**Introduction**

- Shape matching is the enabling technique for shape comparison, registration, and recognition.
- The goal of matching is to find sparse correspondences of representative points, i.e., features, on input shapes.
- Our feature matching method is motivated by diffusion geometry and high-order graph matching, and is capable of matching partial dynamic shapes undergoing non-rigid deformations.

**Challenges**

- Dynamic scans often generate partial shapes with boundary changes; techniques based on global geometry or surface parameterization is infeasible.
- Methods based on point-wise or pair-wise similarities cannot handle outliers very well.
- Computational efficiency is required.

**Heat Kernel Tensor (HKT)**

- Geometric relations among features are much more reliable than matching pair-wise feature points.
- We adopt tensor matching [O. Duchenne 2009] and adapt it to manifolds via a diffusion-based relation measurement $d_t(x, y)$:
  
  $$d_t(x, y) = \frac{1}{4} (t \log h_t(x, y))^2$$
  
  $$h_t(x, y) = \sum_k e^{-\Delta^2_k \phi(x) \phi(y)}$$

  - $h_t(x, y)$: the heat kernel from point $x$ to $y$ at time $t$
  - $\{\lambda_k, \phi_k\}$: the $k$-th eigenvalue and eigenfunction of the shape’s Laplace-Beltrami operator

- On shape $M_1$ and $M_2$, a pair $i = (i_1, i_2)$ denotes a candidate match of features $b_i \in M_1$ and $b_{i_2} \in M_2$. We consider a tuple of three candidate matches $(i_1, i_2, k)$ for enhanced geometric compatibility.

- Define distance between $(i_1, i_2, k_1)$ and $(i_2, i_2, k_2)$ as the $L_2$ distance of the angle vectors of the two triangles
  
  $$d_B(i, j, k) = \|\theta_{i_1, i_2, k_1} - \theta_{i_2, i_2, k_2}\|_2$$

  - Affinity of tuple $(i, j, k)$
    
    $$\tau_{i,j,k} = e^{-d_B(i, j, k)^2}$$

- Heat kernel tensor: $T = (\tau_{i,j,k})$

- Tensor-based shape matching: Find assignment matrix $X_{N_1 \times N_2}$ that minimizes
  
  $$\text{score}(X) = \sum_{i,j,k} \tau_{i,j,k} X_{i,j,k} X_{i,j,k} X_{i,j,k}$$

**Matching Hierarchy**

- Segment shapes with low-frequency eigenfunctions
- Cluster subgraphs centered at the local extrema of low-frequency eigenfunctions
- First match the cluster centers, then match features in each subgraph.
- Lead to significant improvement in time performance using this hierarchical approach.

**Experimental Results**

- We propose HKT, a diffusion-driven representation of high-order geometric compatibility.
- We develop a new method for matching partial deformable shapes based on HKT and high-order graph matching which can well handle partial deformable shapes and shapes with outliers.
- We generalize our method with a two-level hierarchy which improves time performance dramatically.
- We demonstrate via multiple experiments the better performance and versatility of our method.