Introduction to Medical Imaging

X-Ray CT -- Introduction

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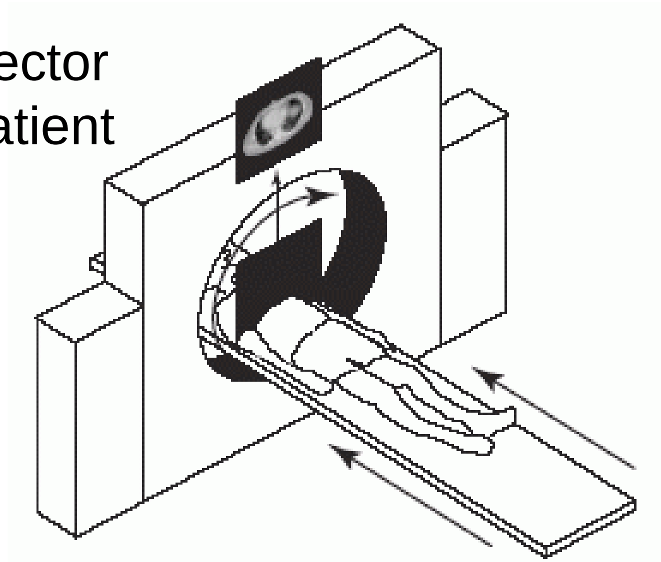
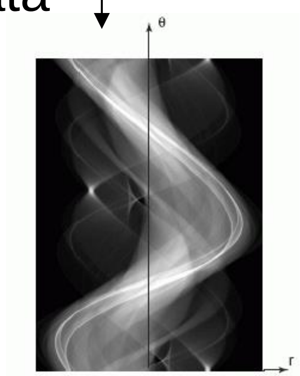
Overview

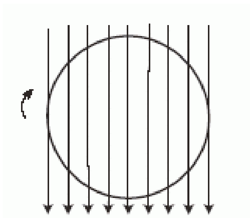
Scanning:
rotate source-detector
pair around the patient



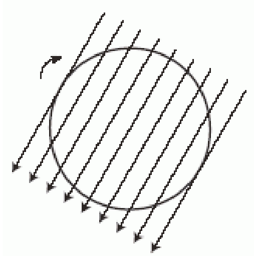
data

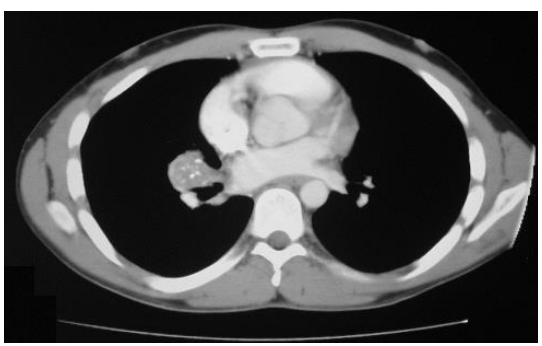


sinogram: a line for every angle



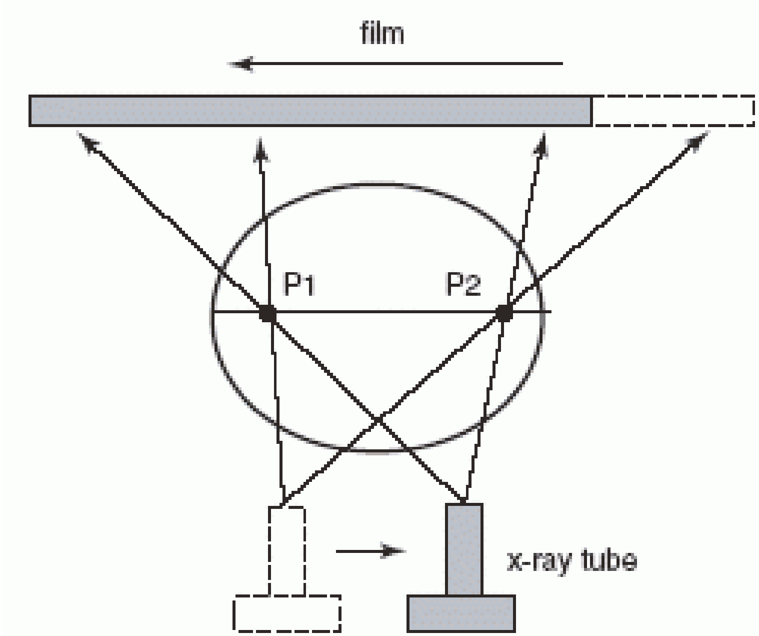
reconstruction routine

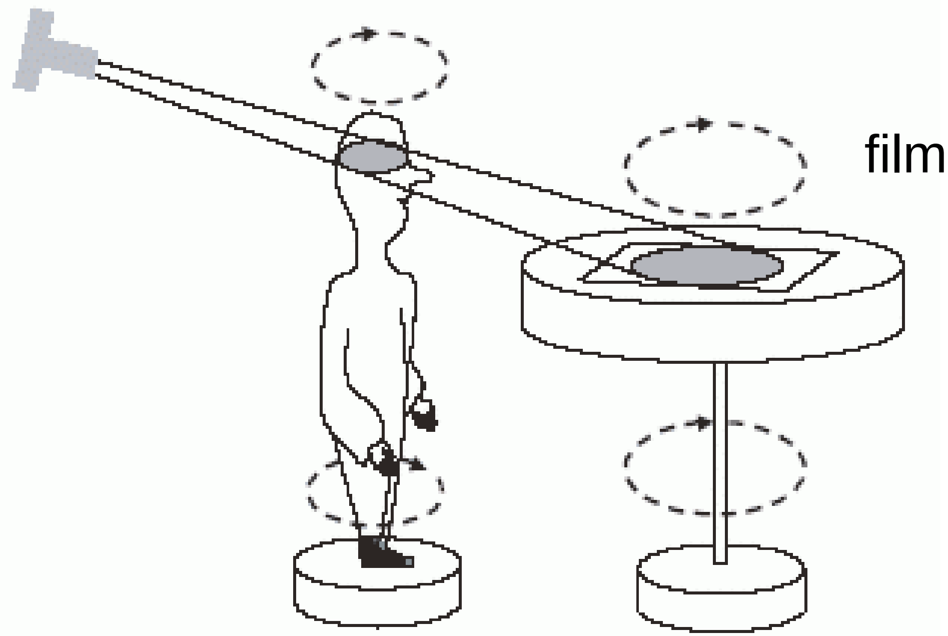




reconstructed cross-sectional slice

Early Beginnings





<u>Linear tomography</u>
only line P1-P2 stays in focus
all others appear blurred

<u>Axial tomography</u>
in principle, simulates the backprojection procedure used in current times

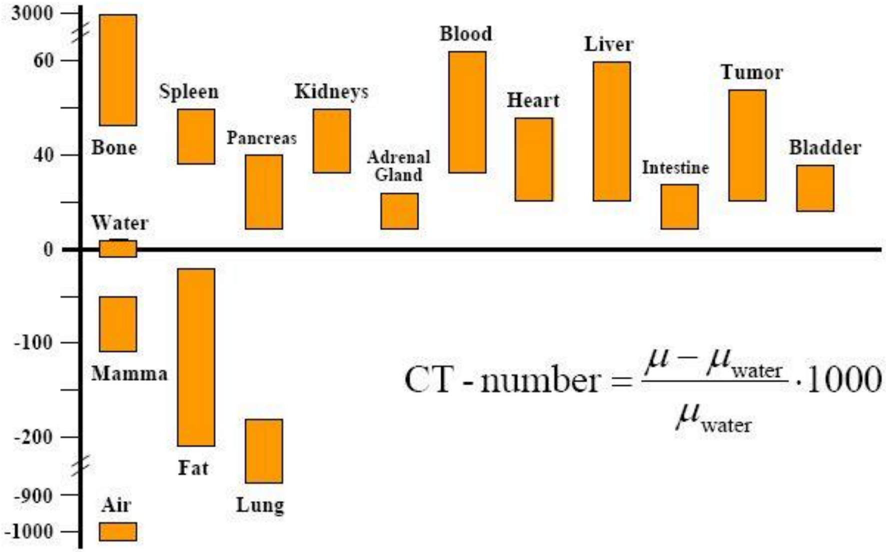
Current Technology

Principles derived by Godfrey Hounsfield for EMI

- based on mathematics by A. Cormack
- both received the Nobel Price in medizine/physiology in 1979
- technology is advanced to this day

Images:

- size generally 512 x 512 pixels
- values in *Hounsfield units* (HU) in the range of –1000 to 1000



$$\text{CT-number} = \frac{\mu - \mu_{\text{water}}}{\mu_{\text{water}}} \cdot 1000$$

μ: linear attenuation coefficient

- due to large dynamic range, windowing must be used to view an image

CT Detectors

Scintillation crystal with photomultiplier tube (PMT)

(scintillator: material that converts ionizing radiation into pulses of light)

- high QE and response time
- low packing density
- PMT used only in the early CT scanners

Gas ionization chambers

- replace PMT
- X-rays cause ionization of gas molecules in chamber
- ionization results in free electrons/ions
- these drift to anode/cathode and yield a measurable electric signal
- lower QE and response time than PMT systems, but higher packing density

Scintillation crystals with photodiode

- current technology (based on solid state or semiconductors)
- photodiodes convert scintillations into measurable electric current
- QE > 98% and very fast response time

Projection Coordinate System

The parallel-beam geometry at angle θ represents a new coordinate system (r,s) in which projection $I_\theta(r)$ is acquired

- rotation matrix R transforms coordinate system (x, y) to (r, s):

$$\begin{pmatrix} r \\ s \end{pmatrix} = R \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

that is, all (x,y) points that fulfill

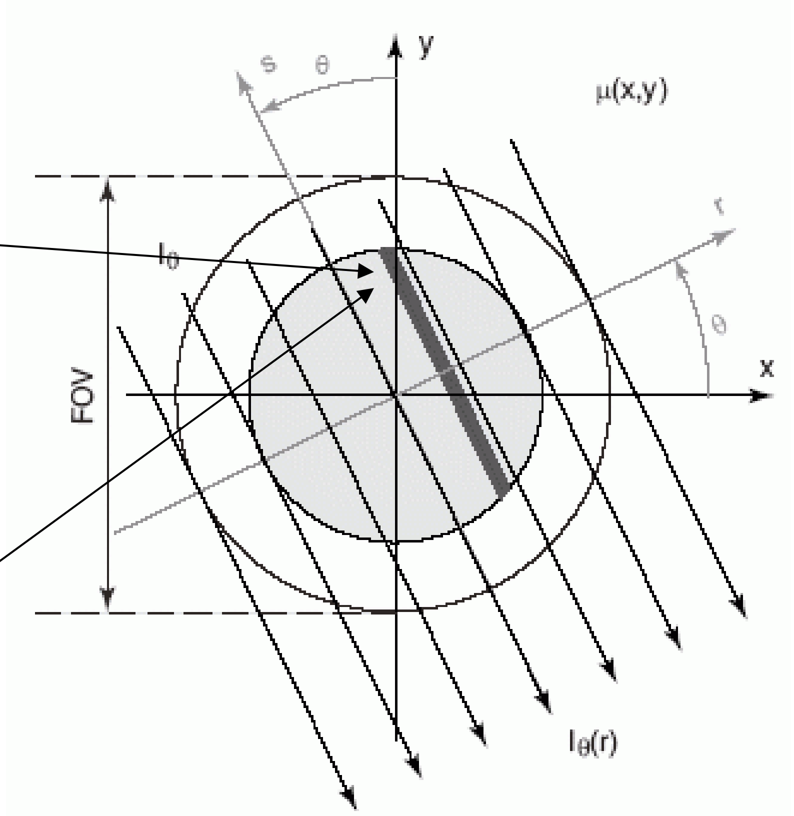
$$r = x\cos(\theta) + y\sin(\theta)$$

are on the (ray) line $L_{(r,\theta)}$

- R^T is the inverse, mapping (r, s) to (x, y)

$$\begin{pmatrix} x \\ y \end{pmatrix} = R^T \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} r \\ s \end{pmatrix}$$

s is the parametric variable along the (ray) line $L_{(r,\theta)}$



Projection

Assuming a fixed angle θ, the measured intensity at detector position r is the integrated density along $L_{(r,\theta)}$:

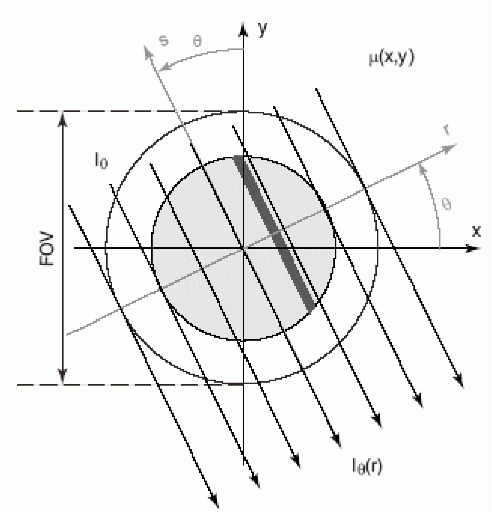
$$I_\theta(r) = I_0 \cdot e^{-\int_{L_{r,\theta}} \mu(x,y)\,ds}$$

$$= I_0 \cdot e^{\int_{L_{r,\theta}} \mu(r\cdot\cos\theta - s\cdot\sin\theta,\, r\cdot\sin\theta + s\cdot\cos\theta)\,ds}$$

For a continuous energy spectrum:

$$I_\theta(r) = \int_0^\infty I_0(E) \cdot e^{\int_{L_{r,\theta}} \mu(r\cdot\cos\theta - s\cdot\sin\theta,\, r\cdot\sin\theta + s\cdot\cos\theta)\,ds}$$

But in practice, is is assumed that the X-rays are monochromatic

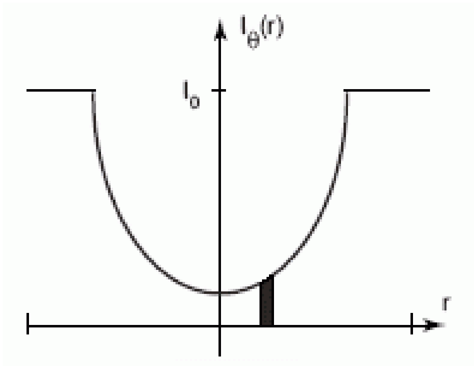


Projection Profile

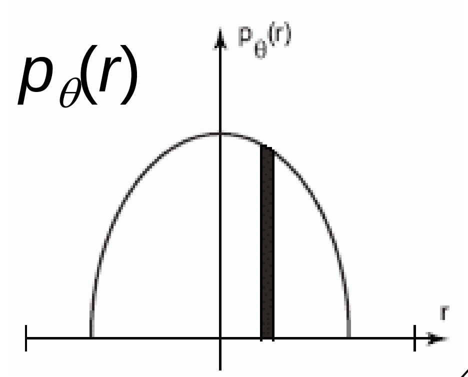
Each intensity profile $I_\theta(r)$ is transformed to into an attenuation profile $p_\theta(r)$:

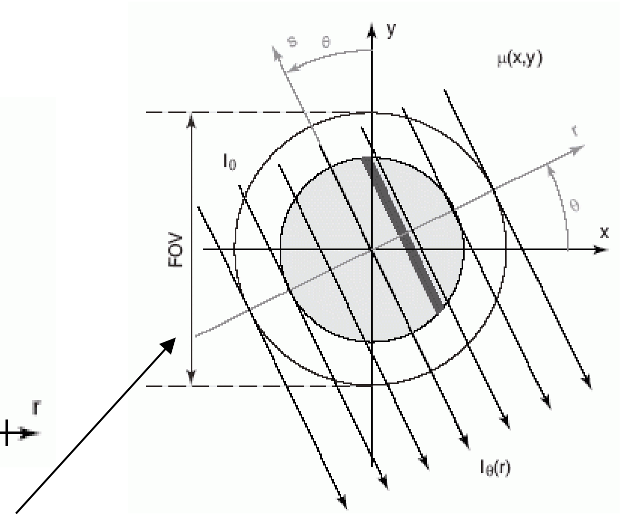
$$p_\theta(r) = -\ln\frac{I_\theta(r)}{I_0} = \int_{L_{r,\theta}} \mu(r\cdot\cos\theta - s\cdot\sin\theta, r\cdot\sin\theta + s\cdot\cos\theta)ds$$

$I_\theta(r)$



$p_\theta(r)$

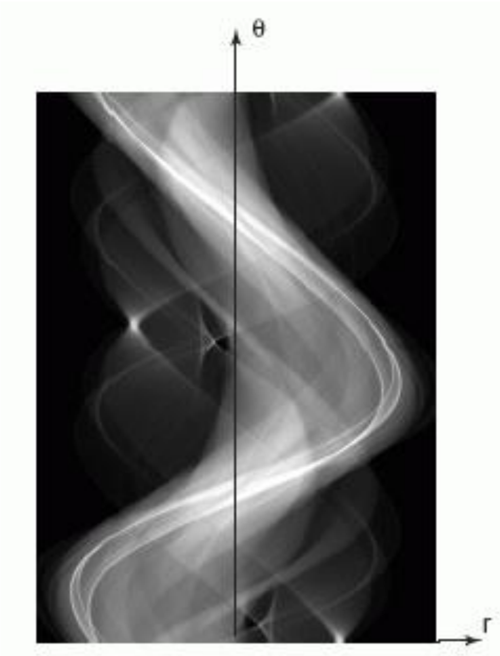




- $p_\theta(r)$ is zero for $|r|$>*FOV*/2 (*FOV* = *F*ield *o*f *V*iew, detector width)
- $p_\theta(r)$ can be measured from (0, 2π)
- however, for parallel beam views (π, 2π) are redundant, so just need to measure from (0, π)

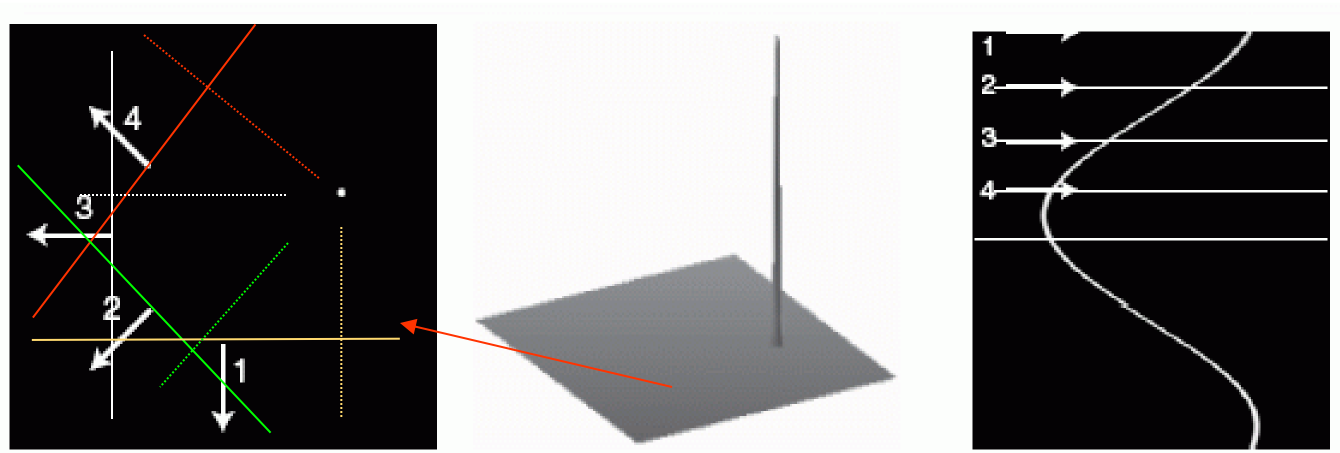
Sinogram

Stacking all projections (line integrals) yields the *sinogram,* a 2D dataset $p(r,\theta)$



To illustrate, imagine an object that is a single point:

- it then describes a sinusoid in $p(r,\theta)$:



projections point object sinogram

- note, the sinogram is very similar to the Hough transform where a single point also gives rise to a sinusoid

Radon Transform

The transformation of any function $f(x,y)$ into $p(r,\theta)$ is called the *2D Radon Transform*

$$p(r,\theta) = R\{f(x,y)\}$$

$$= \int_{-\infty}^{\infty} f(r\cdot\cos\theta - s\cdot\sin\theta, r\cdot\sin\theta + s\cdot\cos\theta)ds$$

The Radon transform has the following properties:

- $p(r,\theta)$ is periodic in θ with period 2π

$$p(r,\theta) = p(r,\theta+2\pi)$$

- $p(r,\theta)$ is symmetric in θ with period π

$$p(r,\theta) = p(-r,\theta\pm\pi)$$

Sampling (1)

In practice, we only have a limited

- number of views, M
- number of detector samples, N
- for example, M=1056, N=768

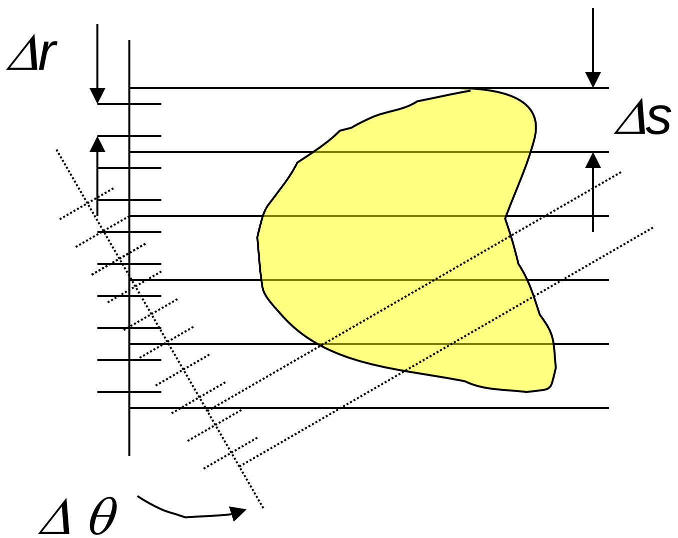
This gives rise to a discrete sinogram $p(n\Delta r, m\Delta\theta)$

- a matrix with M rows and N columns

  Δr is the detector sampling distance

  $\Delta\theta$ is the rotation interval between subsquent views
- assume also a beam of width Δs

Sampling theory will tell us how to choose these parameters for a given desired object resolution



Sampling (2)

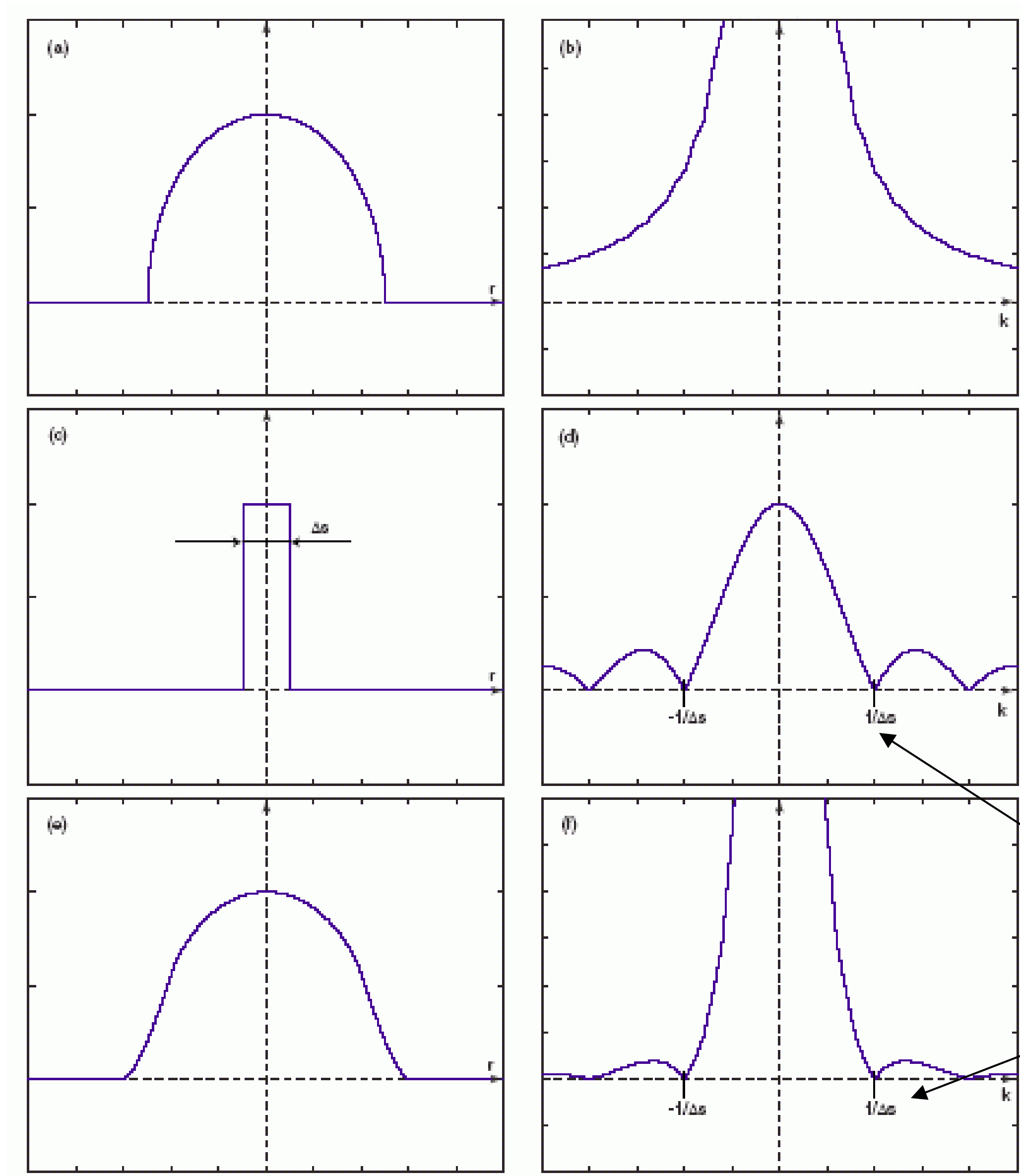
spatial domain | frequency domain

projection $p_\theta(r)$

*

beam aperture Δs

smoothed projection



sinc function

$1 / \Delta s$

Sampling (3)

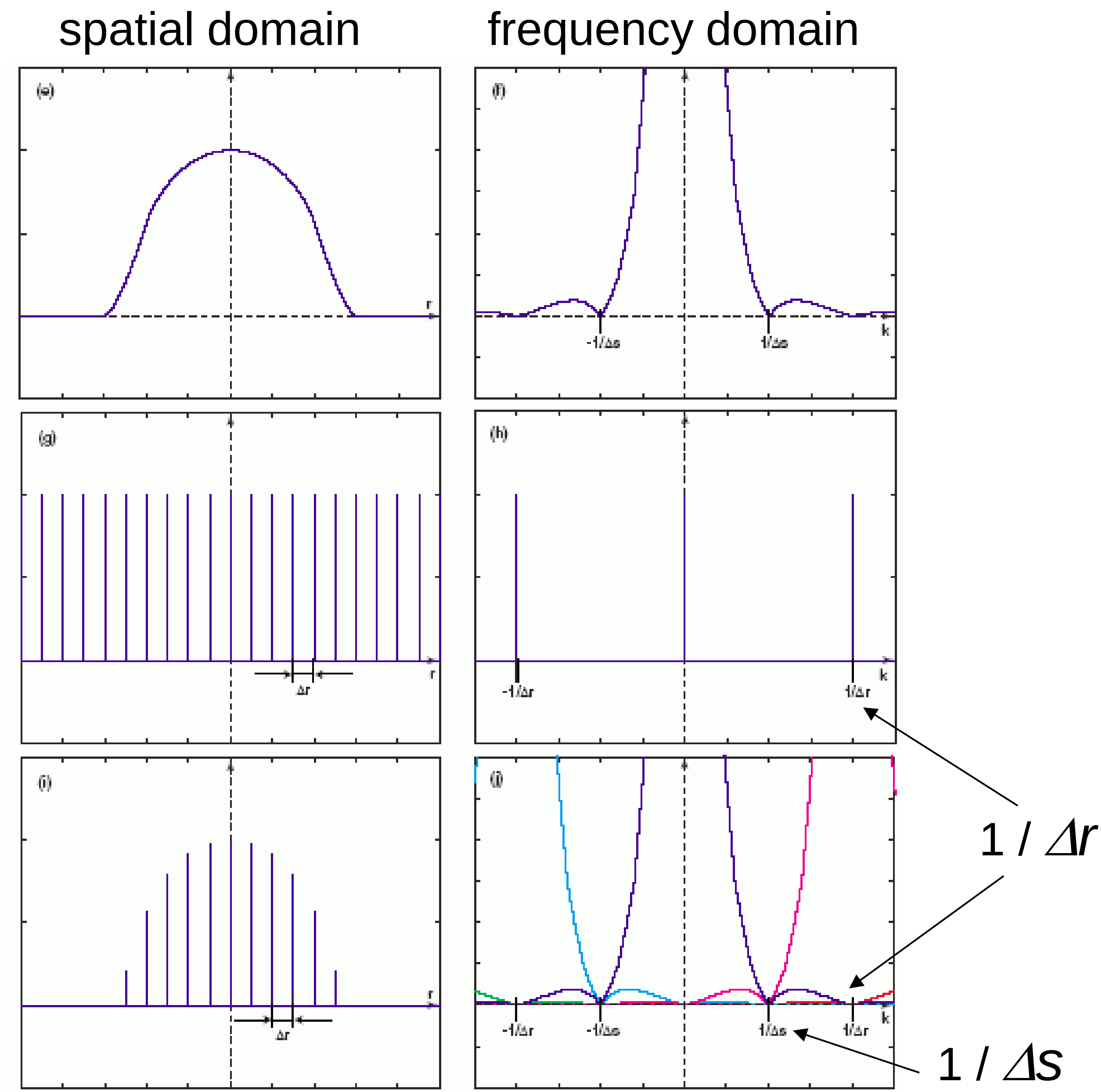
smoothed projection

•

sampling at *Δr*

↓

sampled projection



Limiting Aliasing

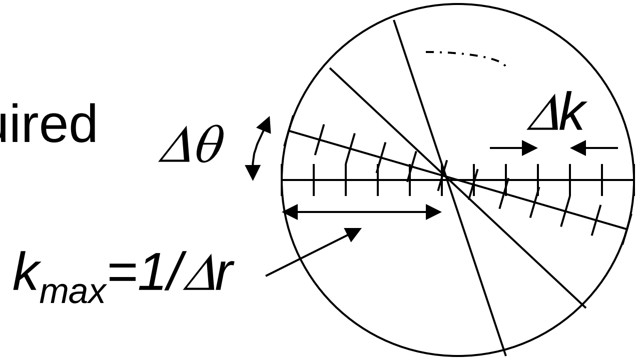
Aliasing within the sinogram lines (projection aliasing):

- to limit aliasing, we must separate the aliases in the frequency domain (at least coinciding the zero-crossings):

$$\frac{1}{\Delta r} \geq \frac{2}{\Delta s} \quad \rightarrow \quad \Delta r \leq \frac{\Delta s}{2}$$

- thus, at least 2 samples per beam are required

Aliasing across the sinogram lines (angular aliasing):



sinogram in the frequency domain
(2 projections with *N*=12 samples each are shown)

$$\Delta\theta = \frac{\pi k_{\max}}{M}$$

M: number of views, evenly distributed around the semi-circle

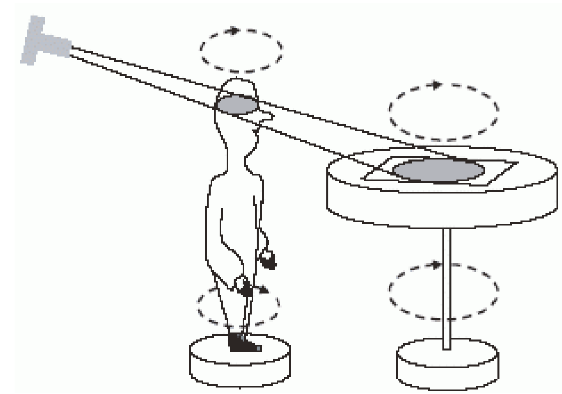
$$\Delta k = \frac{k_{\max}}{N/2}$$

N: number of detector samples, give rise to *N* frequency domain samples for each projection

for uniform sampling: $\Delta\theta = \Delta k \quad \rightarrow \quad \frac{\pi k_{\max}}{M} = \frac{k_{\max}}{N/2} \quad \rightarrow \quad M = \pi \frac{N}{2}$

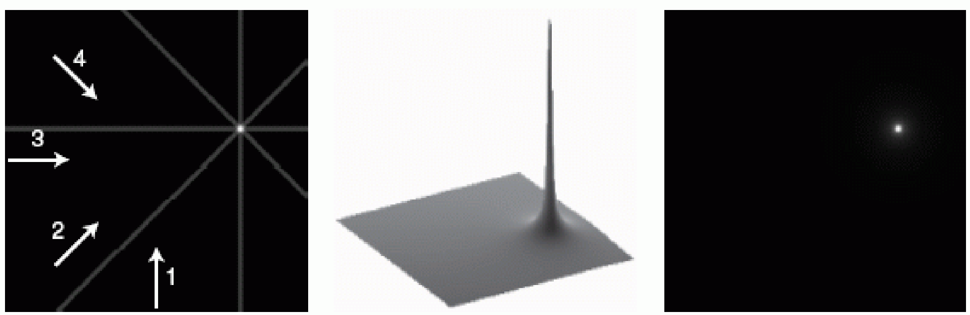
Reconstruction: Concept

Given the sinogram $p(r,\theta)$ we want to recover the object described in (x,y) coordinates



Recall the early axial tomography method

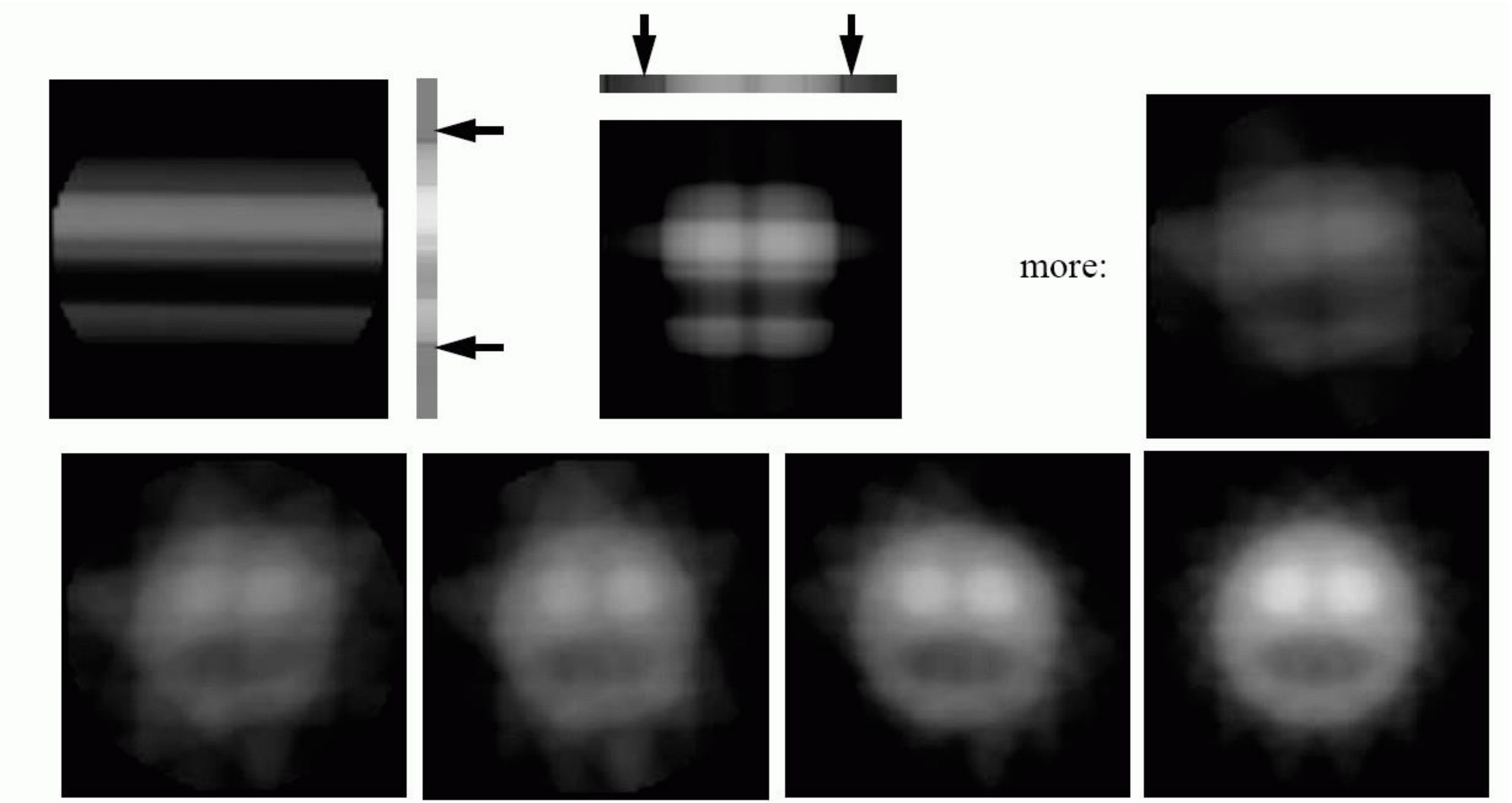
- basically it worked by subsequently “smearing” the acquired $p(r,\theta)$ across a film plate
- for a simple point we would get:



This is called *backprojection*:

$$b(x,y) = B\{p(r,\theta)\} = \int_0^{\pi} p(x\cdot\cos\theta + y\cdot\sin\theta, \theta)\, d\theta$$

Backprojection: Illustration



Backprojection: Practical Considerations

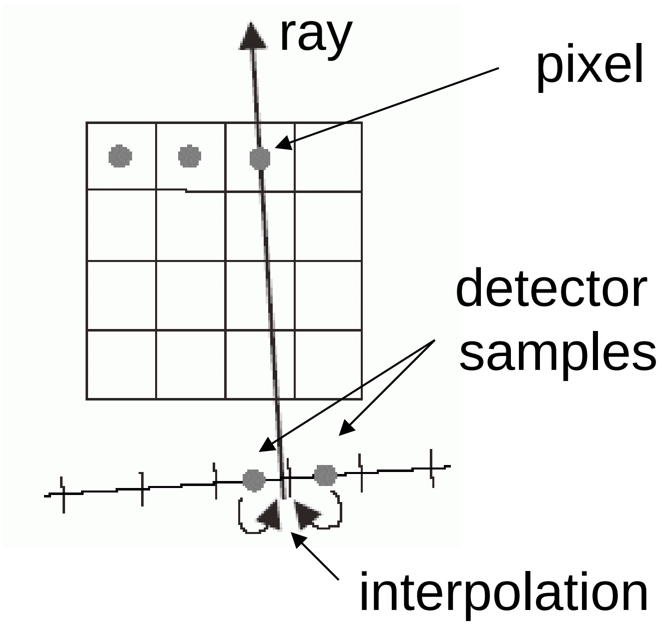
A few issues remain for practical use of this theory:

- we only have a finite set of *M* projections and a discrete array of *N* pixels (x_i, y_j)

$$b(x_i, y_j) = B\{p(r_n, \theta_m)\} = \sum_{m=1}^{M} p(x_i \cdot \cos\theta_m + y_j \cdot \sin\theta_m, \theta_m)$$

- to reconstruct a pixel (x_i, y_j) there may not be a ray $p(r_n,\theta_n)$ (detector sample) in the projection set

  → this requires interpolation (usually linear interpolation is used)

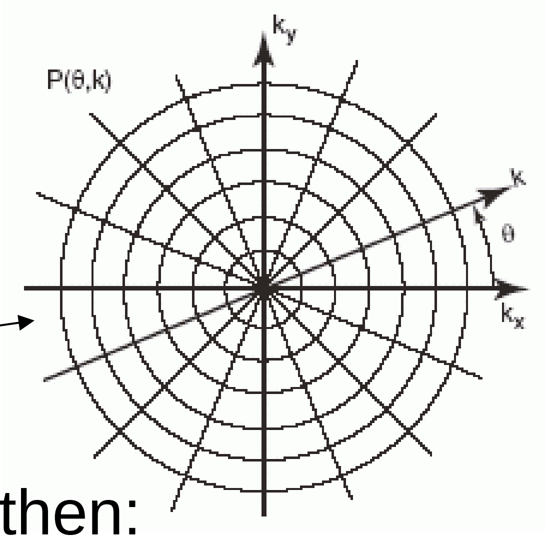


- the reconstructions obtained with the simple backprojection appear blurred (see previous slides)

The Fourier Slice Theorem

To understand the blurring we need more theory → the *Fourier Slice Theorem* or *Central Slice Theorem*

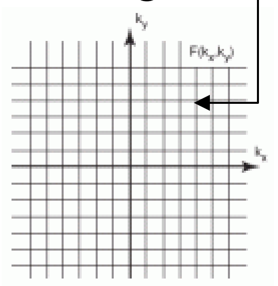
- it states that the Fourier transform $P(\theta,k)$ of a projection $p(r,\theta)$ is a line across the origin of the Fourier transform $F(k_x,k_y)$ of function $f(x,y)$



polar grid

A possible reconstruction procedure would then:

- calculate the 1D FT of all projections $p(r_m,\theta_m)$, which gives rise to $F(k_x,k_y)$ sampled on a polar grid (see figure)
- resample the polar grid into a cartesian grid (using interpolation)
- perform inverse 2D FT to obtain the desired $f(x,y)$ on a cartesian grid



However, there are two important observations:

- interpolation in the frequency domain leads to artifacts
- at the FT periphery the spectrum is only sparsely sampled

Filtered Backprojection: Concept

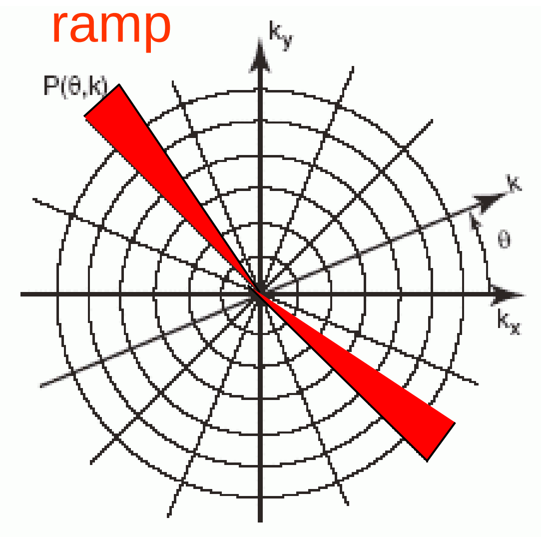
To account for the implications of these two observations, we modify the reconstruction procedure as follows:

- filter the projections to compensate for the blurring
- perform the interpolation in the spatial domain via backprojection

  → hence the name *Filtered Backprojection*

Filtering – for now a more practical explanation:

- we need a way to equalize the contributions of all frequencies in the FT's polar grid
- this can be done by multiplying each $P(\theta,k)$ by a ramp function → this way the magnitudes of the existing higher-frequency samples in each projection are scaled up to compensate for their lower amount
- the ramp is the appropriate scaling function since the sample density decreases linearly towards the FT's periphery



Filtered Backprojection: Equation and Result

1D Fourier transform of $p(r,\theta)$ → $P(k,\theta)$

ramp-filtering

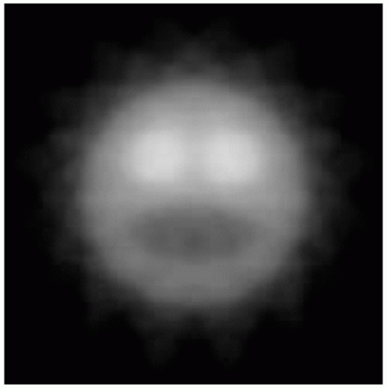
$$f(x,y) = \int_0^{\pi} \left(\int_{-\infty}^{\infty} P(k,\theta) \cdot |k| \cdot e^{i2\pi kr} dk \right) d\theta$$

backprojection for all angles

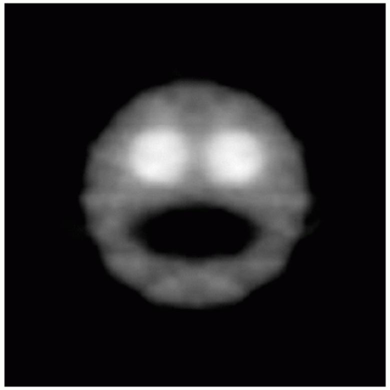
inverse 1D Fourier transform → $p(r,\theta)$

Recall the previous (blurred) backprojection illustration

- now using the filtered projections:

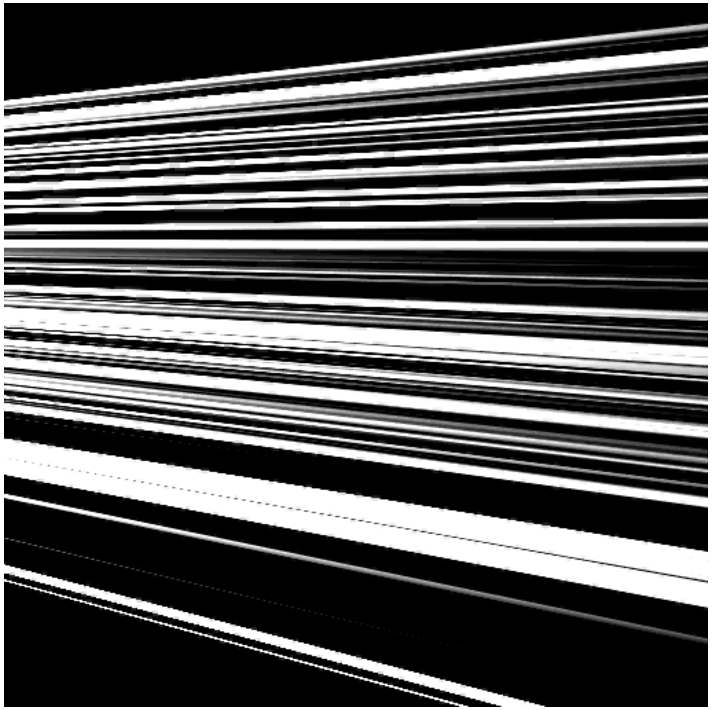


not filtered



filtered

Filtered Backprojection: Illustration



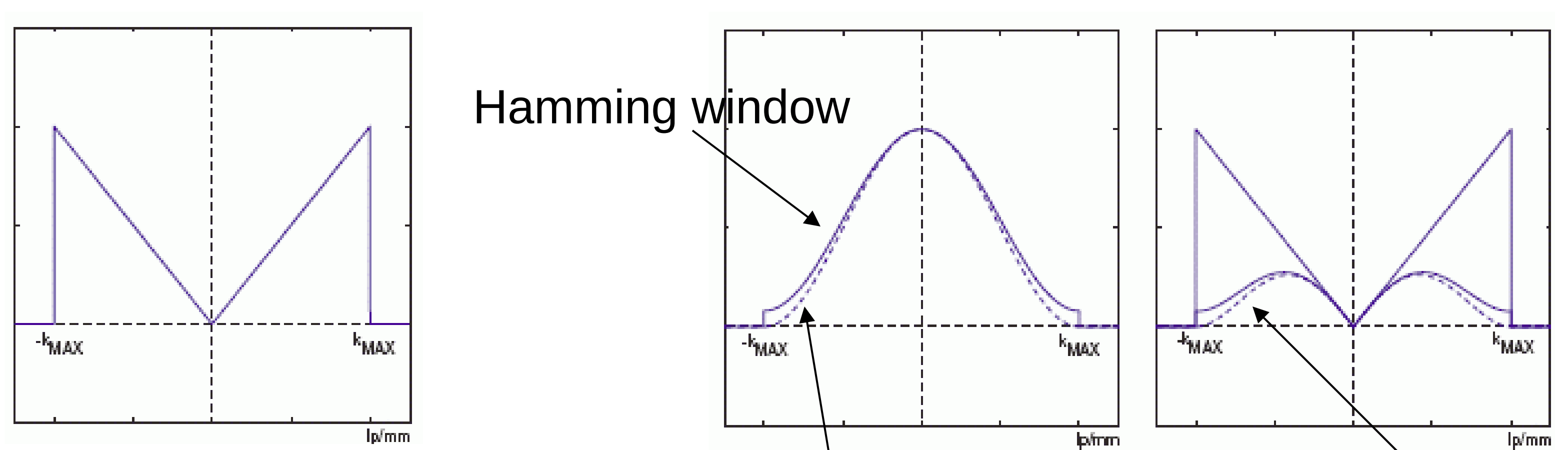
Filters

There are various filters:

- all filters have large spatial extent → convolution would be expensive
- therefore the filtering is usually done in the frequency domain → the required two FT's plus the multiplication by the filter function has lower complexity

Popular filters:

- Ram-Lak: original ramp filter limited to interval [$\pm k_{max}$]
- Ram-Lak with Hanning/Hamming smoothing window: de-emphasizes the higher spatial frequencies to reduce aliasing and noise



Ram-Lak

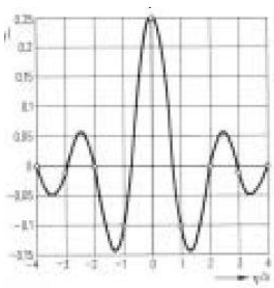
Hanning window

Windowed Ram-Lak

Frequency domain:

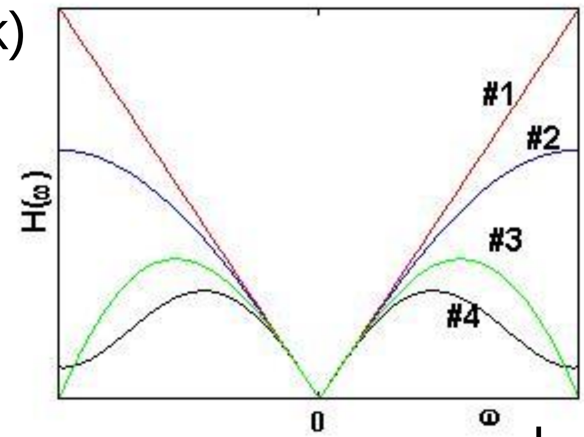
- #1: Ram-Lak

$$H(k) = |k|$$



spatial domain

H(k)



k_{max}

frequency domain

- #2: Shepp-Logan

$$H(k) = |k| \cdot \mathrm{sinc}\left(\frac{\pi k}{2k_{\mathrm{max}}}\right)$$

- #3: cosine $H(k) = \cos(k / k_{\mathrm{max}})$

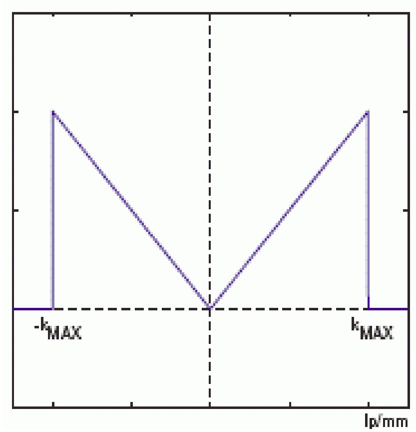
- #4: Hamming (α=0.54) and Hanning (α=0.5)

$$H(k) = \alpha + (1-\alpha)\cos\left(\frac{\pi k}{k_{\mathrm{max}}}\right)$$

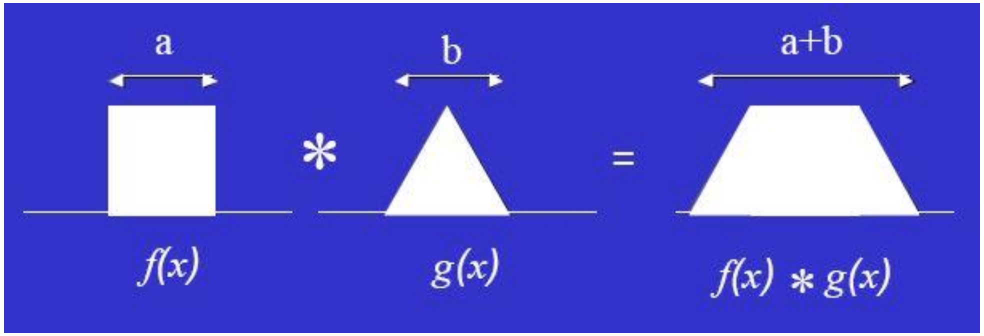
Filtering: Details

The filter seems to set the DC term to zero:

- but the reconstructed image has all positive values with a non-zero average
- how can that be explained?



Recall convolution theory

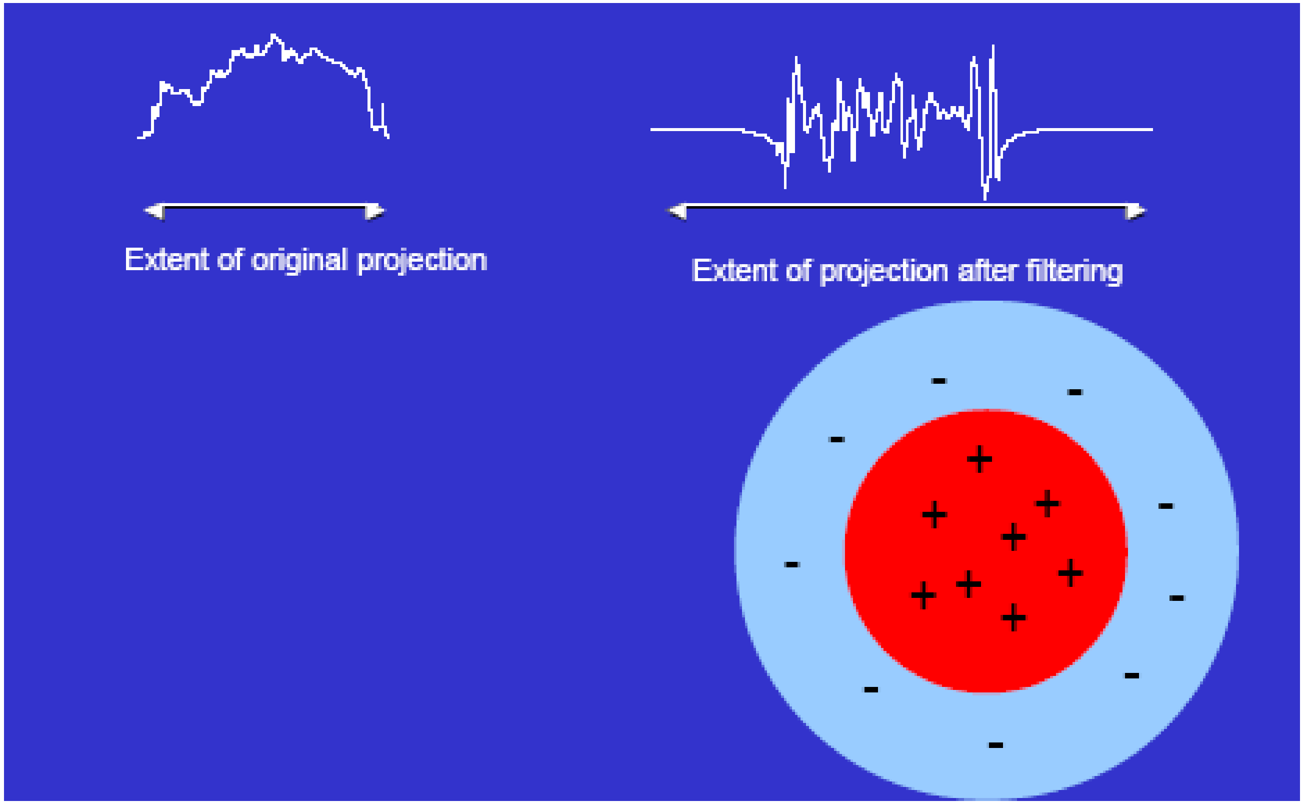


- the length of a convolved signal is the sum of the supports of the individual functions
- since convolution in the spatial domain is equivalent to multiplication in the frequency domain, there is a direct correspondance

Filering: Details

Thus:

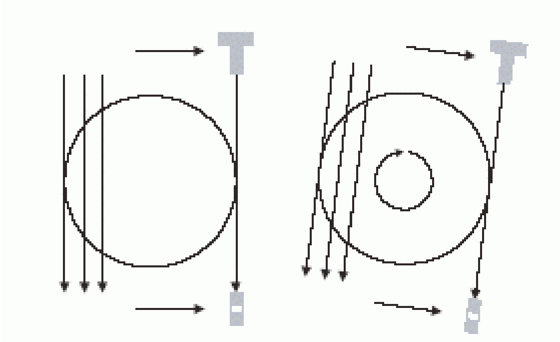
- the extent of the projection doubles with filtering
- and the average value of this image is indeed zero
- but we are only interested in the inner part
- so it all works out!



Beam Geometry

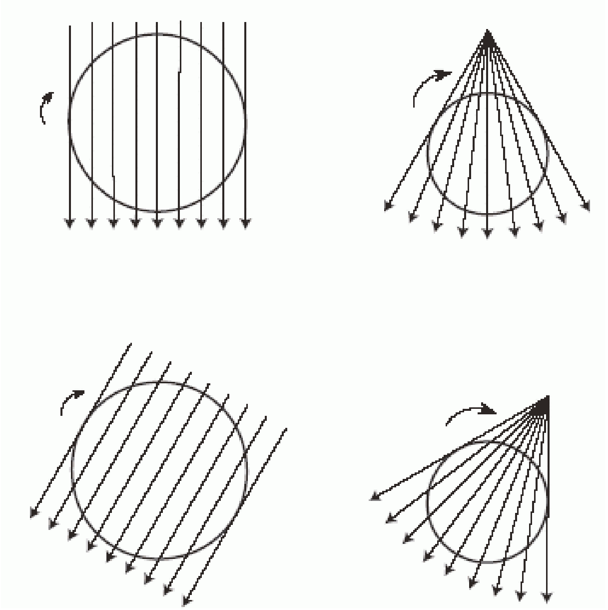
The parallel-beam configuration is not practical

- it requires a new source location for each ray

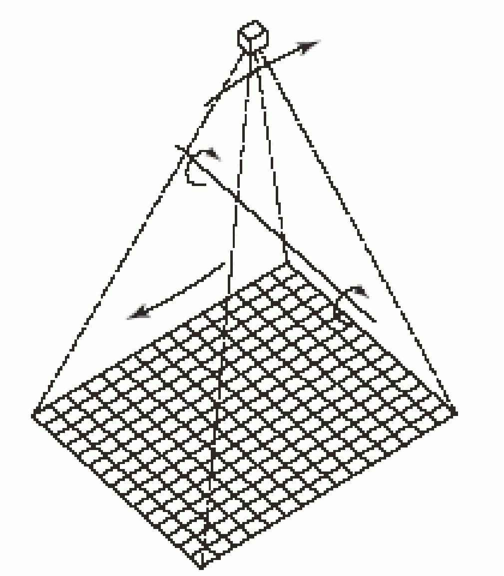


We'd rather get an image in "one shot"

- the requires *fan-beam* acquisition

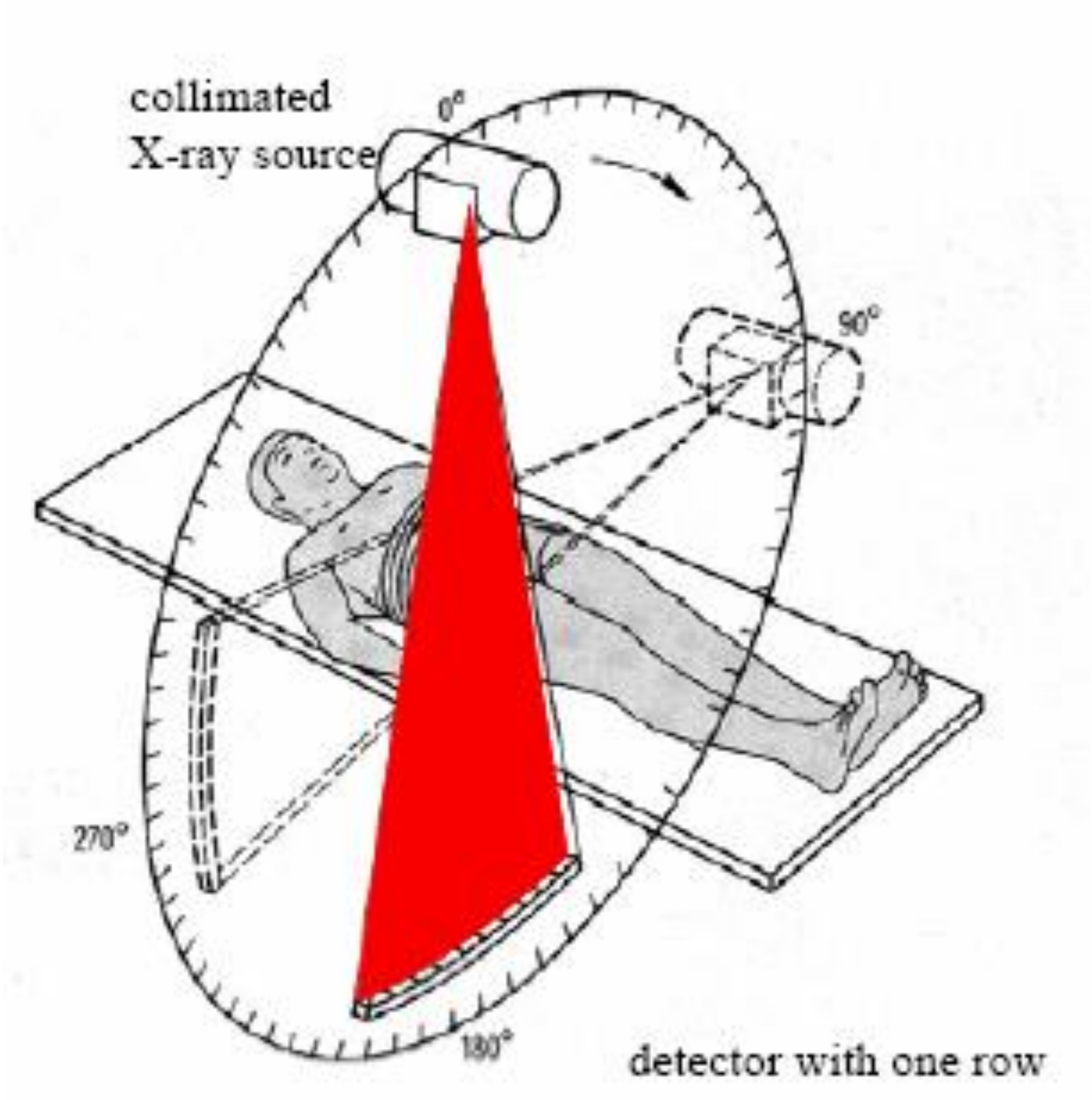


parallel-beam fan-beam



cone-beam in 3D

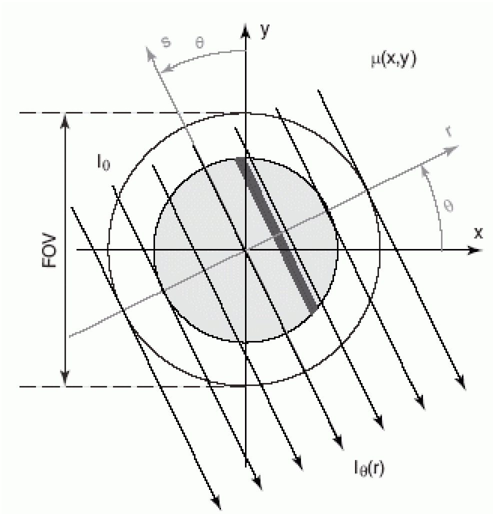
Fan Beam Acquisition



from: Dr. Günter Lauritsch, Siemens

Fan-Beam Mathematics

Rewrite the parallel-beam equations into the fan-beam geometry



Recall:

- filtering:

$$p^*(r,\theta) = \int_{-FOV/2}^{+FOV/2} p(r',\theta)q(r-r')dr'$$

- backprojection:

$$f(x,y) = \int_0^{\pi} p^*(r,\theta)d\theta \quad \text{with} \quad r = x\cos\theta + y\sin\theta$$

- and combine:

$$f(x,y) = \int_0^{2\pi} \int_{-FOV/2}^{+FOV/2} p(r',\theta)q(x\cos\theta + y\sin\theta - r')dr'd\theta$$

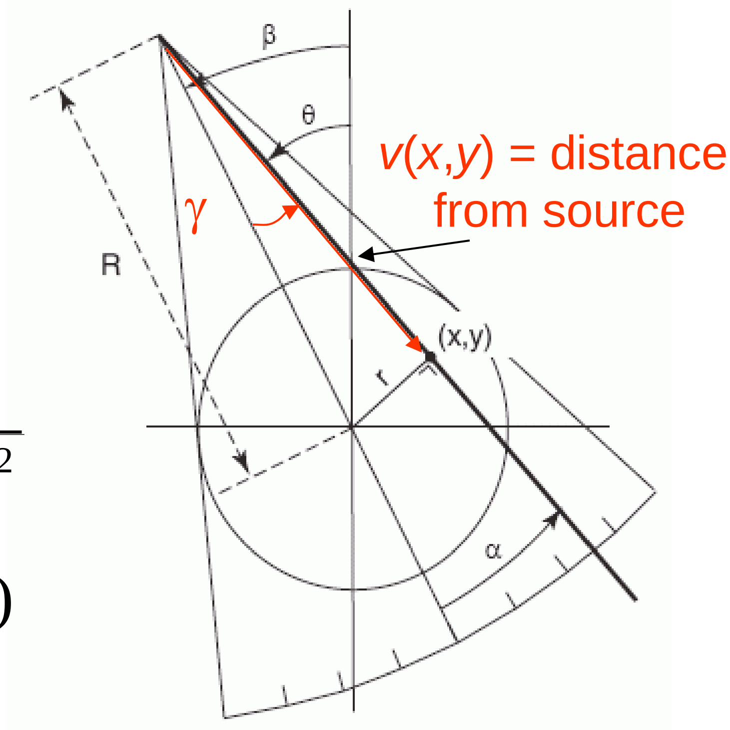
Fan-Beam Mathematics

$$f(x,y)=\int_0^{2\pi}\int_{-FOV/2}^{+FOV/2} p(r',\theta)q(x\cos\theta+y\sin\theta-r')dr'd\theta$$

- with change of variables:

$$\theta=\alpha+\beta \qquad r=R\sin\alpha$$

- for a voxel (x,y): v = distance, γ = angle



$$v=\sqrt{(x\cos\beta+y\sin\beta)^2+(x\sin\beta-y\cos\beta+R)^2}$$

$$\gamma=\text{atan}((x\cos\beta+y\sin\beta)/(x\sin\beta-y\cos\beta+R))$$

the projection at β

$$f(x,y)=\int_0^{2\pi}\frac{1}{v^2(x,y)}\int_{-fan-angle/2}^{+fan-angle/2} R\cos\alpha\cdot p(\alpha,\beta)\cdot\left(\frac{\gamma-\alpha}{2\sin(\gamma-\alpha)}\right)q(\gamma-\alpha)d\alpha\, d\beta$$

3. weighting during backprojection

1. projection pre-weighting

2. filter

Fan-Beam Mathematics

See chapter in Kak-Slaney (posted on the class website) for equations associated with flat detectors

So, reconstruction from fan-beam data involves

- a pre-weighting of the projection data, depending on α
- a pre-weighting of the filter (here we used the spatial domain filters)
- a backprojection along the fan-beam rays (interpolation as usual)
- a weighting of the contributions at the reconstructed pixels, depending on their distance $v(x, y)$ from the source

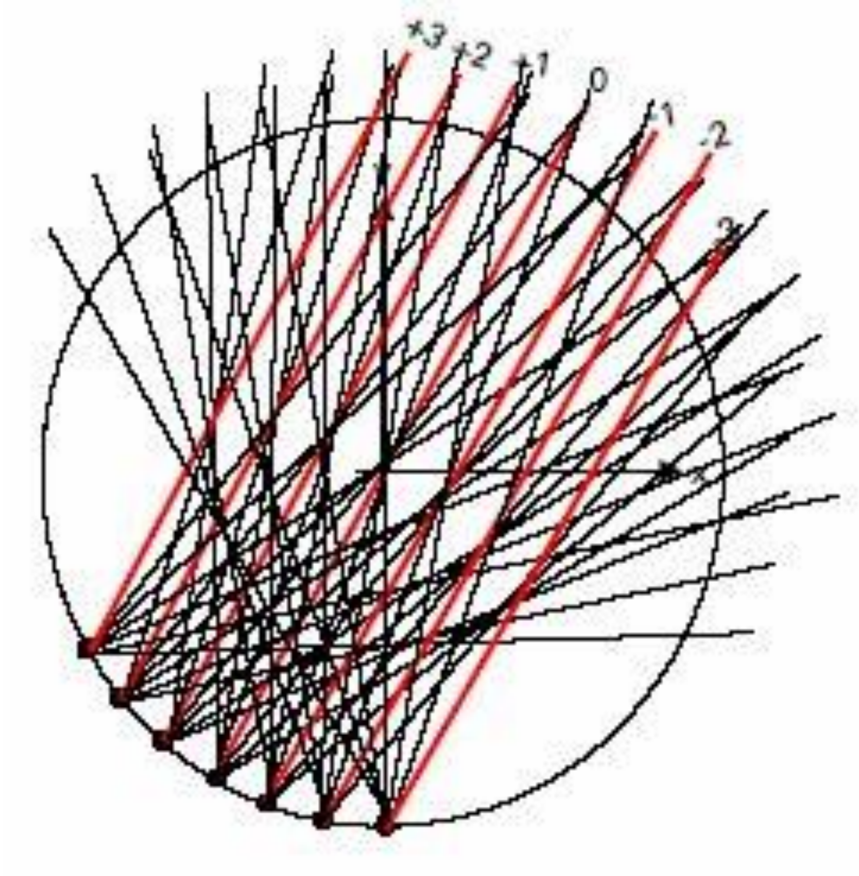
There are also iterative algorithms

- these pose the reconstruction problem as a system of linear equations
- solution via iterative solvers
- more on this to come in the nuclear medicine lectures

Fan-Beam Mathematics

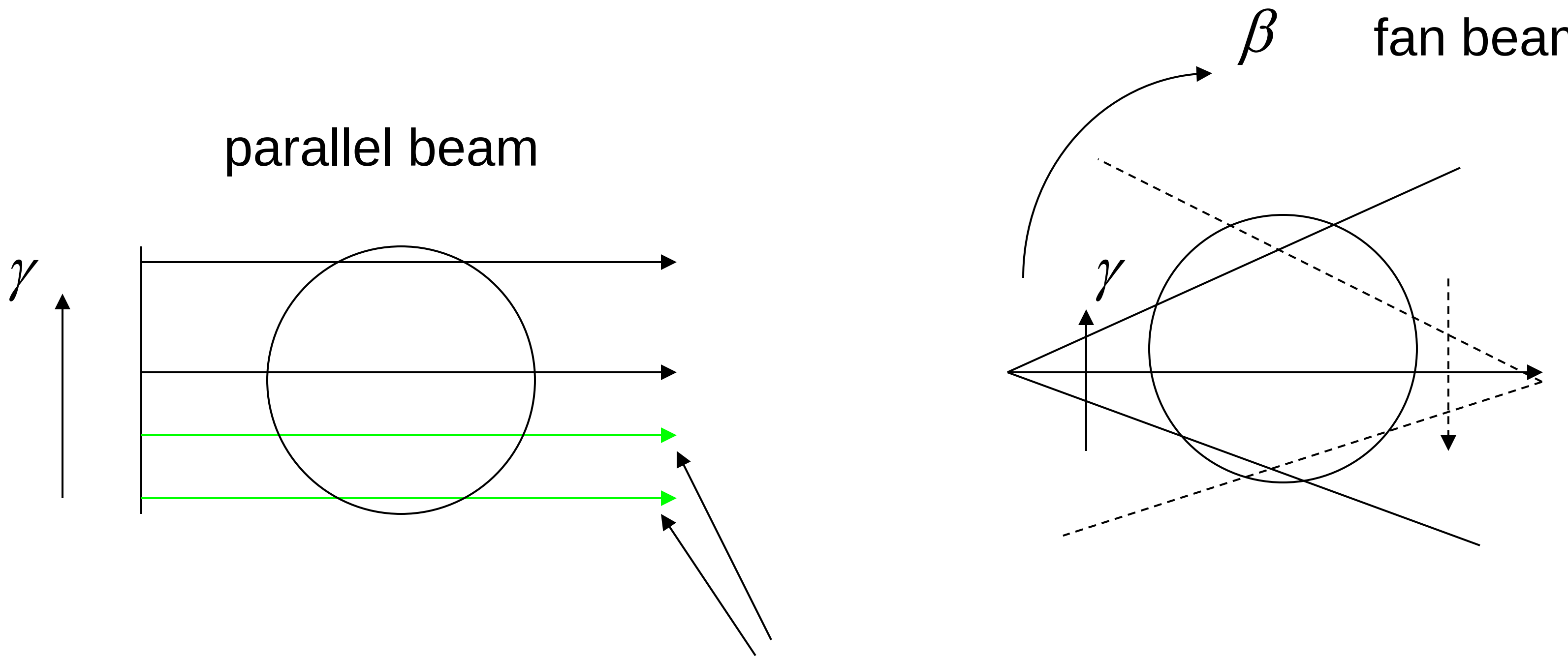
Alternatively, one could also “rebin” the data into a parallel-beam configuration

- however, this requires an additional interpolation since there is no direct mapping into a uniform parallel-beam configuration



Fan-Beam Mathematics

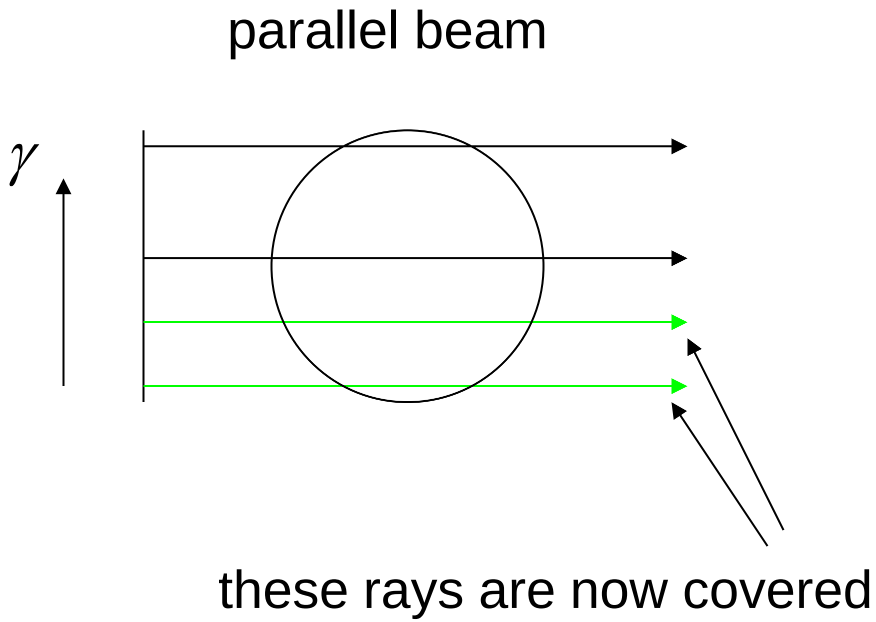
Problem: fan-beam does not fill the sinogram adequately

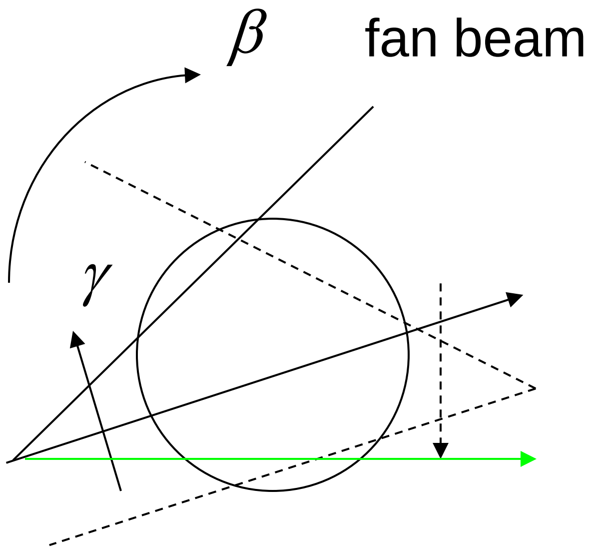


these rays (and others) are not covered by any fan-beam view

Fan-Beam Mathematics

Solution: extend the source-detector trajectory by the fan half-angle on both ends

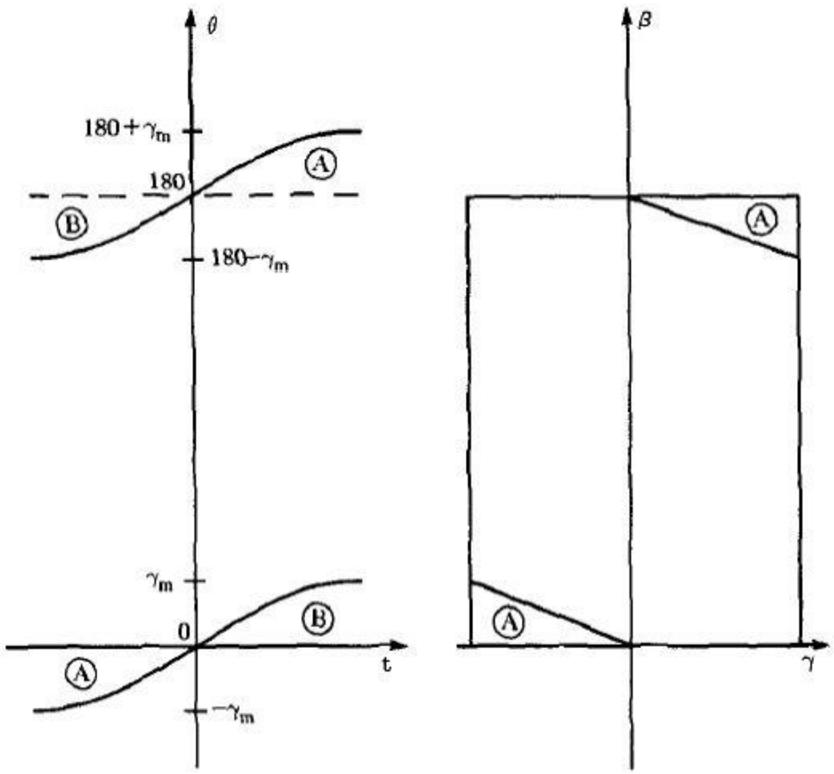




Fan-Beam Mathematics

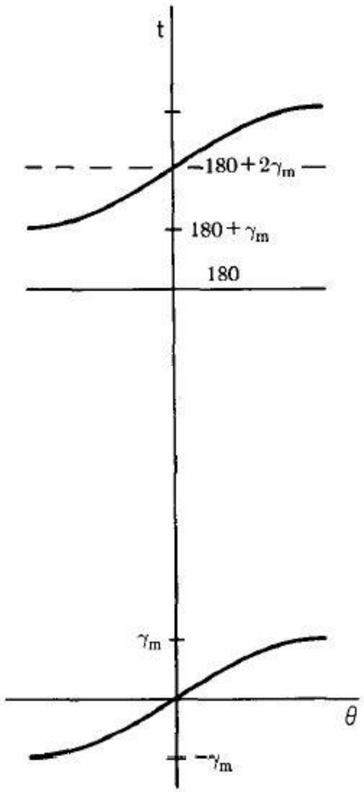
More formally

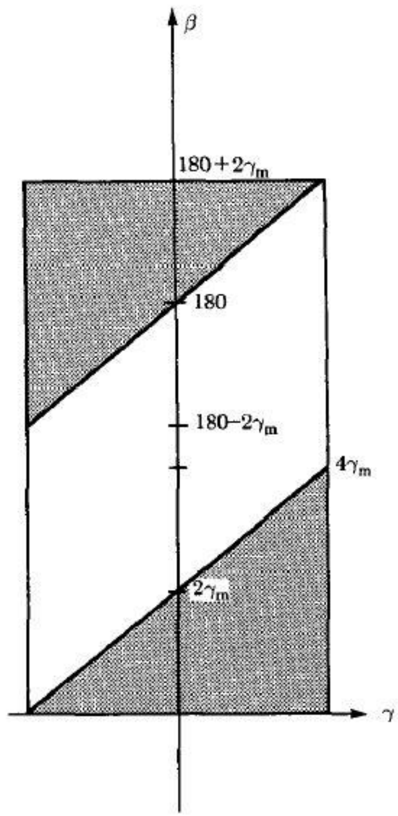
- region A is covered twice, while region B is not covered at all

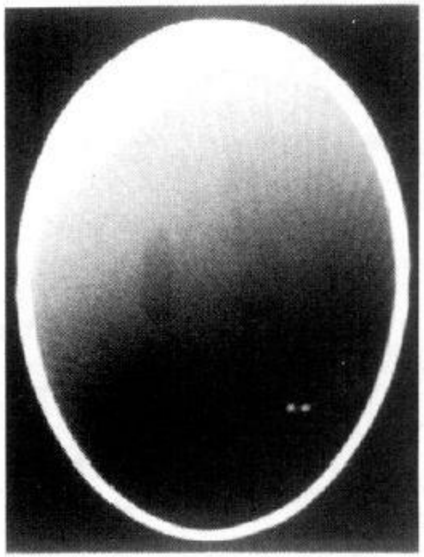


Extending the trajectory fills the space

- but some areas are filled twice, which causes problems







Fan-Beam Mathematics

Simply setting these regions to zero will result in heavy streak artifacts

- recall the filtering step?

Need to use a smoother window

- a smooth window is both continuous and has a continuous derivative at the boundary of single and double-overlap regions

$$\left.\frac{\partial w_\beta(\gamma)}{\partial \beta}\right|_{\beta=2\gamma_m+2\gamma}=0$$

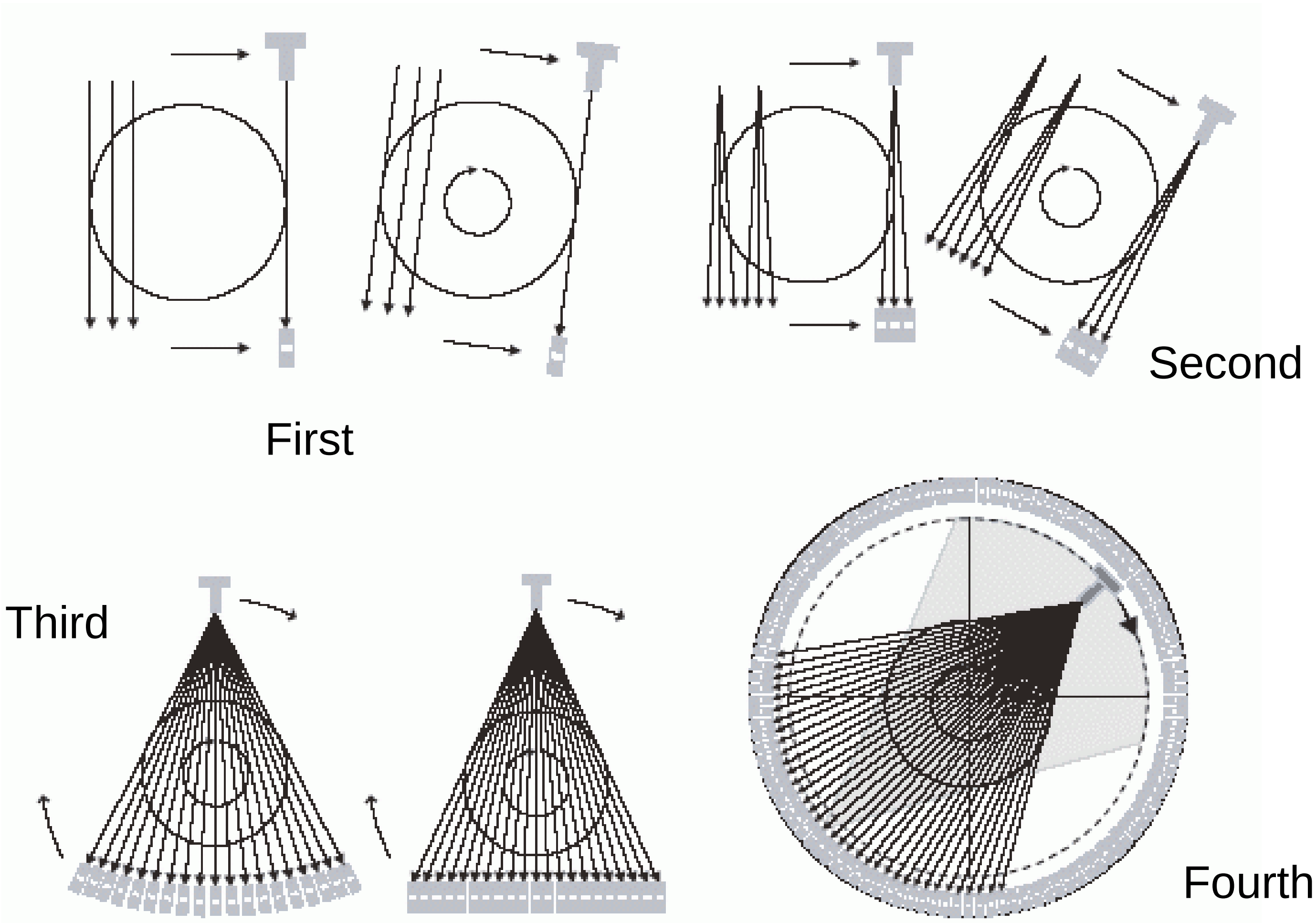
$$\left.\frac{\partial w_\beta(\gamma)}{\partial \beta}\right|_{\beta=180^\circ+2\gamma}=0.$$

- the window weights for the same rays at opposite sides of the sinogram must be 1. $w_{\beta_1}(\gamma_1)+w_{\beta_2}(\gamma_2)=1$

- the Parker window fullfills these conditions:

$$w_\beta(\gamma)=\begin{cases}\sin^2\left[\dfrac{45^\circ\beta}{\gamma_m-\gamma}\right], & 0\le\beta\le 2\gamma_m-2\gamma\\ 1, & 2\gamma_m-2\gamma\le\beta\le 180^\circ-2\gamma\\ \sin^2\left[45^\circ\,\dfrac{180^\circ+2\gamma_m-\beta}{\gamma+\gamma_m}\right], & 180^\circ-2\gamma\le\beta\le 180^\circ+2\gamma_m.\end{cases}$$

Scanner Generations

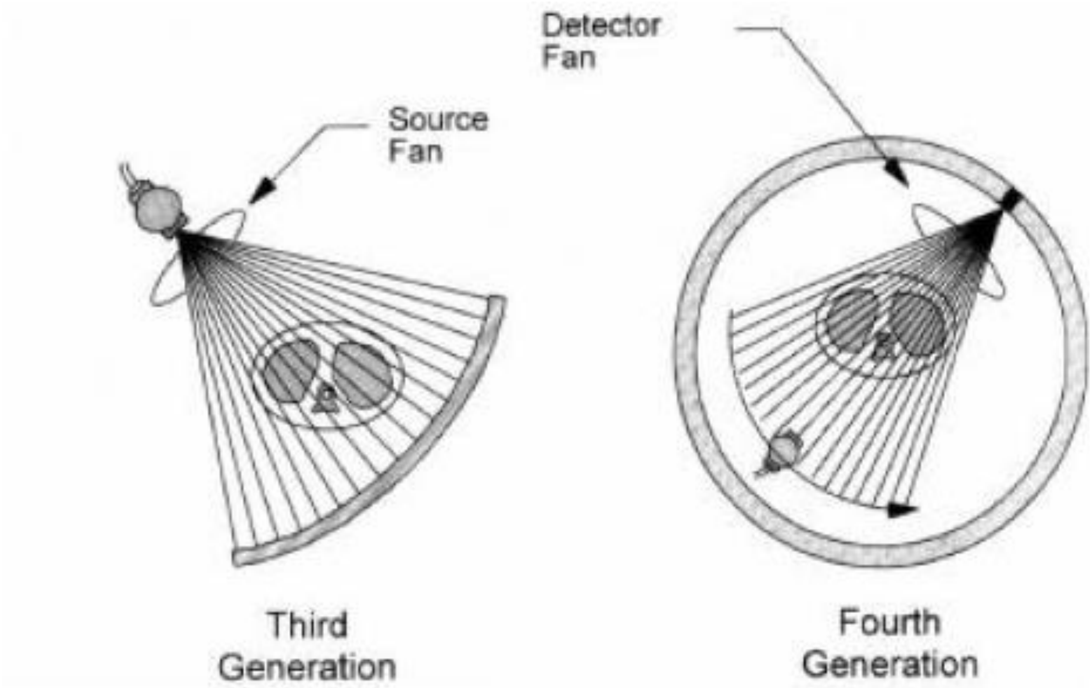


Third generation most popular since detector geometry is simplest

- collimation is feasible which eliminates scattering artifacts

Fan Beam Scanners

The 3rd and 4th generation scanners:



However, in the 3rd generation scanner:

- the detector width (the beam aperture Δs) = detector spacing Δr
- recall our earlier discussion on sampling constraints where we found that:

$$\frac{1}{\Delta r} \geq \frac{2}{\Delta s} \quad \rightarrow \quad \Delta r \leq \frac{\Delta s}{2}$$

Fan Beam Data Acquisition: Practice

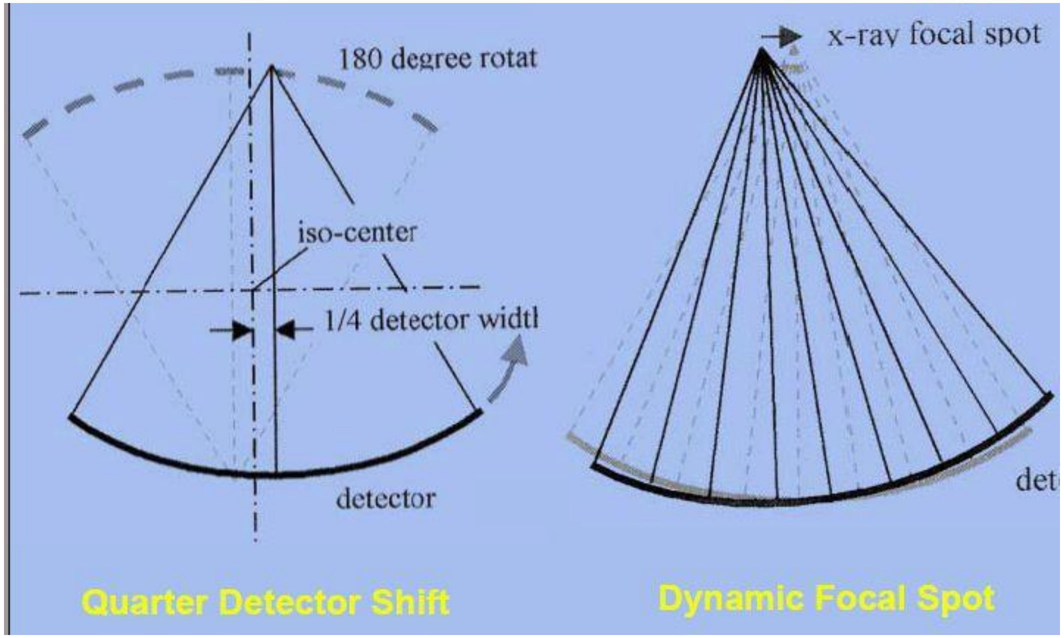
So, we should acquire 2 samples per detector width

- a symmetrical rotation configuration violates this requirement
- the consequence is ray aliasing:

Ray Aliasing Remedies

For 3rd generation scanners:

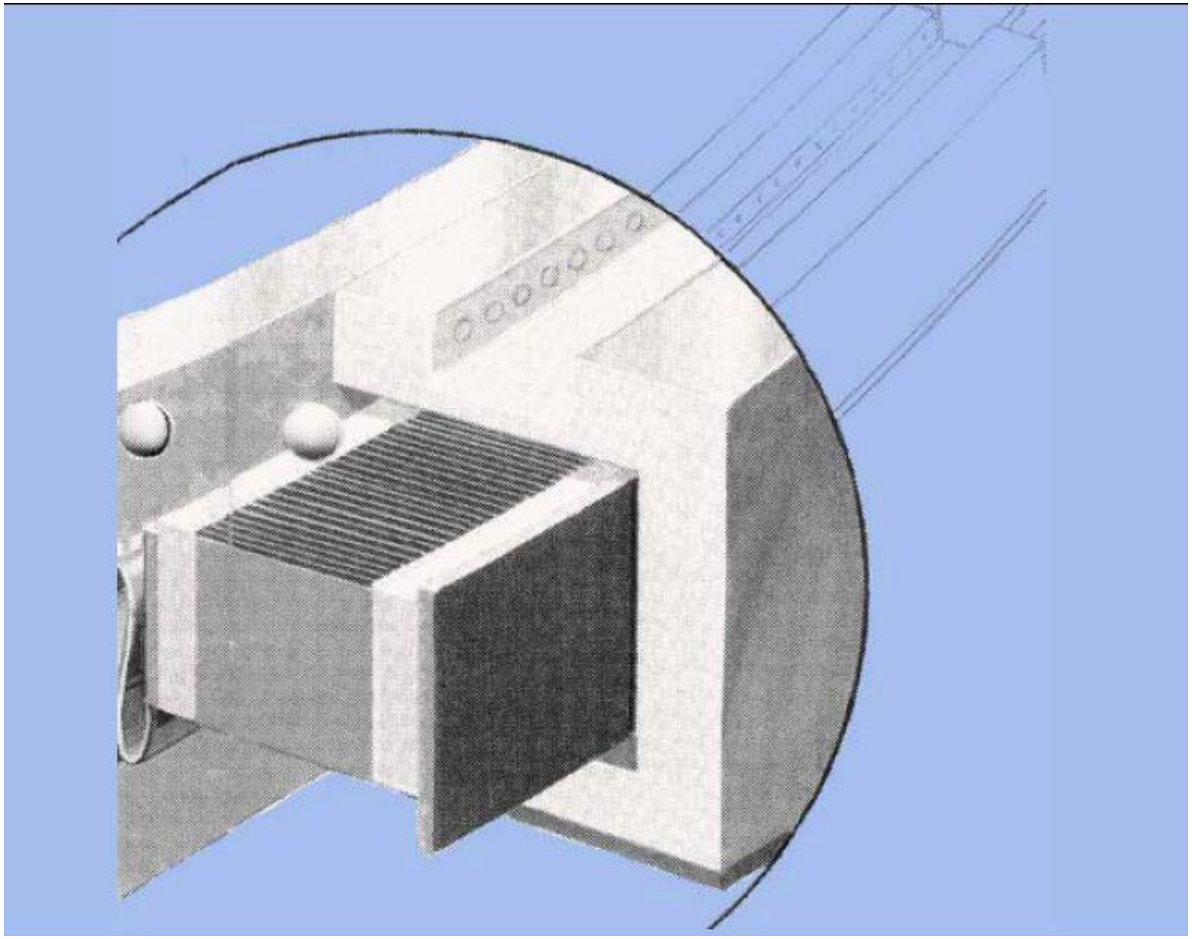
- ¼ detector shift
- dynamic focal spot
- both double the density of the sinogram with little technical overhead



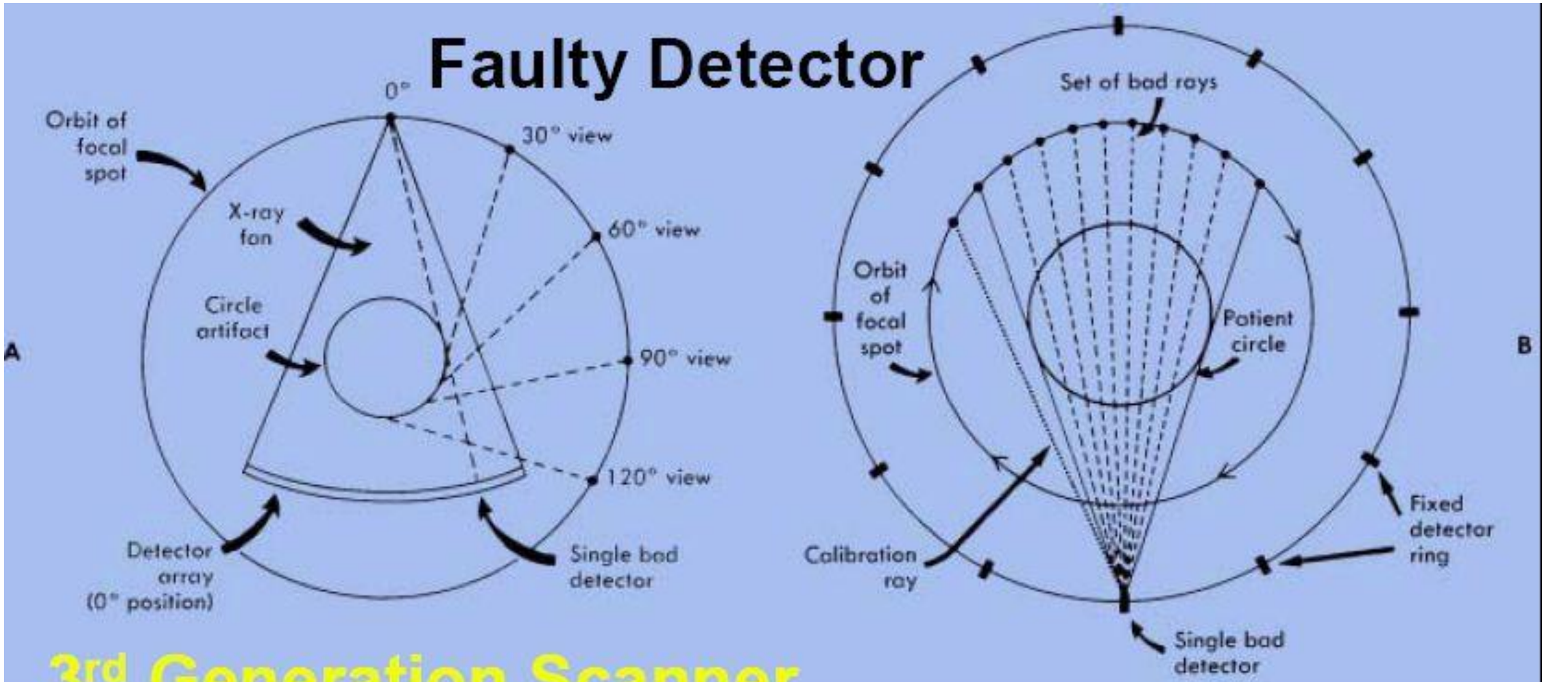
For 4rd generation scanners:

- move the X-ray tube at slower speeds
- this increases the number of ray samples

A Detector Element



Artifacts Related to Faulty Detectors



3rd Generation Scanner Ring Artifact

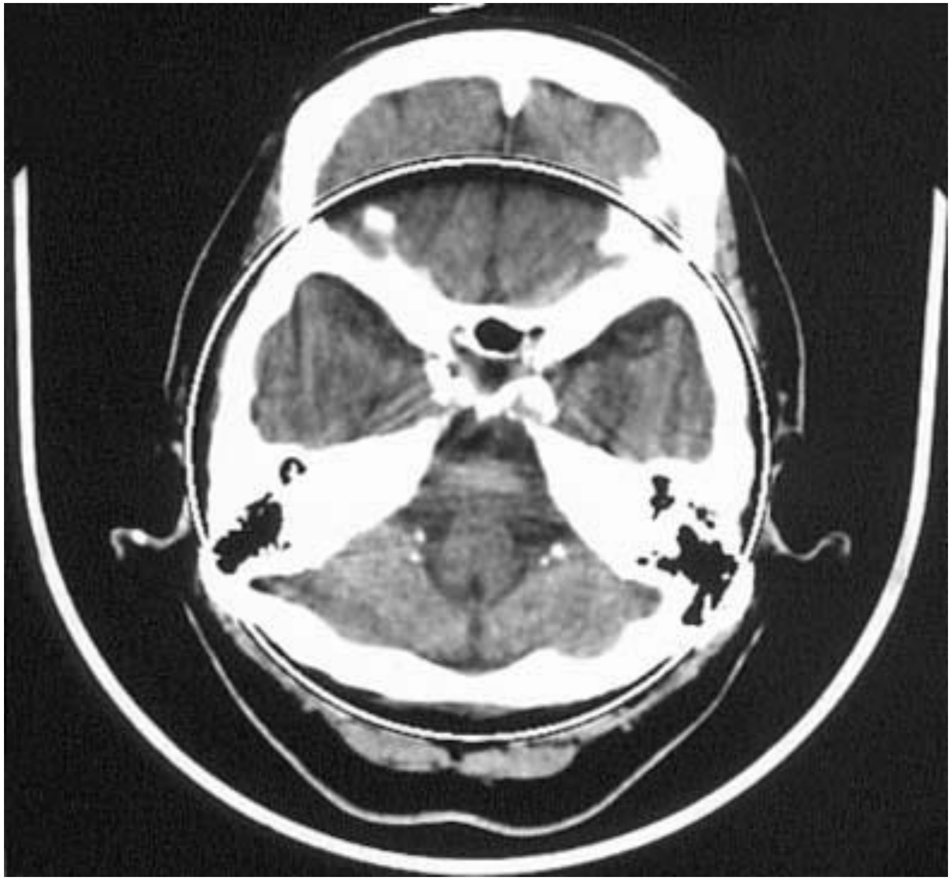
Rays Between Tube And Bad Detector Trace A Tangent To A Circle And Create A Ring On Image

4th Generation Scanner

Image Fog

Ring Artifacts

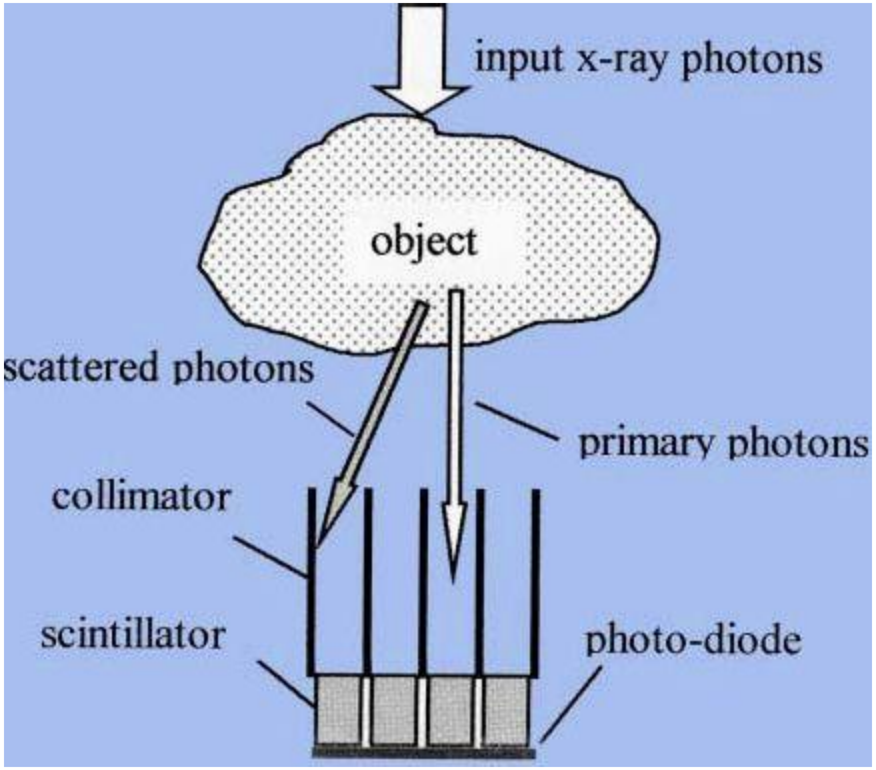
See larger ring just at the edge of the skull



A Note on Collimation

Collimation ensures that we know the ray direction at each detector bin (perpendicular to the local tangent)

- this enables reconstruction theory



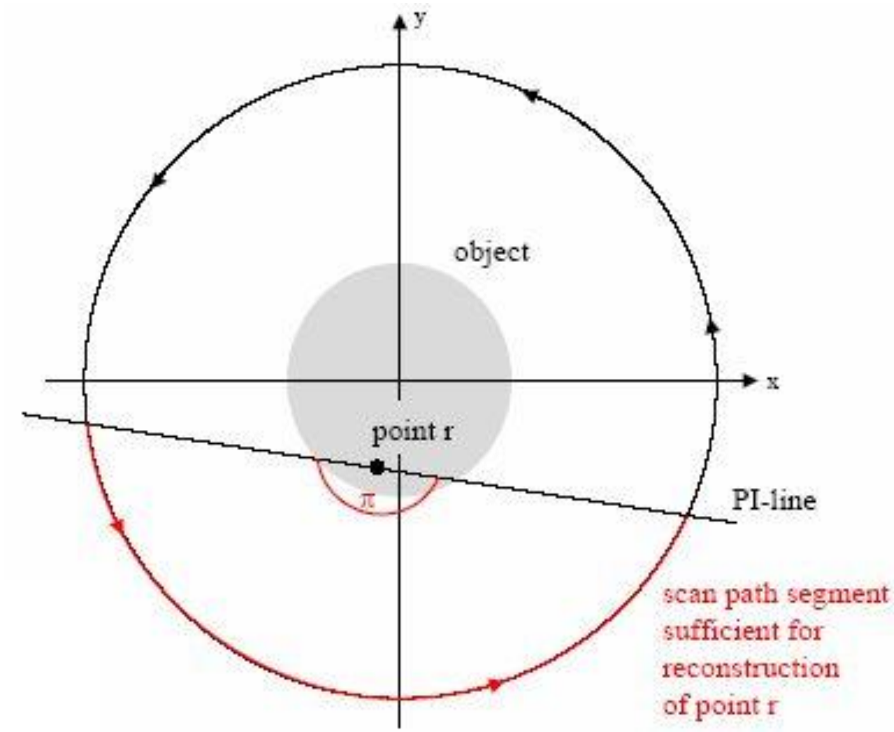
Short-Scan CT

Requirement:

- an object point *r* can be reconstructed exactly if it sees a scan path segment of an angular range π

Consequence:

- an smaller Region of Interest (ROI) can be reconstructed without acquiring complete data of the object (super short-scan).



Short-Scan CT

Specific algorithms are needed for reconstruction from a super short-scan

- F. Noo et al., BMP 2002
- H. Kudo et al., IEEE NSS 2002