SML

CSE 307 – Principles of Programming Languages

Stony Brook University

http://www.cs.stonybrook.edu/~cse307

Functional Programming

- *Function evaluation* is the basic concept for a programming paradigm that has been implemented in ***functional programming languages***.
- The language ML ("Meta Language") was originally introduced in the 1970's as part of a theorem proving system, and was intended for describing and implementing proof strategies in the Logic for Computable Functions (LCF) theorem prover (whose language, pplambda, a combination of the first-order predicate calculus and the simply typed polymorphic lambda calculus, had ML as its metalanguage)
- Standard ML of New Jersey (SML) is an implementation of ML.
- The basic mode of computation in SML is the use of the definition and application of functions.

Install Standard ML

- Download from:
  - http://www.smlnj.org
- Start Standard ML:
  - Type **`sml`** from the shell (run command line in Windows)
- Exit Standard ML:
  - **`Ctrl-Z`** under Windows
  - **`Ctrl-D`** under Unix/Mac

Standard ML

- The basic cycle of SML activity has three parts:
  - read input from the user,
  - evaluate it,
  - print the computed value (or an error message).

First SML example

- SML prompt:

```
-
```

- Simple example:

```
- 3;
val it = 3 : int
```

- The first line contains the SML prompt, followed by *an expression* typed in by the user and ended by *a semicolon*.
- The second line is SML's response, indicating the *value* of the input expression and its *type*.

Interacting with SML

- SML has a number of built-in operators and data types.
  - it provides the standard arithmetic operators

```
- 3+2;
val it = 5 : int
```

- The Boolean values true and false are available, as are logical operators such as **not** (negation), **andalso** (conjunction), and **orelse** (disjunction).

```
- not(true);
val it = false : bool
- true andalso false;
val it = false : bool
```

Types in SML

- As part of the evaluation process, SML determines the type of the output value using methods of *type inference*.
- Simple types include *int*, *real*, *bool*, and *string*.
- One can also associate identifiers with values

```
- val five = 3+2;
val five = 5 : int
```

and thereby establish a new value binding

```
- five;
val it = 5 : int
```

Function Definitions in SML

- The general form of a function definition in SML is:

```
fun <identifier> (<parameters>) =
                <expression>;
```

- For example,

```
- fun double(x) = 2*x;
val double = fn : int -> int
```

declares **double** as a function from integers to integers, i.e., of type **int → int**

- Apply a function to an argument of the wrong type results in an error message:

```
- double(2.0);
Error: operator and operand don't agree ...
```

Function Definitions in SML

- The user may also **explicitly** indicate types:

```
- fun max(x:int,y:int,z:int) =
    if ((x>y) andalso (x>z)) then x
    else (if (y>z) then y else z);
val max = fn : int * int * int -> int

- max(3,2,2);
val it = 3 : int
```

Recursive Definitions

- The use of recursive definitions is a main characteristic of functional programming languages, and these languages encourage the use of recursion over iterative constructs such as while loops:

```
- fun factorial(x) = if x=0 then 1
    else x*factorial(x-1);
val factorial = fn : int -> int
```

- The definition is used by SML to evaluate applications of the function to specific arguments.

```
- factorial(5);
val it = 120 : int
- factorial(10);
val it = 3628800 : int
```

Example: Greatest Common Divisor

- The greatest common divisor (gcd) of two positive integers can defined recursively based on the following observations:

```
gcd(n, n) = n,
gcd(m, n) = gcd(n,m), if m < n, and
gcd(m, n) = gcd(m - n, n), if m > n.
```

- These identities suggest the following recursive definition:

```
- fun gcd(m,n):int = if m=n then n
      else if m>n then gcd(m-n,n)
      else gcd(m,n-m);
  val gcd = fn : int * int -> int
- gcd(12,30);      - gcd(1,20);      - gcd(125,56345);
val it = 6 : int   val it = 1 : int   val it = 5 : int
```

More recursive functions

```
- fun exp(b,n) = if n=0 then 1.0
      else b * exp(b,n-1);
val exp = fn : real * int -> real

- exp(2.0,10);
val it = 1024.0 : real
```

Tuples in SML

- In SML tuples are finite sequences of arbitrary but fixed length, where different components need not be of the same type.

```
- (1, "two");
val it = (1,"two") : int * string
- val t1 = (1,2,3);
val t1 = (1,2,3) : int * int * int
- val t2 = (4,(5.0,6));
val t2 = (4,(5.0,6)) : int * (real * int)
```

- The components of a tuple can be accessed by applying the built-in functions #i, where i is a positive number.

```
- #1(t1);
val it = 1 : int
- #2(t2);
val it = (5.0,6) : real * int
```

If a function #i is applied to a tuple with fewer than i components, an error results.

Polymorphic functions

```
- fun id x = x;
val id = fn : 'a -> 'a
- (id 1, id "two");
val it = (1,"two") : int * string
- fun fst(x,y) = x;
val fst = fn : 'a * 'b -> 'a
- fun snd(x,y) = y;
val snd = fn : 'a * 'b -> 'b
- fun switch(x,y) = (y,x);
val switch = fn : 'a * 'b -> 'b * 'a
```

Polymorphic functions

- `'a` means "*any type*", while `''a` means "*any type that can be compared for equality*" (see the `concat` function later which compares a polymorphic variable list with `[]`).
- There will be a "*Warning: calling polyEqual*" that means that you're comparing two values with polymorphic type for equality.
  - Why does this produce a warning? Because it's less efficient than comparing two values of known types for equality.
  - How do you get rid of the warning? By changing your function to only work with a specific type instead of any type.
    - Should you do that or care about the warning? Probably not. In most cases having a function that can work for any type is more important than having the most efficient code possible, so you should just ignore the warning.

Lists in SML

- A list in SML is a finite sequence of objects, all of the same type:

```
- [1,2,3];
val it = [1,2,3] : int list
- [true,false,true];
val it = [true,false,true] : bool list
- [[1,2,3],[4,5],[6]];
val it = [[1,2,3],[4,5],[6]] :
                         int list list
```

  - The last example is a list of lists of integers.

Lists in SML

- All objects in a list must be of the same type:

```
- [1,[2]];
Error: operator and operand don't agree
```

- An empty list is denoted by one of the following expressions:

```
- [];
val it = [] : 'a list
- nil;
val it = [] : 'a list
```

- Note that the type is described in terms of a type variable `'a`. Instantiating the type variable, by types such as int, results in (different) empty lists of corresponding types.

Operations on Lists

- SML provides various functions for manipulating lists.
  - The function hd returns the first element of its argument list.

```
- hd[1,2,3];
val it = 1 : int
- hd[[1,2],[3]];
val it = [1,2] : int list
```

Applying this function to the empty list will result in an error.

  - The function tl removes the first element of its argument lists, and returns the remaining list.

```
- tl[1,2,3];
val it = [2,3] : int list
- tl[[1,2],[3]];
val it = [[3]] : int list list
```

  - The application of this function to the empty list will also result in an error.

Operations on Lists

- Lists can be constructed by the (binary) function :: (read cons) that adds its first argument to the front of the second argument.

```
- 5::[];
val it = [5] : int list
- 1::[2,3];
val it = [1,2,3] : int list
- [1,2]::[[3],[4,5,6,7]];
val it = [[1,2],[3],[4,5,6,7]] : int list list
```

The arguments must be of the right type (such that the result is a list of elements of the same type):

```
- [1]::[2,3];
Error: operator and operand don't agree
```

Operations on Lists

- Lists can also be compared for equality:

```
- [1,2,3]=[1,2,3];
val it = true : bool

- [1,2]=[2,1];
val it = false : bool

- tl[1] = [];
val it = true : bool
```

Defining List Functions

- Recursion is particularly useful for defining functions that process lists.
  - For example, consider the problem of defining an SML function that takes as arguments two lists of the same type and returns the concatenated list.
- In defining such list functions, it is helpful to keep in mind that a list is either
  - an empty list **`[]`** or
  - of the form **`x::y`**

Concatenation

- In designing a function for concatenating two lists **`x`** and **`y`** we thus distinguish two cases, depending on the form of **`x`**:
  - If **`x`** is an empty list **`[]`**, then concatenating **`x`** with **`y`** yields just **`y`**.
  - If **`x`** is of the form **`x1::x2`**, then concatenating **`x`** with **`y`** is a list of the form **`x1::z`**, where **`z`** is the result of concatenating **`x2`** with **`y`**.
    - We can be more specific by observing that **`x = hd(x)::tl(x)`**

Concatenation

```
- fun concat(x,y) = if x=[] then y
  else hd(x)::concat(tl(x),y);
val concat = fn : ''a list * ''a list -> ''a list
```

- Applying the function yields the expected results:

```
- concat([1,2],[3,4,5]);
val it = [1,2,3,4,5] : int list
- concat([],[1,2]);
val it = [1,2] : int list
- concat([1,2],[]);
val it = [1,2] : int list
```

Length

- The following function computes the length of its argument list:

```
- fun length(L) = if (L=nil) then 0
                  else 1+length(tl(L));
val length = fn : ''a list -> int

- length[1,2,3];
val it = 3 : int
- length[[5],[4],[3],[2,1]];
val it = 4 : int
- length[];
val it = 0 : int
```

doubleall

- The following function doubles all the elements in its argument list (of integers):

```
- fun doubleall(L) =
          if L=[] then []
          else (2*hd(L))::doubleall(tl(L));
val doubleall = fn : int list -> int list

- doubleall[1,3,5,7];
val it = [2,6,10,14] : int list
```

Reversing a List

- Concatenation of lists, for which we gave a recursive definition, is actually a built-in operator in SML, denoted by the symbol @.
- We use this operator in the following recursive definition of a function that reverses a list.

```
- fun reverse(L) =
      if L = nil then nil
      else reverse(tl(L)) @ [hd(L)];
val reverse = fn : ''a list -> ''a list
- reverse [1,2,3];
val it = [3,2,1] : int list
```

This method is not efficient: $O(n^2)$

Reversing a List

- This way (using an accumulator) is better: `O(n)`

```
- fun reverse_helper(L,L2) =
 if L = nil then L2
  else reverse_helper(tl(L),hd(L)::L2);

- fun reverse(L) = reverse_helper(L,[]);
```

Removing List Elements

- The following function removes all occurrences of its first argument from its second argument list.

```
- fun remove(x,L) = if (L=[]) then []
        else if x=hd(L)then remove(x,tl(L))
        else hd(L)::remove(x,tl(L));
val remove = fn : ''a * ''a list -> ''a list

- remove(1,[5,3,1]);
val it = [5,3] : int list

- remove(2,[4,2,4,2,4,2,2]);
val it = [4,4,4] : int list
```

Removing Duplicates

- The remove function can be used in the definition of another function that removes all duplicate occurrences of elements from its argument list:

```
- fun removedupl(L) =
 if (L=[]) then []
 else hd(L)::removedupl(remove(hd(L),tl(L)));
val removedupl = fn : ''a list -> ''a list

- removedupl([3,2,4,6,4,3,2,3,4,3,2,1]);
val it = [3,2,4,6,1] : int list
```

Definition by Patterns

- In SML functions can also be defined via patterns.
- The general form of such definitions is:

```
fun <identifier>(<pattern1>) = <expression1>
| <identifier>(<pattern2>) = <expression2>
| ...
| <identifier>(<patternK>) = <expressionK>;
```

where the identifiers, which name the function, are all the same, all patterns are of the same type, and all expressions are of the same type.

- Example:

The patterns are inspected in order and the first match determines the value of the function.

```
- fun reverse(nil) = nil
  | reverse(x::xs) = reverse(xs) @ [x];
val reverse = fn : 'a list -> 'a list
```

Sets with lists in SML

```
fun member(X,L) =
      if L=[] then false
      else if X=hd(L) then true
      else member(X,tl(L));
```

OR with patterns:

```
fun member(X,[]) = false
      | member(X,Y::Ys) =
            if (X=Y) then true
            else member(X,Ys);
member(1,[1,2]); (* true *)
member(1,[2,1]); (* true *)
member(1,[2,3]); (* false *)
```

Sets - UNION

```
fun union(L1,L2) =
    if L1=[] then L2
    else if member(hd(L1),L2)
        then union(tl(L1),L2)
        else hd(L1)::union(tl(L1),L2);
union([1,5,7,9],[2,3,5,10]);
    (* [1,7,9,2,3,5,10] *)
union([],[1,2]);
    (* [1,2] *)
union([1,2],[]);
    (* [1,2] *)
```

Sets - UNION patterns

```
fun union([],L2) = L2
    | union(X::Xs,L2) =
        if member(X,L2) then union(Xs,L2)
        else X::union(Xs,L2);
union([1,5,7,9],[2,3,5,10]);
      (* [1,7,9,2,3,5,10] *)
union([],[1,2]);
      (* [1,2] *)
union([1,2],[]);
      (* [1,2] *)
```

Sets - Intersection ∩

```
fun intersection(L1,L2) =
    if L1=[] then []
    else if member(hd(L1),L2)
    then hd(L1)::intersection(tl(L1),L2)
    else intersection(tl(L1),L2);

intersection([1,5,7,9],[2,3,5,10]);
    (* [5] *)
```

Sets - ∩ patterns

```
fun intersection([],L2) = []
    | intersection(L1,[]) = []
    | intersection(X::Xs,L2) =
        if member(X,L2)
        then X::intersection(Xs,L2)
        else intersection(Xs,L2);

intersection([1,5,7,9],[2,3,5,10]);
    (* [5] *)
```

Sets – subset

```
fun subset(L1,L2) = if L1=[] then true
     else if L2=[] then false
     else if member(hd(L1),L2)
          then subset(tl(L1),L2)
          else false;
subset([1,5,7,9],[2,3,5,10]);
     (* false *)
subset([5],[2,3,5,10]);
     (* true *)
```

Sets – subset patterns

```
fun subset([],L2) = true
    | subset(L1,[]) = if(L1=[])
        then true
        else false
    | subset(X::Xs,L2) =
        if member(X,L2)
            then subset(Xs,L2)
            else false;
subset([1,5,7,9],[2,3,5,10]);
    (* false *)
subset([5],[2,3,5,10]);
    (* true *)
```

Sets – equals

```
fun setEqual(L1,L2) =
     subset(L1,L2) andalso subset(L2,L1);
setEqual([1,5,7],[7,5,1,2]);
     (* false *)
setEqual([1,5,7],[7,5,1]);
     (* true *)
```

Sets – minus patterns

```
fun minus([],L2) = []
    | minus(X::Xs,L2) =
        if member(X,L2)
            then minus(Xs,L2)
            else X::minus(Xs,L2);

minus([1,5,7,9],[2,3,5,10]);
    (* [1,7,9] *)
```

Sets - Cartesian product

```
fun product_one(X,[]) = []
    | product_one(X,Y::Ys) =
          (X,Y)::product_one(X,Ys);
product_one(1,[2,3]);
    (* [(1,2),(1,3)] *)
fun product([],L2) = []
    | product(X::Xs,L2) =
          union(product_one(X,L2),
                product(Xs,L2));
product([1,5,7,9],[2,3,5,10]);
    (* [(1,2),(1,3),(1,5),(1,10),(5,2),
  (5,3),(5,5),(5,10),(7,2),(7,3),...] *)
```

Sets – Powerset

```
fun insert_all(E,L) =
     if L=[] then []
     else (E::hd(L)) :: insert_all(E,tl(L));
insert_all(1,[[],[2],[3],[2,3]]);
 (* [ [1], [1,2], [1,3], [1,2,3] ] *)
fun powerSet(L) =
     if L=[] then [[]]
     else powerSet(tl(L)) @
          insert_all(hd(L),powerSet(tl(L)));

powerSet([]);
powerSet([1,2,3]);
powerSet([2,3]);
```

Records

- Records are structured data types of heterogeneous elements that are labeled

```
- {x=2, y=3};
```

- The order does not matter:

```
- {make="Toyota", model="Corolla", year=2017,
  color="silver"}
= {model="Corolla", make="Toyota", color="silver",
  year=2017};
val it = true : bool

- fun full_name{first:string,last:string,
  age:int,balance:real}:string =
   first ^ " " ^ last;
      (* ^ is the string concatenation operator *)
val full_name=fn:{age:int, balance:real, first:string,
  last:string} -> string
```

User defined data types

```
- datatype shape = Rectangle of real*real
  | Circle of real
  | Line of (real*real)list;

datatype shape
   = Circle of real
   | Line of (real * real) list
   | Rectangle of real * real
```

Higher-Order Functions

- In functional programming languages functions can be used in definitions of other, so-called higher-order, functions.
  - The following function, **`map`**, applies its first argument (a function) to all elements in its second argument (a list of suitable type):

```
- fun map(f,L) = if (L=[]) then []
 else f(hd(L))::(map(f,tl(L)));
val map = fn : (''a -> 'b) * ''a list -> 'b list
```

  - We may apply **`map`** with any function as argument:

```
- fun square(x) = (x:int)*x;
val square = fn : int -> int
- map(square,[2,3,4]);
val it = [4,9,16] : int list
```

Higher-Order Functions

- More map examples
  - Anonymous functions:

```
- map(fn x=>x+1, [1,2,3,4,5]);

val it = [2,3,4,5,6] : int list

- fun incr(list) = map (fn x=>x+1, list);

val incr = fn : int list -> int list

- incr[1,2,3,4,5];

val it = [2,3,4,5,6] : int list
```

McCarthy's 91 function

- McCarthy's 91 function:

```
- fun mc91(n) = if n>100 then n-10
  else mc91(mc91(n+11));

val mc91 = fn : int -> int

- map mc91 [101, 100, 99, 98, 97, 96];

val it = [91,91,91,91,91,91] : int list
```

Filter

- Filter: keep in a list only the values that satisfy some logical condition/boolean function

```
- fun filter(f,l) =
   if l=[] then []
     else if f(hd l)
       then (hd l)::(filter (f, tl l))
       else filter(f, tl l);

val filter = fn : ('a -> bool) * 'a list -> 'a list

- filter((fn x => x>0), [~1,0,1]);

val it = [1] : int list
```

Permutations

```
- fun myInterleave(x,[]) = [[x]]
 | myInterleave(x,h::t) =
       (x::h::t)::(
             map((fn l => h::l), myInterleave(x,t)));

- myInterleave(1,[]);
val it = [[1]] : int list list

- myInterleave(1,[2]);
val it = [[1,2],[2,1]] : int list list

- myInterleave(1,[2,3]);
val it = [[1,2,3],[2,1,3],[2,3,1]] : int list list
```

Permutations

```
- fun appendAll(nil) = nil
| appendAll(z::zs) = z @ (appendAll(zs));

- appendAll([[[1,2]],[[2,1]]]);
val it = [[1,2],[2,1]] : int list list

- fun permute(nil) = [[]]
| permute(h::t) = appendAll(
    map((fn l => myInterleave(h,l)), permute(t)));

- permute([1,2,3]);
val it = [[1,2,3],[2,1,3],[2,3,1],[1,3,2],[3,1,2],
            [3,2,1]] : int list list
```

Currying

```
- fun f(a)(b)(c) = a+b+c;
val f = fn : int -> int -> int -> int
val f = fn : int -> (int -> (int -> int))
  OR
- fun f a b c = a+b+c;
- val inc1 = f(1);
val inc1 = fn : int -> int -> int
val inc1 = fn : int -> (int -> int)
- val inc12 = inc1(2);
val inc12 = fn : int -> int
- inc12(3);
val it = 6 : int
```

Composition

- Composition is another example of a higher-order function:

```
- fun comp(f,g)(x) = f(g(x));
val comp = fn : ('a -> 'b) * ('c -> 'a) -> 'c -> 'b
- val f = comp(Math.sin, Math.cos);
val f = fn : real -> real
      SAME WITH:
- val g = Math.sin o Math.cos;
      (*  Composition "o" is predefined  *)
val g = fn : real -> real
- f(0.25);
val it = 0.824270418114 : real
- g(0.25);
val it = 0.824270418114 : real
```

Mutually recursive function definitions

```
- fun odd(n) = if n=0 then false
                else even(n-1)
 and
      even(n) = if n=0 then true
                else odd(n-1);
val odd = fn : int -> bool
val even = fn : int -> bool

- even(1);
val it = false : bool
- odd(1);
val it = true : bool
```

Sorting

- We next design a function for sorting a list of integers:
  - The function is recursive and based on a method known as Merge-Sort.
  - To sort a list L:
    - first split L into two disjoint sublists (of about equal size),
    - then (recursively) sort the sublists, and
    - finally merge the (now sorted) sublists.
  - This recursive method is known as **Merge-Sort**
  - It requires suitable functions for
    - splitting a list into two sublists AND
    - merging two sorted lists into one sorted list

Splitting

- We split a list by applying two functions, take and skip, which extract alternate elements; respectively, the elements at odd-numbered positions and the elements at even-numbered positions (if any).
- The definitions of the two functions mutually depend on each other, and hence provide an example of mutual recursion, as indicated by the SML-keyword and:

```
- fun take(L) =
      if L = nil then nil
      else hd(L)::skip(tl(L))
and
 skip(L) =
      if L=nil then nil
      else take(tl(L));
val take = fn : ''a list -> ''a list
val skip = fn : ''a list -> ''a list
- take[1,2,3,4,5,6,7];
val it = [1,3,5,7] : int list
- skip[1,2,3,4,5,6,7];
val it = [2,4,6] : int list
```

Merging

- Merge pattern definition:

```
- fun merge([],M) = M
  | merge(L,[]) = L
  | merge(x::xl,y::yl) =
        if (x:int)<y then x::merge(xl,y::yl)
        else y::merge(x::xl,yl);
val merge = fn : int list * int list -> int list

- merge([1,5,7,9],[2,3,5,5,10]);
val it = [1,2,3,5,5,5,7,9,10] : int list
- merge([],[1,2]);
val it = [1,2] : int list
- merge([1,2],[]);
val it = [1,2] : int list
```

Merge Sort

```
- fun sort(L) =
  if L=[] then []
  else if tl(L)=[] then L
  else merge(sort(take(L)),sort(skip(L)));

val sort = fn : int list -> int list
```

string and char

```
- "a";
val it = "a" : string
- #"a";
val it = #"a" : char
- explode("ab");
val it = [#"a",#"b"] : char list
- implode([#"a",#"b"]);
val it = "ab" : string
- "abc" ^ "def" = "abcdef";
val it = true : bool
- size ("abcd");
val it = 4 : int
```

string and char

```
- String.sub("abcde",2);
val it = #"c" : char
- substring("abcdefghij",3,4);
val it = "defg" : string
- concat ["AB"," ","CD"];
val it = "AB CD" : string
- str(#"x");
val it = "x" : string
```

The program of Young McML

```
fun tartan_column(i,j,n) =
  if j=n+1 then "\n"
  else if (i+j) mod 2=1 then
      concat(["* ",tartan_column(i,j+1,n)])
  else concat(["+ ",tartan_column(i,j+1,n)]);
fun tartan_row(i,n) =
  if i=n+1 then ""
  else concat([tartan_column(i,1,n),
           tartan_row(i+1,n)]);
fun tartan(n) = tartan_row(1,n);
print(tartan(30));
```