

Introduction to Medical Imaging

Lecture 11: MRI Physics

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The Essential Element for MRI: Hydrogen

In MRI only hydrogen is used for imaging: ^1H

- the hydrogen atom is a component of water: H_2O
- the body consists of 2/3 water \rightarrow a lot of potential signal

The hydrogen atom has only one proton

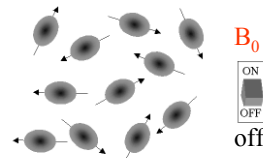
- this proton has a spin
- it rotates around its own axis which makes it act as a tiny magnet



Alignment of Protons

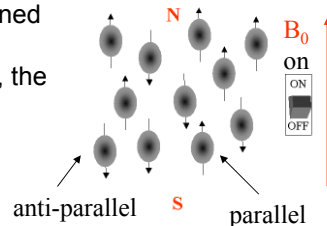
There are millions of protons in human tissue

- they are randomly oriented in the absence of an external magnetic field



An MRI magnet has a strong magnetic field, B_0 (measured in Tesla)

- it causes the protons to align themselves in the direction of B_0
- some align parallel to B_0 , some anti-parallel
- parallel alignment has the higher energy state
- the higher B_0 the more protons will be aligned parallel
- the more protons are in parallel alignment, the higher the *net magnetization* M_{z0}



$$M_{z0} \sim \sum \text{parallel protons} - \sum \text{anti-parallel protons}$$

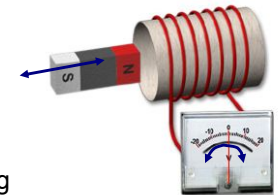
Measuring the Net Magnetization M_{z0}

We suspect that M_{z0} is related to the amount of hydrogen

- but how do we measure M_{z0} ?

A way to measure a magnetic field is via electromagnetic induction

- moving the magnet in and out of the coil induces an alternating current which can be measured
- the faster we move the magnet, the more current is induced
- the problem with M_{z0} is that it is not changing and therefore cannot be measured via induction



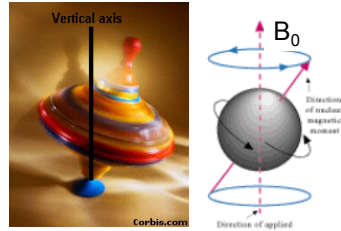
Need a way to turn M_{z0} into an alternating magnet field

- then the stronger M_{z0} , the more current would be induced
- also need to perform the measurements orthogonal to B_0

Turn M_{z0} into such an orthogonal, alternating magnet field by adding a precession component

Proton Spin Precession: Introduction

Equivalent to a spinning top



Now the magnetic field has

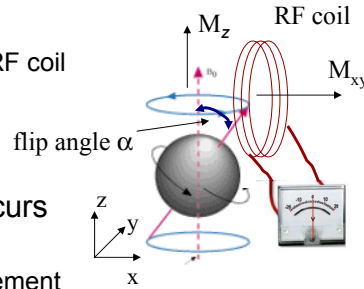
- a longitudinal (along B_0) component M_z
- a transverse component ($\perp B_0$) M_{xy}

Due to the precession M_{xy} oscillates in a sinusoidal fashion

- can be measured via induction in an RF coil
- will induce a sinusoidal current at frequency ω_0
- the magnitude is $M_{xy} = M_{z0} \sin \alpha$

The highest amount of induction occurs when the *flip angle* is 90°

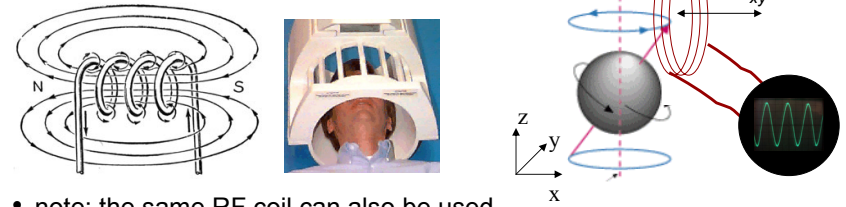
- then $M_{xy} = M_{z0} \rightarrow$ the desired measurement



How To Create The Precession

We need to add a magnetic field B_{xy} orthogonal to B_0

- this will pull the spinning proton into a precession
- generated by RF pulse (range: 10 - 100MHz)



- note: the same RF coil can also be used for the measurement of the resulting M_{xy}

B_{xy} needs to alternate at *Larmor frequency* $\omega_0 = \gamma B_0$

- then we obtain resonance \rightarrow the magnetic force is applied synchronous to the proton position on the precession circle
- also, the longer the RF signal is left on, the wider the precession
- to get the highest measured signal, one needs to keep B_{xy} on until the flip angle is 90°

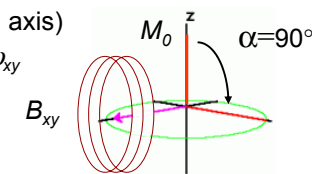
More Formally

The magnet field B_{xy} acts in a similar manner than B_0

- it also causes a spin (around the RF coil axis)
- this spin has also a Larmor frequency, ω_{xy} (orthogonal to ω_0):

$$\omega_{xy} = \gamma B_{xy}$$

- since $B_{xy} \ll B_0 \rightarrow \omega_{xy} \ll \omega_0$



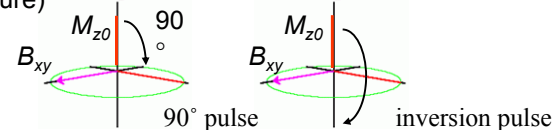
Depending how long B_{xy} is left on (or how large it is), we can rotate M_{z0} into different orientation angles α

- the angle α is called the *flip angle* $\alpha = \int_0^t \gamma B_{xy} d\tau = \gamma B_{xy} t = \omega_{xy} t$

Trade-offs:

- for fast imaging it is desirable to keep t short
- this requires doubling B_{xy} which quadruples the power (and the heat and tissue temperature)

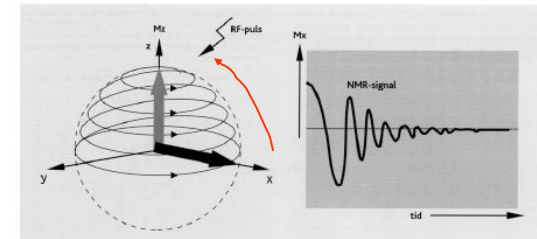
Important flip angles:



Relaxation

The tilt (flip) is an unstable situation

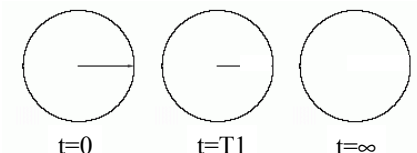
- the proton will rotate back to its original position along the z-axis
- the measured RF signal will decay and eventually go to zero



(also note the sinusoidal form of the induced signal)

- this decay is called *T1-relaxation*

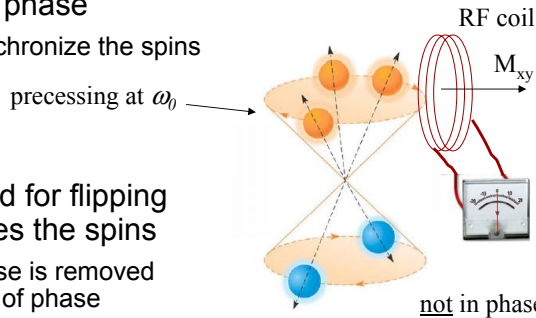
transverse component:



The Net Magnetization M_{xy}

In order to measure a signal of sufficient amplitude, all protons must precess in phase

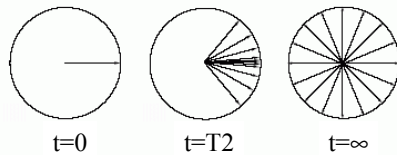
- we need to synchronize the spins



The RF pulse used for flipping also synchronizes the spins

- once the RF pulse is removed the spins go out of phase
- this is called *T2-relaxation*

transverse component:



Spin-Spin Relaxation (T_2)

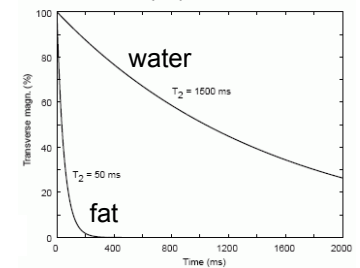
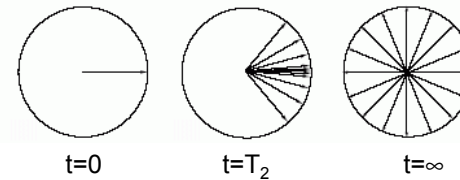
Relaxation due to the gradual disappearance of M_{z0} 's transverse component M_{xy}

- in practice, each spin experiences a slightly different magnet field due to the locally different chemical environments (protons can belong to H_2O , $-OH$, $-CH$, ...)
- this results the spins to rotate at slightly different angular frequencies
- and as a consequence a loss of phase coherence (*dephasing*) occurs
- the time constant for the exponential decay is called *spin-spin relaxation time T_2* :

$$M_{xy}(t) = M_{xy}(0)e^{-\frac{t}{T_2}}$$

T_2 is very tissue-dependent

90° RF pulse 37% dephased no M_{xy} left



Spin-Lattice Relaxation (T_1)

In spin-spin relaxation there is no loss of flip angle

- the system became only disordered and unsynchronized

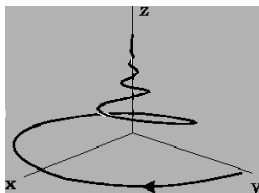
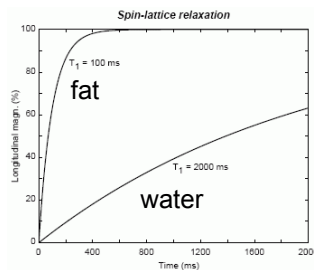
In spin-lattice relaxation, the flip angle actually changes

- the longitudinal component M_z will grow from $M_{z0} \cos \alpha$ to M_{z0}
- the energy shift is caused by the (small) heat released through the lattice molecule vibrations
- the time constant for the exponential decay is called *spin-lattice relaxation time T_1* :

$$M_z(t) = M_{z0} \cos \alpha e^{-\frac{t}{T_1}} + M_{z0} (1 - e^{-\frac{t}{T_1}})$$

will return to the equilibrium value, M_0

Note: T_1 is typically always greater than T_2



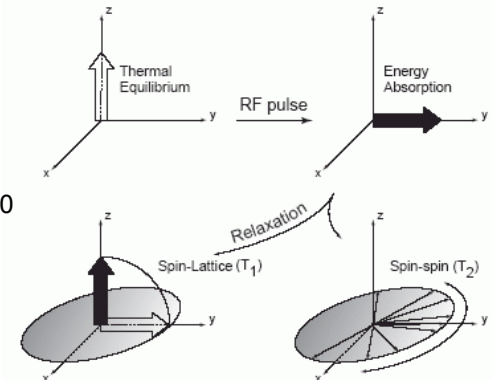
Summary: Energy Absorption and Relaxation

Combining the T_1 and T_2 effects into a single equation (the Bloch relaxation equation):

$$M_{xy} = M_{xy0} e^{-\frac{t}{T_1}} e^{-\frac{t}{T_2}}$$

M_{xy} is the measured transverse component at some time $t > 0$

M_{xy0} is the (maximal) transverse component at $t=0$



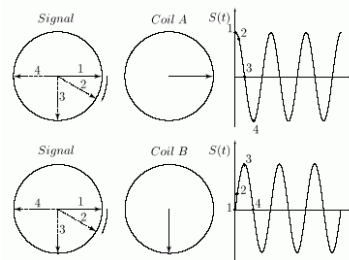
Complex Exponential Representation

To improve SNR, we use two coils, one aligned with the x-axis and one aligned with the y-axis (*quadrature* scheme)

- the detected signal can then be represented as follows:

$$s_x(t) = Ae^{-\frac{t}{T_2}} \cos(-\omega_0 t)$$

$$s_y(t) = Ae^{-\frac{t}{T_2}} \sin(-\omega_0 t)$$



- thus, coil x gives the real part and coil y the imaginary part of a complex-valued signal:

$$s(t) = Ae^{-\frac{t}{T_2}} e^{-i\omega_0 t}$$

- expressed in a rotating reference frame:

$$s(t) = Ae^{-\frac{t}{T_2}}$$