ```prolog
  quicksort([], []).
  quicksort([X0|Xs], Ys) :-
    partition(X0, Xs, Ls, Gs),
    quicksort(Ls, Ys1),
    quicksort(Gs, Ys2),
    append(Ys1, [X0|Ys2], Ys).

  partition(Pivot, [], [], []).
  partition(Pivot, [X|Xs], [X|Ys], Zs) :-
    X <= Pivot,
    partition(Pivot, Xs, Ys, Zs).
  partition(Pivot, [X|Xs], Ys, [X|Zs]) :-
    X > Pivot,
    partition(Pivot, Xs, Ys, Zs).
  ```
We want to define `select/3`, to remove a given element from a list (similar to `select/3` in XSB's `basics` library)

- `select(2, [1,2,3], X)` should succeed with `X = [1,3]`.
- `select(X, [1,2,3], [1,3])` should succeed with `X = 2`.
- `select(2, X, [1,3])` should succeed with `X=[2,1,3]; X=[1,2,3]; X=[1,3,2]; fail`
Algorithm:

- When $X$ is selected from $[X|Ys]$, $Ys$ results.
- When $X$ is selected from the tail of $[H|Ys]$, $[H|Zs]$ results, where $Zs$ is the result of taking $X$ out of $Ys$. 

The program:

\[
\text{select}(X, [], _) :- \text{fail}. \% \text{not needed}
\]
\[
\text{select}(X, [X|Ys], Ys).
\]
\[
\text{select}(X, [Y|Ys], [Y|Zs]) :-
\]
\[
\quad \text{select}(X, Ys, Zs).
\]

Example queries:

?- \text{select}(s, [l,i,s,t], Z).
\[
Z = [l, i, t]
\]

?- \text{select}(X, [l,i,s,t], Z).

?- \text{select}(s, Y, [l,i,t]).

?- \text{select}(X, Y, [l,i,s,t]).
?- select(1, B, [2,3]).

x = [1,2,3];
x = [2,1,3];
x = [2,3,1];
no
Permutations

- Define `permute/2`, to find a permutation of a given list.
  - E.g. `permute([1,2,3], X)` should return 
    
    X=[1,2,3] and upon backtracking, X=[1,3,2], X=[2,1,3], X=[2,3,1], X=[3,1,2], and 
    X=[3,2,1].
  
  - Hint: What is the relationship between the permutations of
    [1,2,3] and the permutations of [2,3]?
  
<table>
<thead>
<tr>
<th>permute([2,3],Y)</th>
<th>permute([1,2,3],Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[2,3]</td>
<td>[1,2,3]</td>
</tr>
<tr>
<td></td>
<td>[2,1,3]</td>
</tr>
<tr>
<td></td>
<td>[2,3,1]</td>
</tr>
<tr>
<td>[3,2]</td>
<td>[1,3,2]</td>
</tr>
<tr>
<td></td>
<td>[3,1,2]</td>
</tr>
<tr>
<td></td>
<td>[3,2,1]</td>
</tr>
</tbody>
</table>
The program:

\[
\text{permute}([], []).
\]

\[
\text{permute}([X|Xs], Ys) :-
\quad \text{permute}(Xs, Zs),
\quad \text{select}(X, Ys, Zs).
\]

Example query:

\[
?- \text{permute}([1,2,3], X).
\]

X=[1,2,3]; X=[2,1,3]; X=[2,3,1]; X=[1,3,2]; X=[3,1,2]; X=[3,2,1]; no
The Issue of Efficiency

• Define a predicate, `rev/2` that finds the reverse of a given list.
  - E.g. `rev([1,2,3],X)` should succeed with `X=[3,2,1]
  - Hint: what is the relationship between the reverse of `[1,2,3]` and the reverse of `[2,3]?` Answer: `append([3,2],[1],[3,2,1])`

\[\text{rev}([],[]).\]
\[\text{rev}([X|Xs],Ys) :- \text{rev}(Xs,Zs),\]
\[\text{append}(Zs,[X],Ys).\]

• How long does it take to evaluate `rev([1,2,...,n],X)`?
  - \(T(n) = T(n-1) + \text{time to add 1 element to the end of an } n-1 \text{element list} = T(n-1) + n - 1 = T(n-2) + n - 2 + n - 1 = ... = T(1) + 2 + ... + n - 2 + n - 1 = O(n^2)\)
Making rev/2 faster

• Keep an accumulator: a stack all elements seen so far.
  • i.e. a list, with elements seen so far in reverse order.

• The program:

rev(L1, L2) :- rev(L1, [], L2).

rev([X|Xs], AccBefore, AccAfter) :-
    rev(Xs, [X|AccBefore], AccAfter).

rev([], Acc, Acc). % Base case

• Example query:

?- rev([1,2,3], [], X).

which calls rev([2,3], [1], X)
which calls rev([3], [2,1], X)
which calls rev([], [3,2,1], X)
Tree Traversal

• Assume you have a binary tree, represented by

  • `node/3` facts: for internal nodes: `node(a,b,c)`
    means that `a` has `b` and `c` as children.

  • `leaf/1` facts: for leaves: `leaf(a)` means that `a`
    is a leaf.

• Example:
  
  ```prolog
  node(5, 3, 6).
  node(3, 1, 4).
  leaf(1).
  leaf(4).
  leaf(6).
  ```
Tree Traversal

• Write a predicate `preorder/2` that traverses the tree (starting from a given node) and returns the list of nodes in pre-order
Tree Traversal

preorder(Root, [Root]) :-
   leaf(Root).
preorder(Root, [Root|L]) :-
   node(Root, Child1, Child2),
   preorder(Child1, L1),
   preorder(Child2, L2),
   append(L1, L2, L).

- The program takes $O(n^2)$ time to traverse a tree with $n$ nodes because of the **append** at the end.
Difference Lists

• The lists in Prolog are singly-linked; hence we can access the first element in constant time, but need to scan the entire list to get the last element.

• However, unlike functional languages like Lisp or SML, we can use variables in data structures:

  • We can exploit this to make lists “open tailed”
Difference Lists

• When $x = [1, 2, 3 | y]$, $x$ is a list with 1, 2, 3 as its first three elements, followed by $y$.
  
• Now if $y = [4 | z]$ then $x = [1, 2, 3, 4 | z]$
  
• We can think of $z$ as “pointing to” the end of $x$.

• We can now add an element to the end of $x$ in constant time!!
  
  - (e.g. $z = [5 | w]$)

• Open-tailed lists are also called difference lists in Prolog.
Difference Lists: Conventions

- A difference list is represented by two variables: one referring to the entire list, and another to its (uninstantiated) tail.
  - e.g. $X = [1,2,3|Z]$.

- Most Prolog programmers use the notation $\text{List-Tail}$ to denote a list $\text{List}$ with tail $\text{Tail}$.
  - e.g. $X-Z$

- Note that “−” is used as a data structure symbol (not used here for arithmetic).
Difference Lists

- Append 2 open ended lists:
  \[ \text{dappend}(X,T,Y,T_2,L,T_3) : - T = Y, \]
  \[ T_2 = T_3, \]
  \[ L = X. \]

\[- \text{dappend}([1,2,3|T],T,[4,5,6|T_2],T_2,L,T_3). \]
\[ L = [1,2,3,4,5,6|T_2] \]

- Simplified version:
  \[ \text{dappend}(X,T, T,T_2, X,T_2). \]

- More simplified notation (with "/"): 
  \[ \text{dappend}(X-T, T-T_2, X-T_2). \]

\[- \text{dappend}([1,2,3|T]-T, [4,5,6|T_2]-T_2, L-T_3). \]
Difference Lists

• Add an element at the end of a list:

\[ \text{add}(L-T, X, L2-T2) :- \]
\[ T = [X|T2], \]
\[ L = L2. \]

?- \text{add}([1,2,3|T]-T, 4, L-T2).
L = [1,2,3,4|T2]

• Alternative:

\[ \text{add}(L-T, X, L2-T2) :- \]
\[ \text{dappend}(L-T, [X|T3]-T3,L-T3). \]
Difference Lists

- Check if a list is a palindrome:

\[
\text{palindrome}(X) :- \\
\quad \text{palindromeHelp}(X-[]).
\]

\[
\text{palindromeHelp}(A-A).
\]
\[
\text{palindromeHelp}([\_\_|A]-A).
\]
\[
\text{palindromeHelp}([C|A]-D) :- \\
\quad \text{palindromeHelp}(A-B), \\
\quad B=[C|D].
\]

?- \text{palindrome}([1,2,3,2,1]).
yes
?- \text{palindrome}([1,2,3,4,5]).
no
Tree Traversal, Revisited

preorder1(Node, List, Tail) :-
    node(Node, Child1, Child2),
    List = [Node|List1],
    preorder1(Child1, List1, Tail1),
    preorder1(Child2, Tail1, Tail).

preorder1(Node, [Node|Tail], Tail) :-
    leaf(Node).

preorder(Node, List) :-
    preorder1(Node, List, []).

• The program takes $O(n)$ time to traverse a tree with $n$ nodes.
Difference Lists: Conventions

- The preorder traversal program may be written as:

```prolog
preorder1(Node, [Node|L]-T) :-
    node(Node, Child1, Child2),
    preorder1(Child1, L-T1),
    preorder1(Child2, T1-T),
    preorder1(Node, [Node|T]-T).
```
Graphs in Prolog

- There are several ways to represent graphs in Prolog:
  - represent each edge separately as one clause (fact):
    
    ```prolog
    edge(a,b).
    edge(b,c).
    ```
  - isolated nodes cannot be represented, unless we have also `node/1` facts
  - the whole graph as one data object: as a pair of two sets (nodes and edges): `graph([a,b,c,d,f,g], [e(a,b), e(b,c), e(b,f)])`
  - list of arcs: `[a-b, b-c, b-f]`
  - adjacency-list: `[n(a, [b]), n(b, [c,f]), n(d, [])]`
Graphs in Prolog

- Path from one node to another one:
  - a predicate `path(G, A, B, P)` to find an acyclic path `P` from node `A` to node `B` in the graph `G`.
  - The predicate should return all paths via backtracking.
- We will solve it using the graph as a data object, like in `graph([a, b, c, d, f, g], [e(a, b), e(b, c), e(b, f)])`
Graphs in Prolog

• **adjacent** for directed edges:

```prolog
adjacent(X,Y,graph(_,Es)) :-
    member(e(X,Y),Es).
```

• **adjacent** for undirected edges (no distinction between the two vertices associated with each edge):

```prolog
adjacent(X,Y,graph(_,Es)) :-
    member(e(X,Y),Es).
adjacent(X,Y,graph(_,Es)) :-
    member(e(Y,X),Es).
```
Graphs in Prolog

Path from one node to another one:

\[
\text{path}(G,A,B,P) :- \\
\quad \text{pathHelper}(G,A,[B],P).
\]

% Base case
\[
\text{pathHelper}(\_\_\_\_,A,[A\|P1],[A\|P1]).
\]

\[
\text{pathHelper}(G,A,[Y\|P1],P) :- \\
\quad \text{adjacent}(X,Y,G), \\
\quad \\text{\texttt{\^{}}} \text{~} \text{member}(X,[Y\|P1]), \\
\quad \text{pathHelper}(G,A,[X,Y\|P1],P).
\]
Graphs in Prolog

- Cycle from a given node in a directed graph:

  - a predicate `cycle(G,A,P)` to find a closed path (cycle) `P` starting at a given node `A` in the graph `G`.
  - The predicate should return all cycles via backtracking

\[
\text{cycle}(G,A,P) :\neg \\
\text{adjacent}(A,B,G), \\
\text{path}(G,B,A,P1), \\
P = [A|P1].
\]
Complete program in XSB

:- import member/2 from basics.

adjacent(X,Y,graph(_,Es)) :-
   member(e(X,Y),Es).

path(G,A,B,P) :-
   pathHelper(G,A,[B],P).

pathHelper(_,A,[A|P1],[A|P1]).
pathHelper(G,A,[Y|P1],P) :-
   adjacent(X,Y,G),
   member(X,[Y|P1]),
   pathHelper(G,A,[X,Y|P1],P).

cycle(G,A,P) :-
   adjacent(A,B,G),
   path(G,B,A,P1),
   P = [A|P1].

?- Graph = graph([a,b,c,d,f,g], [e(a,b), e(b,c), e(c,a), e(a,e), e(e,a)]),
cycle(Graph,a,P),
writeln(P),
fail;
true.

(c) Paul Fodor (CS Stony Brook) and Elsevier
Aggregates in XSB

- `setof(Template, Goal, Set)` is the set of all instances of `Template` such that `Goal` is provable.
- `bagof(Template, Goal, Bag)` has the same semantics as `setof/3` except that the third argument returns an unsorted list that may contain duplicates.
- `findall(Template, Goal, List)` is similar to predicate `bagof/3`, except that variables in `Goal` that do not occur in `Template` are treated as existential, and alternative lists are not returned for different bindings of such variables.
- `tfindall(Template, Goal, List)` is similar to predicate `findall/3`, but the `Goal` must be a call to a single tabled predicate.
Aggregates in XSB

\[ p(1,1). \]
\[ p(1,2). \]
\[ p(2,1). \]
\[ ?- \text{setof}(Y, p(X,Y), L). \]
\[ L=[1,2] \]
\[ ?- \text{bagof}(Y, p(X,Y), L). \]
\[ L=[1,2] \]
\[ ?- \text{findall}(Y, p(X,Y), L). \]
\[ L=[1,2,1] \]
XSB Prolog

- Negation: `\+` is negation-as-failure
- Another negation called `tnot` (*TABLEING = memoization*)
  - Use: `... :- ...`, `tnot(foobar(X))`.
  - All variables under the scope of `tnot` must also occur to the left of that scope in the body of the rule in other positive relations:
    - Ok: `...:-p(X,Y),tnot(foobar(X,Y)),...`
    - Not ok: `...:-p(X,Z),tnot(foobar(X,Y)),...`
- XSB also supports Datalog:
  ```prolog
  :- auto_table.
  ```
  at the top of the program file
XSB Prolog

- Read/write from and to files:
  - Edinburgh style:

    ?- tell('a.txt'),
        write('Hello, World!'), told.

    ?- see('a.txt'), read(X), seen.
XSB Prolog

• Read/write from and to files:
  • ISO style:

?- open('a.txt', write, X),
   write(X,'Hello, World!'),
   close(X).
Cut (logic programming)

- Cut (! in Prolog) is a goal which always succeeds, but cannot be backtracked past.

  \[
  \text{max}(X, Y, Y) :- X \leq Y, !.
  \]
  \[
  \text{max}(X, _, X).
  \]

- cut says “stop looking for alternatives”

- no check is needed in the second rule anymore because if we got there, then \( X \leq Y \) must have failed, so \( X > Y \) must be true.

- red cut: if someone deletes !, then the rule is incorrect

- Green cut

  \[
  \text{max}(X, Y, Y) :- X \leq Y, !.
  \]
  \[
  \text{max}(X, Y, X) :- X > Y.
  \]

- by explicitly writing \( X > Y \), it guarantees that the second rule will always work even if the first one is removed by accident or changed (cut is deleted)
No backtracking pass the guard, but ok after:

\[
\begin{align*}
  p(a) & . \ p(b) . \\
  q(a) & . \ q(b) . \ q(c) . \\
  \text{?- } & p(X) , \! . \\
  X=a & ; \\
  \text{no} & \\
  \text{?- } & p(X) , \! , q(Y) . \\
  X=a, & y=a ; \\
  X=a, & y=b ; \\
  X=a, & y=c ; \\
  \text{no} & 
\end{align*}
\]
Testing types

- **atom(X)**
  Tests whether X is bound to a symbolic atom.
  
  ```
  ?- atom(a).
  yes
  ?- atom(3).
  no
  ```

- **integer(X)**
  Tests whether X is bound to an integer.

- **real(X)**
  Tests whether X is bound to a real number.
Testing for variables

- **is_list(L)**
  Tests whether \( L \) is bound to a list.

- **ground(G)**
  Tests whether \( G \) has unbound logical variables.

- **var(X)**
  Tests whether \( X \) is bound to a Prolog variable.
Control / Meta-predicates

- **call(P)**
  Force P to be a goal; succeed if P does, else fail.

- **clause(H,B)**
  Retrieves clauses from memory whose head matches H and body matches B. H must be sufficiently instantiated to determine the main predicate of the head.

- **copy_term(P,NewP)**
  Creates a new copy of the first parameter (with new variables).
  - It is used in iteration through non-ground clauses, so that the original calls are not bound to values.
Write a Prolog relation
map(BinaryRelation, InputList, OutputList)
which applies a binary relation on each of the elements of the list InputList as the first argument and collects the second argument in the result list.

Example:
?- map(inc1(X,Y), [5,6], R).
returns R=[6,7]
where inc1(X,Y) was defined as:

inc1(X,Y) :-
    Y is X+1.
Control / Meta-predicates

map(_BinaryCall,[],[]).
map(BinaryCall,[X|T],[Y|T2]) :-
    copy_term(BinaryCall, BinaryCall2),
    BinaryCall2 =.. [_F,X,Y],
    call(BinaryCall2),
    map(BinaryCall, T, T2).

incl(X,Y) :-
    Y is X+1.
?- map(incl(X,Y), [5,6], R),
    writeln(R),
    fail;
    true.
Control / Meta-predicates

\[
square(X,Y) :- \\
    Y \text{ is } X \times X. \\
\]

?- \text{map}(\text{square}(E, E2), [2,3,1], R), \\
    \text{writeln}(R), \\
    \text{fail}; \\
    \text{true.} \\
[4,9,1]
Control / Meta-predicates

- Use the relation `map` to implement a relation `pairAll(E, L, L2)` which pairs the element `E` with each element of the list `L` to obtain `L2`.

Examples:

?- pairAll(1, [2,3,1], L2).
returns `L2=[[1,2],[1,3],[1,1]]`

?- pairAll(1, [], L2).
returns `L2=[]`. 
Control / Meta-predicates

\[\text{pair}(E2, \ (_E1,E2))\].

\[\text{pairAll}(E,L,L2):=\]
\[\text{map}(\text{pair}(E2,\ (E,E2)),\ L,\ L2)\].

?\[-\text{pairAll}(1,\ [2,3,1],\ R)\].
\[R = [(1,2),(1,3),(1,1)]\]
Assert and retract

- **asserta**(C)
  Assert clause **C** into database above other clauses with the same predicate.

- **assertz**(C), **assert**(C)
  Assert clause **C** into database below other clauses with the same predicate.

- **retract**(C)
  Retract **C** from the database. **C** must be sufficiently instantiated to determine the predicate.
Prolog terms

- **functor (E, F, N)**
  
  E must be bound to a functor expression of the form 'f(...)'. F will be bound to 'f', and N will be bound to the number of arguments that f has.

- **arg (N, E, A)**
  
  E must be bound to a functor expression, N is a whole number, and A will be bound to the Nth argument of E.
Prolog terms and clauses

- =.. converts between term and list. For example,

?- parent(a,X) =.. L.
L = [parent, a, _X001]

?- [1] =.. X.
X = [.,1,[]]