Introduction to Medical Imaging

Lecture 12: MRI Imaging

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What We have Learned So Far…

Steady state:
• with just an axial B-field, the nuclei only spin along the z-direction which cannot be measured
• the nuclei also spin in an unsynchronized fashion, which sets their net effect to zero

Disturbed state:
• a transversal RF pulse (at Larmor frequency $\omega_0$) can bring the nuclei spins out of their z-alignment, creating a transversal component in the net magnetization vector
• this transversal RF pulse also synchronizes the spins
• this component can then be measured using (possibly the same) RF coils, whose axes are in the transversal plane (the xy-plane)

A remaining challenge is:
• we receive a single sinusoidal signal (at Larmor frequency $\omega_0$) characterized by amplitude $A$ and phase shift $\theta_s \to 2$ knowns
• what we want is the density of $nx \cdot ny \cdot nz$ individual voxels $\to nx \cdot ny \cdot nz$ unknowns
• how do we solve such an equation?

The General Idea

Before going into the math, we shall pursue a more intuitive route to provide an explanation of the used approach
• see also “The basics of MRI” by Joseph Hornak
http://www.cis.rit.edu/htbooks/mri

Recall that the Larmor frequency at which the nuclei spin is dependent on the magnetic field: $\omega_0 = \gamma B_0$

Then how about this:
• if we could simply change the local frequency at each nuclei (or better, voxel) then we could select a voxel just by its frequency
• we would then measure its amplitude to get its density
• we learnt that frequency space is a good space for this

So how is this done?
• we can influence the local frequency by the local magnet field
• the local magnet field can be altered by adding a varying magnet field to the large existing one, $B_0$

Slice Selection

Step 1: slice coding
• select the slice $s$ by coding the nuclei in that slice with a specific $\omega_s$
• this is done by adding a magnetic field linearly rising along $z$ (called gradient field)
• now only the voxels in the selected gradient field spin at $\omega_s$

Step 2: RF pulse at $\omega_s$
• this will only bring the nuclei in slice $s$ into the transversal plane and sync them
• we could then measure a signal but it would still be a composite of all nuclei in the slice

Hence, more tagging is needed within the slice
The Phase Encoding Concept

Once the slice has been selected we can impose another magnetic gradient field, say along $x$

- now the nuclei spin at different frequencies, varying along $x$

After releasing the gradient, the nuclei spin at the slice's Larmor frequency again, but an $x$-dependent phase shift angle remains

- we need a method to measure this phase shift
- we also still need a method to distinguish different rows $y$

Decoding

We could measure the RF signal and perform a Fourier transform
- this would give us $ny$ frequencies
- however, we would like to measure $nx \times ny$ voxels

Need to generate more equations
- we can get these by performing $nx$ different phase encodings
- for each phase encoding we use a gradient field with a different slope

Voxel Decoding: Example 1

spatial domain: only one voxel is turned on

$n$ RF measured responses

Phase angles of the Fourier transforms along the frequency encoding direction

Extrapolation into continuous signal (just for illustration)

Fourier transform along the phase encoding direction yields voxel position:
Image Generation

The spectra have amplitudes as well and an image can be generated

or more complex:
Gradient Coils

Field gradients, generated by coils, are essential in MRI.

Examples:
- **x-direction:** surface coils
- **y-direction:** volume (quadrature) coil
- **z-direction:** head volume coil

More Formally... Slice Selection

Usually one selects a slice perpendicular to the z-axis (but any slice orientation is possible).

Use linear magnetic field gradient $G_z$:

$$G = (G_x, G_y, G_z) = (0, 0, \frac{\partial B_y}{\partial z})$$

- the Larmor frequency of a slice is then:
  $$\omega(z) = \gamma (B_y + G_z z)$$
- hence, a slice of thickness $\Delta z$ will resonate at frequencies:
  $$\Delta \omega = \gamma G_z \Delta z$$
- and a RF pulse of freq. bandwidth $BW$ resonates a slice of width:

There is a lower bound on $\Delta z$:

- technical and safety limits on gradient strength $G_z$
- a very thin slice has too low SNR
- typically: $\Delta z = 2 \text{mm} (B_0 = 1.5\text{T})$, $\Delta z = 3 \text{mm} (B_0 = 1\text{T})$

Slice Selection: Example

Dynamic cardiac sequence
- four cardiac chambers together with the heart valves in a plane parallel to the cardiac axis

Position Encoding: Spin Differentiation

The slice selection RF pulse puts all nuclei within $\Delta z$ into a similar synchronized transverse spin (resonance)

- now all nuclei within the slice’s $(x,y)$-plane have a transverse component
- these must be further encoded for spatial localization
- all nuclei rotate at $\omega_0$, in a synchronized fashion

Let’s assume, for now, that we only have one x-beam of voxels

For encoding, we apply a field gradient $G_x$ along $x$:

- this changes the transverse component frequency to:
  $$\omega(x) = \gamma G_x x$$
- in complex notation:
  $$M_x(t) = M_x(0) e^{-i\gamma G_x x t}$$
Position Encoding: Signal Collection

To collect signals from all x-beam voxels, we must integrate across the entire x-beam.

The detected signal is:

\[ s(t) = \int_{x=-\infty}^{\infty} \rho(x) e^{-i\gamma G_t x} \, dx \]

\( \rho(x) \) is the (transverse) net magnetization density in \( x \).

Defining \( k_x = \frac{\gamma}{2\pi} G_t t \), we get:

\[ s(t) = S(k_x) = \int_{x=-\infty}^{\infty} \rho(x) e^{-i2\pi k_x x} \, dx \]

Recall 1D Fourier Transform:

\[ F(k) = \int_{x=-\infty}^{\infty} f(x) e^{-i2\pi k x} \, dx \]

We can interpret this as:

\[ s(t) = S(k_x) = F\{\rho(x)\}(k_x) \]

- \( k_x \) grows as long as the gradient is active (as \( t \) increases).

The K-Theorem (1)

A generalization of what we have just seen

- position vector \( r = (x, y) \) and magnetization density \( \rho(x, y) \) are now 2D vectors
- the angular frequency is given by the dot product: \( \omega(r) = \gamma(G \cdot r) \)
- the measured signal becomes:

\[ s(t) = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} \rho(x, y) e^{-i\gamma (G_x x + G_y y) t} \, dx \, dy \]

- recall the 2D Fourier Transform (and \( k_x = \frac{\gamma}{2\pi} G_t, k_y = \frac{\gamma}{2\pi} G_t \))

\[ F(k_x, k_y) = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} f(x, y) e^{-i2\pi (k_x x + k_y y)} \, dx \, dy \]

- thus, MRI allows us to collect data along any line in \( k \)-space.

The K-Theorem (2)

Generalizing further into 3D:

\[ s(t) = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} \int_{z=-\infty}^{\infty} \rho(x, y, z) e^{-i\gamma (G_x x + G_y y + G_z z) t} \, dx \, dy \, dz \]

Taking into account relaxation effects \( g(t, T_1) \) and \( h(t, T_2) \):

\[ s(t) = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} \int_{z=-\infty}^{\infty} \rho(x, y, z) g(t, T_1) h(t, T_2) e^{-i\gamma (G_x x + G_y y + G_z z) t} \, dx \, dy \, dz \]

MRI seeks techniques where the returned values at \((x, y, z)\) all have the same relaxation effects

- the relaxation effects allow MRI to obtain images of different contrasts, since different types of tissues have different \( g(t, T_1) \) and \( h(t, T_2) \)
- thus, an MRI image is not (necessarily) just a proton density (\( \rho \)) image (but this can be obtained as well).

The K-Theorem (3)

We have seen that the time signal \( s(t) \) is equivalent to the Fourier Transform (FT) of the desired image

- as mentioned, this data collection space is called \( k \)-space
- an inverse FT will produce the desired image

MRI process:

- 2D Fourier transform
- k-space image
- spatial image
Dephasing Phenomena

Dephasing occurs when the vectors precess at different Larmor frequencies
• this results in a reduced (and noisy) receiver coil signal

Distinguish between three types of dephasing phenomena:
1. Spin-spin interaction:
• irreversible, described by time constant $T_2$
2. Magnetic field inhomogeneities:
• reversible, described by time constant $T'_2 < T_2$
• due to inhomogeneous magnetic field and differences in magnetic susceptibility of the tissues (for example, iron is very susceptible and can change the local field significantly)
• can be undone by a 180° RF pulse
3. Magnetic field gradients:
• reversible, time constant $T''_2 < T'_2$
• undone with reversed gradient fields

Rephasing: Runners Equivalent

Rephasing is the process of undoing the effects of dephasing
• two types of rephasing, illustrated by the famous runner example

180° RF pulse
Gradient reversal

Rephasing: 180 RF Pulse

Opposite field gradients of same length and strength

Rephasing: Gradient Reversal

$TE = $ Time to Echo
$TR = $ Repetition Time (can also be chosen for different effects)
large TR ensure that all protons are aligned with the z-axis before the next RF pulse
The Art of Choosing TR and TE

TR and TE can be varied to bring out proton density (PD), T1, and T2 properties of the tissue.

Recall the Bloch relaxation formula: \( \frac{M_{xy}(t)}{M_{xy0}} = 1 - e^{-\frac{TR}{T1}} e^{-\frac{TE}{T2}} \)

where \( M_{xy0} \) is the transversal component at \( t=0 \)

MRI seeks to bring out tissue contrast as manifested by differences in PD, T1, or T2 properties (T1/TR, T2/TE ratios)

- choose TE and TR according to the T1 and T2 constants of the target and adjacent tissues
- pick the imaging protocol that brings out these contrasts best
- this is called T1, T2, PD weighting

Examples

T1-weighting typically brings out anatomy better
T2 weighting shows pathologies better
PD is used when T1/T2 densities are similar
Often, all are obtained and possibly fused

T2 Weighting

Long TR

- to give all tissue time to recover (back to \( M_{z0} \)) and thus eliminate T1 weighting

Long TE

- to give T2 weighting time to develop

\[ M_{xy}^{T2} = M_{xy0}(1-e^{-\frac{TR}{T1}}) e^{-\frac{TE}{T2}} \Rightarrow M_{z0} e^{-\frac{TE}{T2}} \]
T2 Weighting: Examples

TR = 3070ms  TE = 92ms
TR = 4000ms  TE = 132ms

T1 Weighting

Short TR
- to bring out differences in T1 between tissues

Short TE
- so differences in T2 do not have time to appear

\[ M_{T1}^{xy} = M_{xy0}(1 - e^{-\frac{TR}{T1}}) e^{-\frac{TE}{T2}} \Rightarrow M_{z0}(1 - e^{-\frac{TR}{T1}}) \Rightarrow M_{z0} \frac{TR}{T1} \]

Mz0 protons have not recovered to Mz0 yet therefore, the 90° RF pulse will not be able to spin them fully into the xy-plane

T1 Weighting: First Time Around

T1 Weighting: Second Time Around
**T1 Weighting: All Subsequent Rounds**

Flip Magnetization

Short TE Time

Short TR Time

**T1 Weighting: Examples**

TR = 525ms TE = 15ms

**T1 vs. T2 Weighting (1)**

<table>
<thead>
<tr>
<th>Substance</th>
<th>T1 Weighted</th>
<th>T2 Weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water/Vitrious/CSF</td>
<td>black</td>
<td>light grey or white</td>
</tr>
<tr>
<td>Fat</td>
<td>white</td>
<td>light grey</td>
</tr>
<tr>
<td>Muscle</td>
<td>grey</td>
<td>grey</td>
</tr>
<tr>
<td>Air</td>
<td>black</td>
<td>black</td>
</tr>
<tr>
<td>Fatty bone marrow</td>
<td>white</td>
<td>Light grey</td>
</tr>
<tr>
<td>Brain white matter</td>
<td>light grey</td>
<td>grey</td>
</tr>
<tr>
<td>Brain grey matter</td>
<td>grey</td>
<td>Very light grey</td>
</tr>
</tbody>
</table>

**T1 vs. T2 Weighting (2)**

TR = 562ms TE = 20ms

TR = 4000ms TE = 132ms

T1 weighted
- short TR = 500ms
- short TE < 30ms

T2 weighted
- long TR = 1500ms (3xT1 max)
- long TE > 80ms
T1 vs. T2 Weighting (3)

TR=525ms TE=15ms

TR=2500ms TE=85ms

Long TR
• to give all tissue time to recover and thus eliminate T1 weighting

Short TE
• to give no T2 weighting

\[ M_{xy}^{PD} = M_{xy0}(1 - e^{-\frac{TR}{T1}}) e^{-\frac{TE}{T2}} \Rightarrow M_{z0} \]

Proton Density Weighting

Flip Magnetization

Short TE Time

Long TR Time

Brain
CSF

Proton Density vs. T2 (1)

TR = 3070ms
TE = 15ms

TR = 3070ms
TE = 92ms
Proton Density vs. T2 (2)

TR = 3070ms
TE = 15ms

TR = 3070ms
TE = 92ms

Knee with Pigmented Villonodular Synovitis (PVNS)
- example shows that PD and T1 weighting creates contrast for complementary features

Contrast Dependence on TR
Use TR to maximize contrast among tissues of interest

from: Frits Thorsen (U Bergen)

Contrast Dependence on TE
Use TE to maximize contrast among tissues of interest

from: Frits Thorsen (U Bergen)
**Other Techniques**

**Gadolinium injection**
- can reduce T1 in certain tissues and therefore enhance T1 results

**Inversion recovery**
- can provide very strong contrast between tissues with different T1
- involves successive 180° and 90° pulses
- tends to be slower than regular T1

**FLAIR=Fluid Attenuated Inversion Recovery**
- used often to enhance lesions inside the brain in areas close to the fluid surrounding it

And many more...

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**Sequences: Spin-Echo**

- half-Fourier space acquisition: possible since spin density is a real function
- although faster, this is not done in practice since a full k-space acquisition improves SNR

**Spin-Echo: Practical Considerations**

**Gradient-Echo Pulse Sequences**

- slow acquisition speed is a major drawback of Spin-Echo
  - particularly in ρ and T₂*-weighted protocols where TR is long to minimize T₁ effects

- gradient-echo is an alternative technique designed to overcome these limitations:
  - flip angle is less than 90° (usually 20° - 60°)
  - no spin-echo due to absence of 180° rephasing pulse
  - rephasing only by gradient reversal → signal characteristics due to T₂*, not T₂

- could obtain images using the same TE, TR values than Spin-Echo
  - but this would be not faster than Spin-Echo
  - the signal would also be lower due to T₂* effects
  - instead perform faster earlier acquisition → measure T₁ effects

**Phase shift:** \( φ(y) = γG_y y T_{\text{phase-encoding}} \)

In k-space: \( G_y = mg_y \) where \( m \in Z \) and \( g_y = \text{constant} \)

\[ k_y = \frac{γ}{2π} mg_y T_{\text{phase-encoding}} \]
Comparison: 180° RF Pulse vs. Gradient Echo

180° RF compensates for magnetic field inhomogeneity effects
- example: ferromagnetic particles in a knee joint

Spin-Echo compensates for dephasing
- surrounding areas OK (with some geometric distortion)

Gradient-Echo cannot compensate for dephasing:
- surrounding areas have no measurable signal

Gradient-Echo: FLASH
FLASH = Fast Low Angle Shot

typical image size (like in Spin-Echo): 256²

16 short axis cardiac images acquired with 2D FLASH Gradient-Echo sequence

Three-Dimensional Imaging

Direct 3D imaging overcomes the problem with thin-slice selection that plagues 2D methods

Needed for many radiological examinations
- wrist, knee, ankle, ...

Requires a second phase-encoding gradient
$$\phi(y,z) = \gamma(mg_y y \tau_{\text{phase-encoding}} + ng_z z \tau_{\text{slab-selection}})$$

The setting of $n$ selects the slice

Reconstruction by ways of a 3D FT
- obtain 16, 32, 100, ... 256² pixel slices
- assuming a slab thickness of 32 mm, can get 32 1 mm thin slices (impossible with 2D imaging)
- better SNR since each excitation selects all spins within the thick slice and not just a thin slice
- slower speed due to the multiple slices

Acquisition and Reconstruction Time

Clinical practice:
- high-quality images useless if it takes tens of minutes to acquire them
- reconstruction time fast since FT implemented in special chips

Acquisition time $T_A$ determined by:
- total number of excitations
- multiplied by the interval between two successive excitations $T_R$

More concretely:
- 2D pulse sequences: $T_A^{2D} = N_{\text{ph}} N_{\text{ac}} T_R$
- 3D pulse sequences: $T_A^{3D} = N_{\text{ph}} N_{\text{sl}} N_{\text{ac}} T_R$

- $N_{\text{ph}}$: number of phase encoding steps
- $N_{\text{ac}}$: number of acquisitions (to improve SNR sometimes multiple acquisitions are taken)
- $N_{\text{sl}}$: number of slices (for 3D)
3D Clinical Imaging Example: Brain

Assume a 32 x 256 x 256 volume:
- \( N_{ac} = 1, N_s = 32, N_{ph} = 256 \)
- with Spin-Echo, \( TR = 2000 \text{ ms} \) \( \rightarrow TA_{3D} = 4 \text{ hours} \) (infeasible)
- with Gradient-Echo, \( TR = 40 \text{ ms} \) \( \rightarrow TA_{3D} = 6 \text{ minutes} \) (acceptable)

Fast Imaging Sequences

Recall acquisition time equation:
\[
TA_{2D} = \frac{N_{ac} \cdot N_{ph} \cdot TR}{ETL}
\]

Could get faster by:
- decreasing \( TR \) (GE (gradient echo) vs. SE (spin echo) sequences)
- decreasing \( N_{ac} \) (truncated or half Fourier imaging)
- or...generate and acquire multiple echos in one excitation (ETL = Echo Train Length)

If \( ETL > 1 \) then rows in k-space are sampled at different echo times
- however, dephasing of T2 (SE) or T2* (GE) can not be neglected between two different echos, leading to:
\[
S(k_x, k_y) = H(k_x, k_y)S(k_x, k_y)
\]
- this multiplication is a convolution in the spatial domain \( \rightarrow \) blurring

Fast Sequences: TurboSE

SE sequence with 3-33 echos
- immediately after first echo apply new phase-encoding
- this selects a different line in k-space

Practice:
- divide k-space into \( n \) segments
- within each excitation, sample one line in each segment

Example: T2-weighted imaging of a brain
- image size: 256x256
- TR=2500ms
- 4 segments
- acquisition time:
\[
TA_{2D} = \frac{1 \cdot 256 \cdot 2.5}{4} = 160 \text{ secs} < 3 \text{ minutes}
\]

Fast Sequences: HASTE and EPI

Half-Fourier Acquisition with Single-shot Turbo Echo (HASTE)
- all echos are generated within one excitation \( \rightarrow ETL = N_{ph} \)
- and thus: \( TA = TR \) (gives heavily T2-encoded images)
- in fact: \( TA = \text{time required to generate and sample ETL echos} \)
- typically: 100 phase-encoding steps take 1s
- application: (motion-less) liver and lung imaging (single breath-hold)

Echo planar imaging (EPI)
- fastest sequence available today (acquisition time 100ms)
- GE sequence (no 180° pulse required, in contrast to TSE, HASTE)
- T2* dephasing limits number of echos measured above noise level
- maximum image size \( = 128 \times 128 \)
- requires high performance scanners to produce strong gradients
- used in functional MRI
Clinical Equipment

Whole-body 1.5T Philips scanner (1.5 T, super-conducting magnet) standard diagnostic scanner

Open (C-shaped) scanner (0.2 T) helps with claustrophobia open space improves patient handling can be used for MRI-guided procedures SNR low and field homogeneity poor

Operation

head coil

body coil

Comparison: CT vs. MRI

Slice across a male prostate

MRI image: T2-weighted turboSE (TE=120ms, TR=6s)

CT image: over-estimates size of prostate (black outline) since MRI can distinguish between prostate tissue (white outline) and adjacent peripheral structures

MRI Tissue Characterization

(liver) T1-weighted GE T2-weighted (HASTE)

lesion

bilir cyst

hemangioma fades with TE (typical)

hemangioma
Functional MRI (fMRI)

Capitalizes on the oxygenation of cerebral bloodflow
- brain requires oxygen to function (via metabolic processes)
- active brain areas require more oxygen than inactive areas

Oxygen is transported in *hemoglobin* molecules
- in the arteries, hemoglobin carries 4 oxygen molecules → *oxyhemoglobin*
- in the capillaries, hemoglobin delivers part of the oxygen to the neurons and carries only 2 oxygen molecules → *deoxyhemoglobin*

Distinguishing magnetic properties
- *oxyhemoglobin* is diamagnetic
- *deoxyhemoglobin* is paramagnetic → produces field inhomogeneities → these decrease T2* of blood and surrounding tissue

Known as BOLD (Blood Oxygenation-Level Dependent effect)
- brain activation influences oxygen concentration in the blood
- this influences magnetic properties which can be measured with MRI

Functional MRI (fMRI)

When brain cells are active, blood flow must increase to meet neuronal demands
- in fact, blood flow over-compensates
  → too much oxygen in active brain areas
  → T2* is larger in active areas
- GE (gradient echo) techniques are very sensitive to T2* variations
- use EPI (echo planar imaging) for fastest acquisition speed

Typical experimental setup and acquisition protocol:
- individual performs a certain experiment (for example, finger tapping) and MRI is taken
- acquire two images: one at rest and one under task
- subtract these two images to visualize the active areas

Many repetitions needed
- signal difference is very small (2-5%) → low SNR
- alternate periods at rest (30s) with active periods (30s) → 6 min total
- take images every 2-10 s and process statistically

fMRI Applications

Get insight into:
- memory
- object recognition
- language
- visual cortex
- sensorimotor cortex

Diffusion Tensor Imaging (DTI)

Capitalizes on the following facts:
- in different classes of tissue, diffusion of water molecules is different
- also, the diffusion is not isotropic → along fibers the diffusion is much greater than across fibers.

Acquisition and analysis
- diffusion-weighted images with diffusion gradients applied in different directions
- areas with high diffusion in that direction will appear dark
- pick a few diffusion directions for this
- perform eigenvector analysis → tensors
Brain fibers can now be tracked using the local tensors