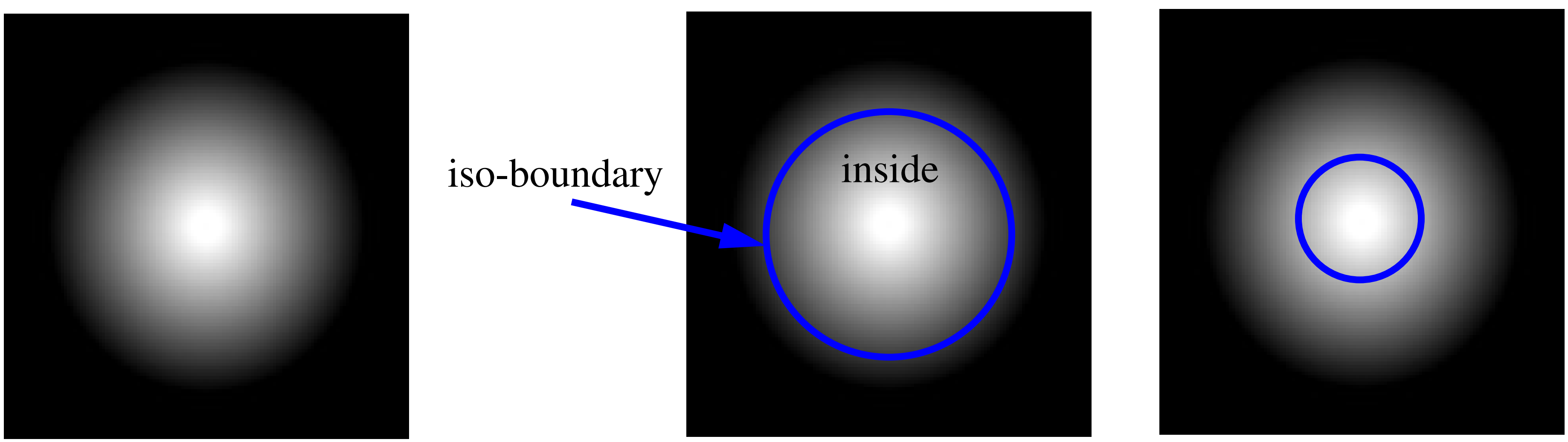
Iso-Surface Rendering

- A closed surface separates ‘outside’ from ‘inside’ (Jordan theorem)
- In iso-surface rendering we say that all voxels with values > some threshold are ‘inside’, and the others are ‘outside’
- The boundary between ‘outside’ and ‘inside’ is the *iso-surface*
- All voxels near the iso-surface have a value close to the *iso-threshold* or *iso-value*
- Example:



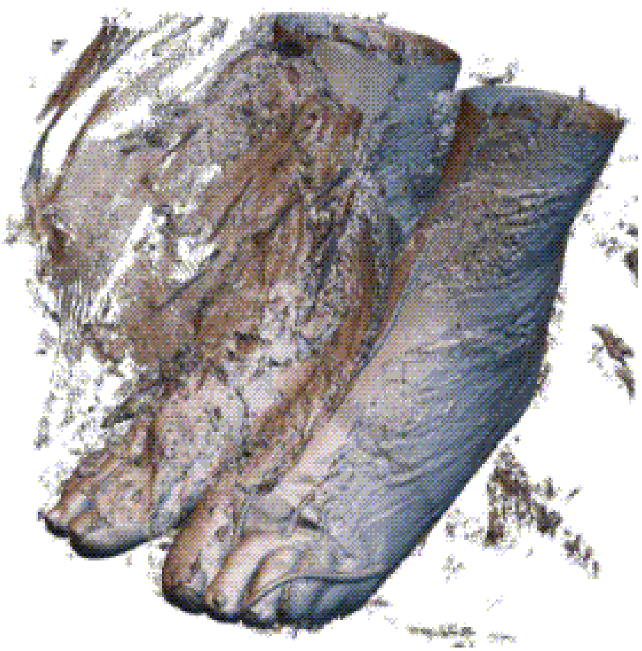
cross-section of a smooth sphere

iso-value = 50
will render a large sphere

iso-value = 200
will render a small sphere

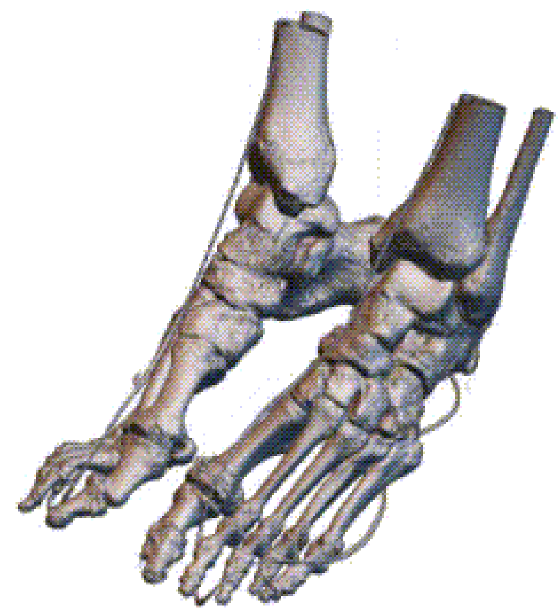
Iso-Surface Rendering - Example

Foot of the Visible Woman



iso-value = 30

iso-value = 80

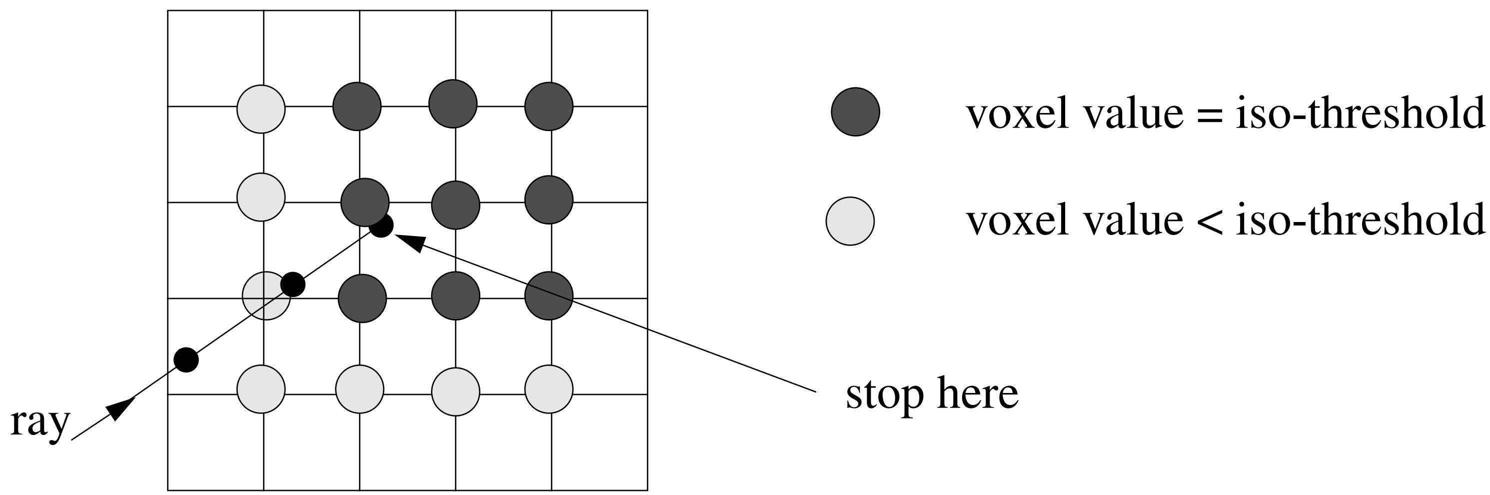


iso-value = 200

Iso-Surface Rendering - Details

- To render an iso-surface we cast the rays as usual...

    but we stop, once we have interpolated a value   iso-threshold

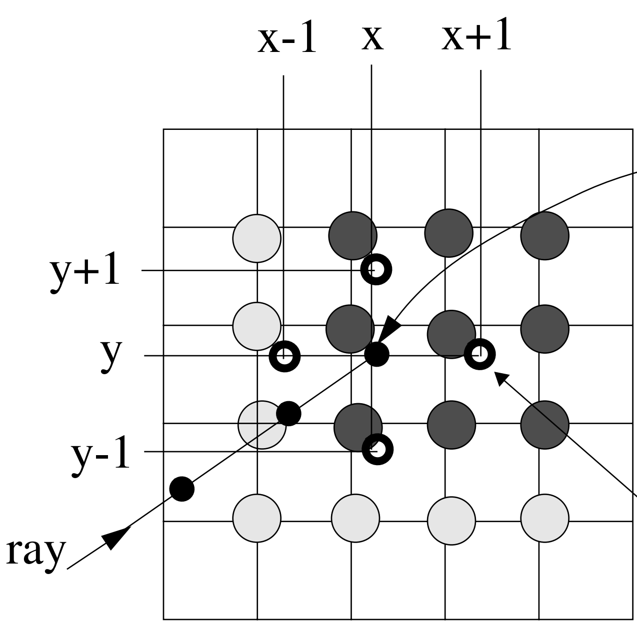


- We would like to illuminate (shade) the iso-surface based on its orientation to the light source
- Recall that we need a normal vector for shading
- The normal vector N is the local gradient, normalized

The Gradient Vector

- The gradient vector $\mathbf{g}=(g_x, g_y, g_z)^T$ at the sample position (x, y, z) is usually computed via central-differencing (for example, g_x is the volume density gradient in the x-direction):

$$g_x = \frac{f(x-1, y, z) - f(x+1, y, z)}{2} \qquad g_y = \frac{f(x, y-1, z) - f(x, y+1, z)}{2} \qquad g_z = \frac{f(x, y, z-1) - f(x, y, z+1)}{2}$$

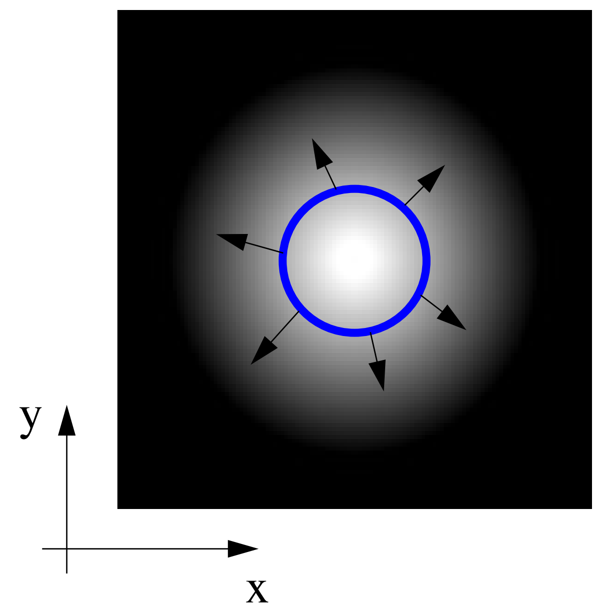


0	0.5	0
-0.5	0	0.5
0	-0.5	0

2D central difference mask

interpolated volume density f(x+1, y, z)

the x and y component of the gradient vector for the smooth sphere



voxel value = iso-threshold

voxel value < iso-threshold

extra sample points interpolated to estimate gradient

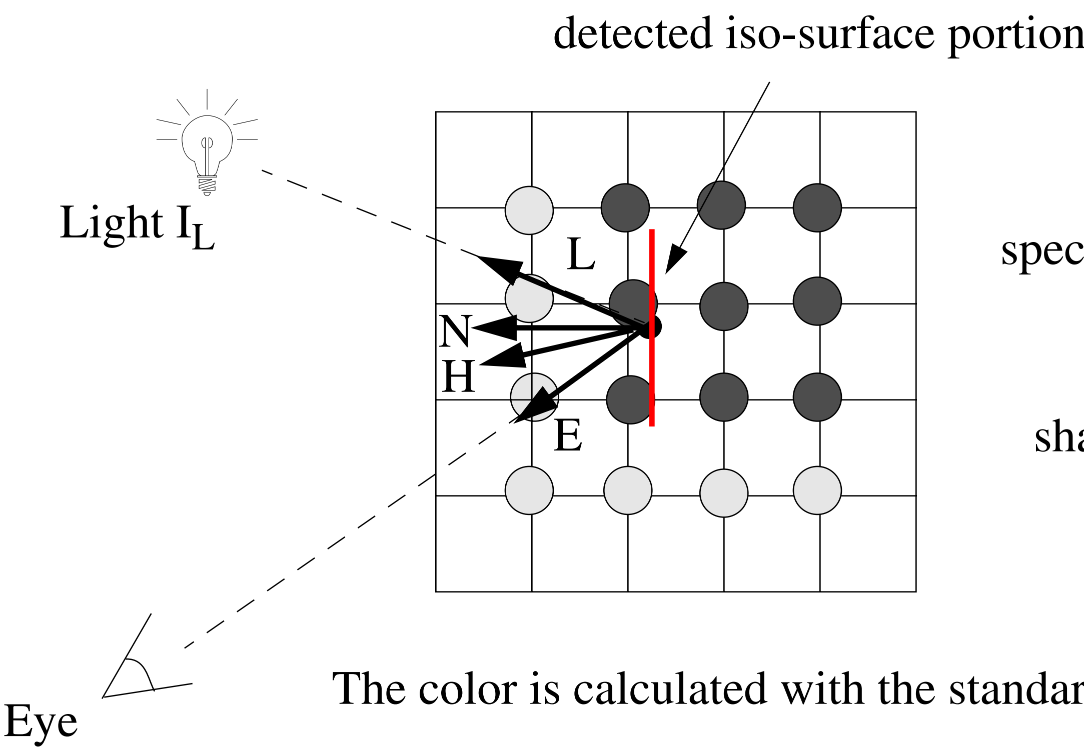
More on gradient filters later....

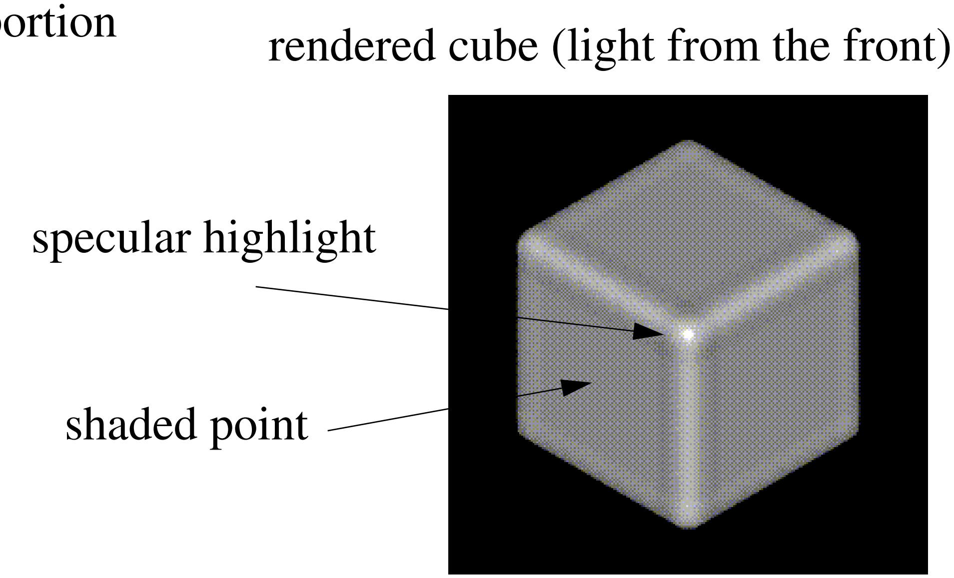
Shading the Iso-Surface

- The normal vector is the *normalized* gradient vector g

  $N = g / |g|$ (normal vector always has unit length)

- Once the normal vector has been calculated we shade the iso-surface at the sample point
- The color so obtained is then written to the pixel that is due to the ray



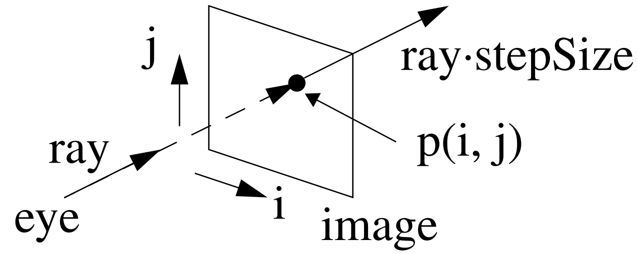


The color is calculated with the standard shading equation:

$$C = C_{obj} (k_a I_A + k_d I_L N \cdot L) + k_s I_L (H \cdot N)^{ns}$$

C_{obj} is obtained by indexing the color transfer function with the interpolated sample value

Iso-Surface Rendering - Algorithm



RenderIsoSurface(Volume V, int stepSize)

```
for each image pixel p(i, j)
    ray = (p(i, j) - eye) / | (p(i, j) - eye) |   /* the ray direction vector, normalized */
    t = 0  /* start at the eye point */
    do forever
        sampleLoc = eye + t · stepSize · ray    /* step along the ray */
        intVal = Interpolate(V, sampleLoc)
        if opacityTransferFunction(intVal) > isoThreshold  /* found the iso-surface */
            /* interpolate 6 samples around sampleLoc and compute the gradient */
            gradVec = ComputeGradientVector(V, sampleLoc)
            /* shade the surface using standard illumination model and color transfer functions */
            {r, g, b} = Shade(gradVec, lightSource, eye, sampleLoc, {R, G, B}TransFunc(intVal))
            value(p(i, j)) = {r, g, b}   /* write color into image pixel p(i, j) */
            break  /* terminate this ray and go to next image pixel */
        t = t + 1      /* iso-surface not found yet, get ready to step to next sample point */
```

Gradient Filter Theory (1)

- The gradient g is the derivative of the signal f

$$g = f_c' = \begin{cases} (h \otimes f_s)' \\ h \otimes f_s' \\ h' \otimes f_s \end{cases} \quad \text{where}$$

f_c: continuous signal
f_s: sampled signal
h: interpolation filter
': the derivative operator
$\otimes$: convolution operator

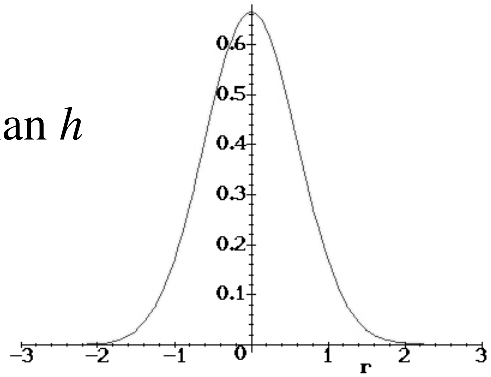
- So we can either:

  - interpolate the signal f_s first and then take the derivative (that is what we did before)

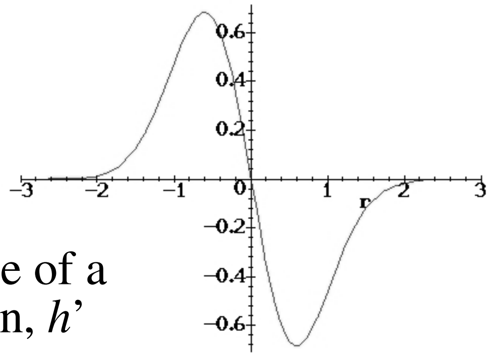
  - take the derivative at the grid positions ($\rightarrow \mathbf{g_s}=(g_x, g_y, g_z)^T$) and then interpolate $\mathbf{g_s}$

  - take the derivative of h ($\rightarrow h'$) and interpolate f_s with h'

Gaussian h

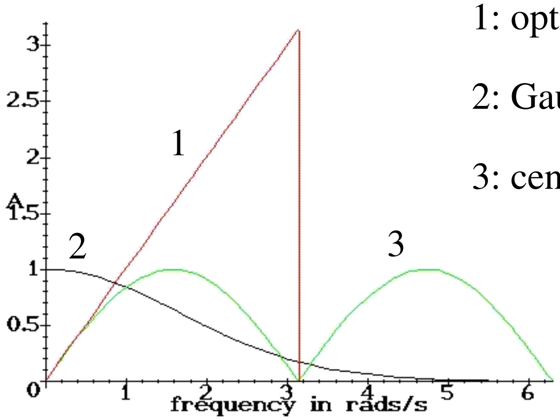


Derivative of a Gaussian, h'



Gradient Filter Theory (2)

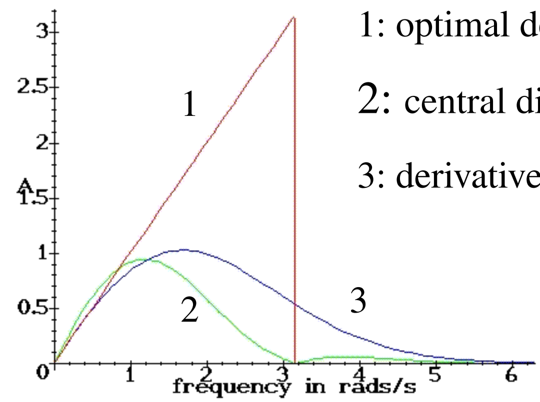
- Quality evaluation is best done in the frequency domain:



1: optimal derivative filter

2: Gaussian (H)

3: central difference (D)



1: optimal derivative filter

2: central difference * Gaussian (DH)

3: derivative of Gaussian (H')

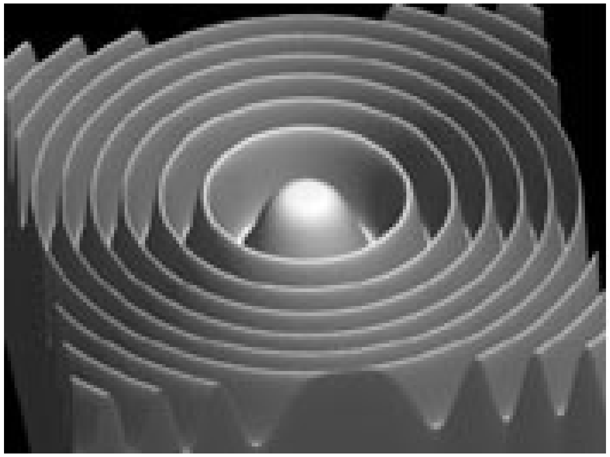
- The optimal gradient filter is a ramp that ends at the cut-off frequency
    - gradients (edges, etc.) are the high frequencies that we want to emphasize
    - homogenous (uniform) regions have low frequencies and need to be suppressed
- The derivate filter H' passes the higher frequencies better then DH but it also passes some aliases
    - more sensitive to noise
    - but promises to estimate crisper edges

Interlude - The Marschner-Lobb Test Function for Filters

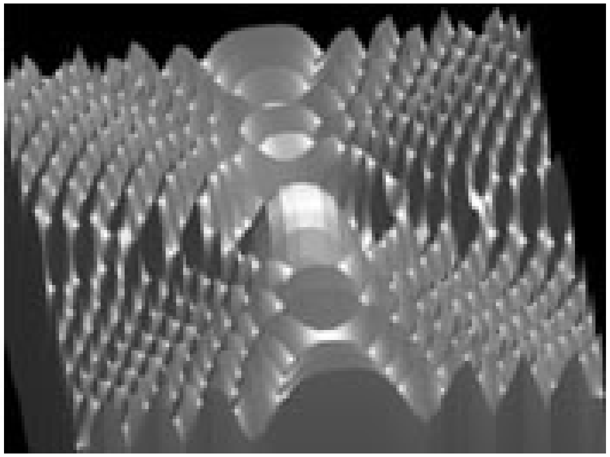
- A common test function to evaluate rendering filters is the Marschner-Lobb function

$$\rho(x, y, z) = 255 \cdot \frac{(1 - \sin(\pi z/2)) + \alpha\left(1 + \rho_r\left(\sqrt{x^2 + y^2}\right)\right)}{2(1 + \alpha)} \qquad \rho_r(r) = \cos\left(2\pi f_M \cos\left(\frac{\pi r}{2}\right)\right) \qquad \begin{array}{l} f_M = 6 \\ \alpha = 0.25 \end{array}$$

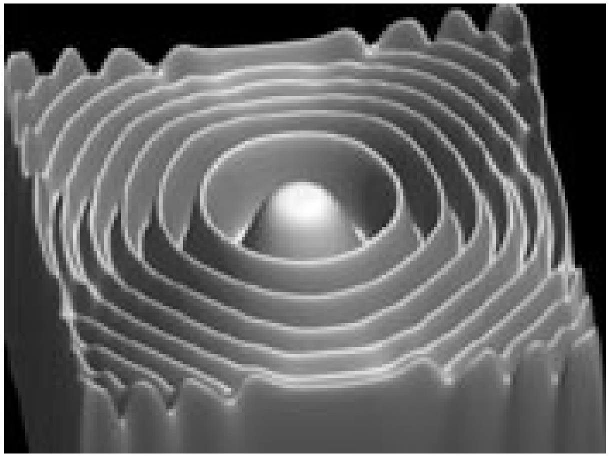
  - when sampled into a 40^3 grid, 99.8% of the frequencies are below the Nyquist rate,
  - the function is interesting since a significant portion of the spectrum is close to Nyquist
  - this makes this function a demanding test for interpolation and gradient filters
- Usually the iso-value is set to 128
- Some images of the ML-function obtained with different interpolation filters:



original function



tri-linear filter



windowed sinc

Gradient Filter Theory (3)

- It turns out that inaccurate estimation of gradients is much more noticeable than inaccurate interpolation
  - this is because the lighting depends greatly on the normal vectors
  - especially specular highlights (due to the exponentiation of $H \cdot N$) are rather sensitive
- To show this, let us take a good filter (family) and vary its quality
- We know that the cubic function is a reasonably good interpolation filter
- To study the sensitivity, we can just vary the coefficient α to sub-optimal values
  - use the cubic filter for interpolation
  - use the derivative of the cubic function as a gradient filter
  - theory has shown that $\alpha = -0.5$ is a good choice for interpolation
  - let us now deviate from that value and see what happens for both interpolation filter and gradient filter

(We will see that the choice of α affects the derivative filter much more than the interpolation)

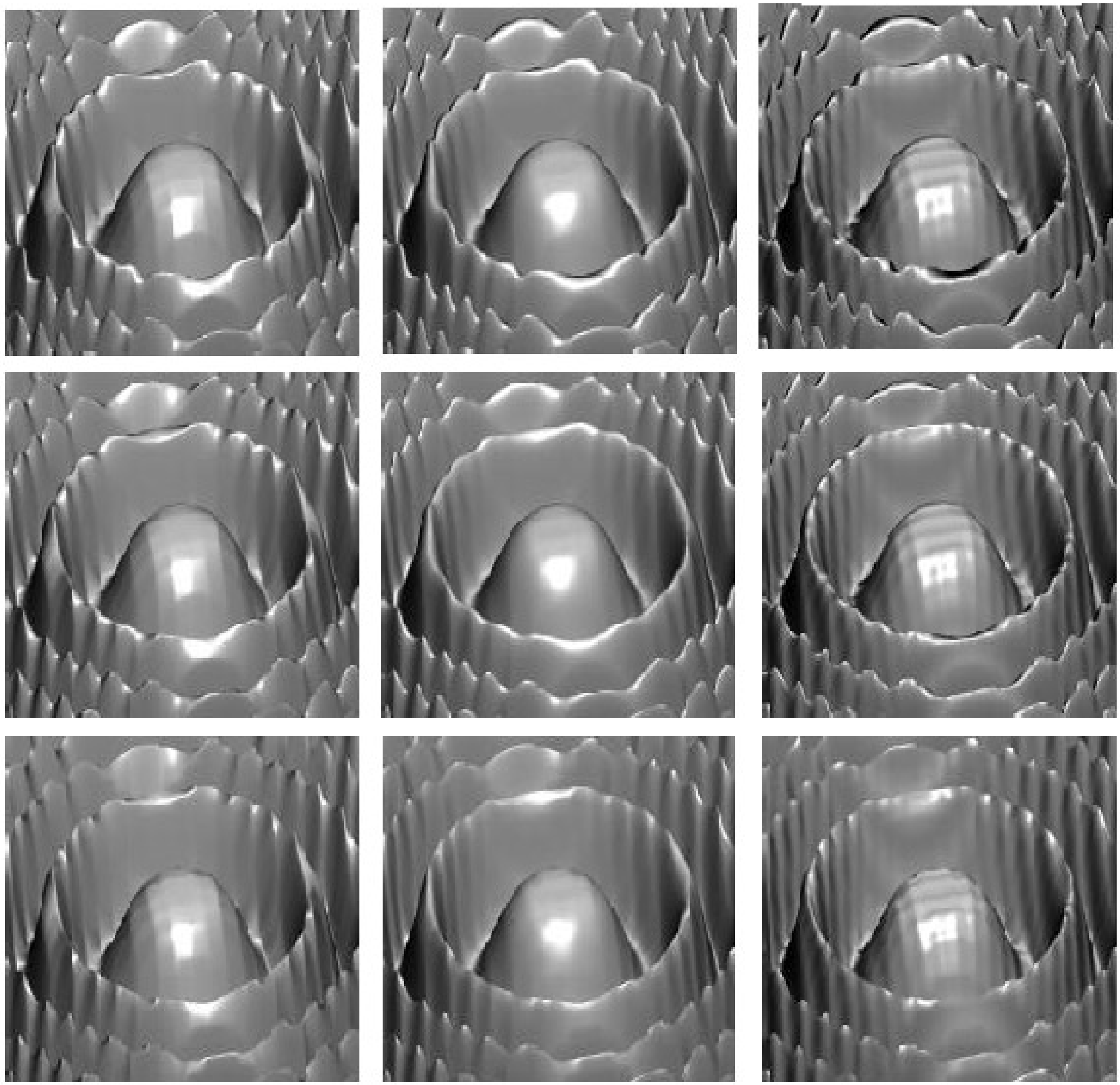
Gradient Filter Theory (4)

Derivative $\alpha = -0.2$ Derivative $\alpha = -0.5$ Derivative $\alpha = -1.0$

Interpolation $\alpha = -0.2$

Interpolation $\alpha = -0.5$

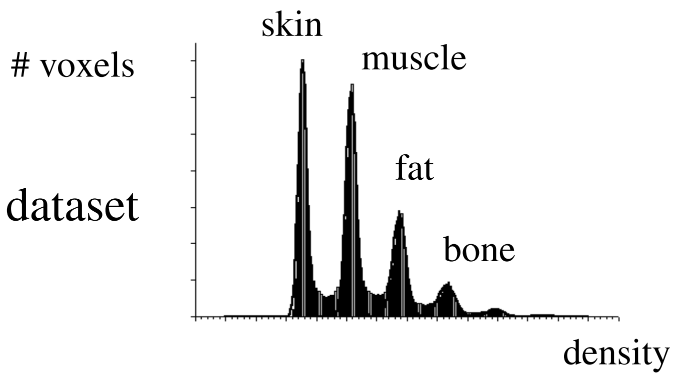
Interpolation $\alpha = -1.0$



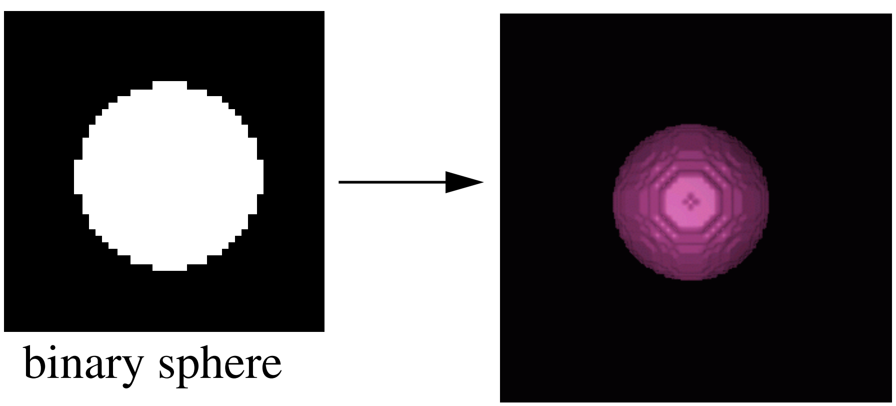
Iso-Surface Rendering - Tips and Tricks (1)

- Finding a good iso-value is not always easy
    - make a histogram of the volume densities and look for peaks (isovalue = onset of peak)

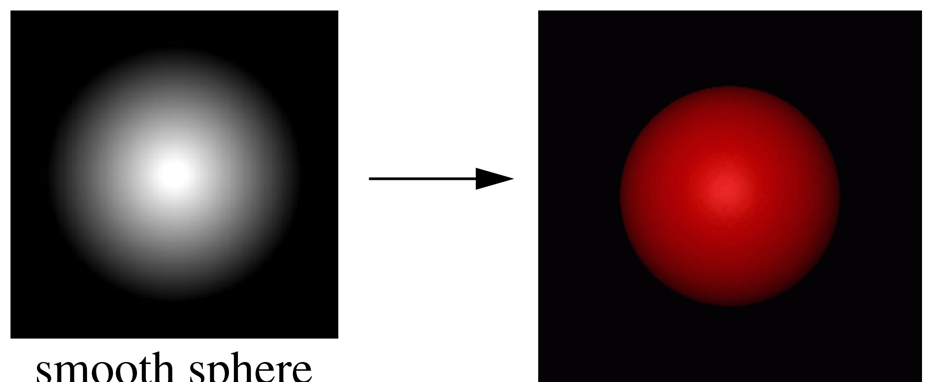
histogram head dataset



- Good shading requires good gradients around iso-surface
    - need smooth degradations at iso-surface for good gradient estimation
    - else get aliasing (example: volumetric binary sphere)



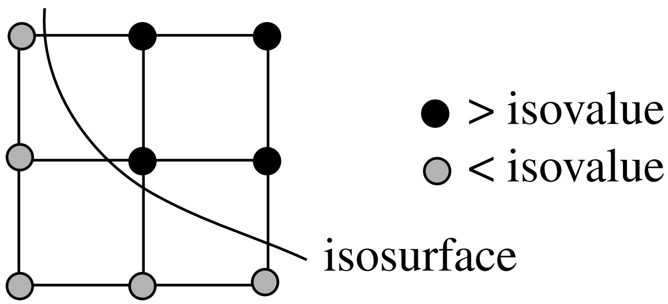
binary sphere



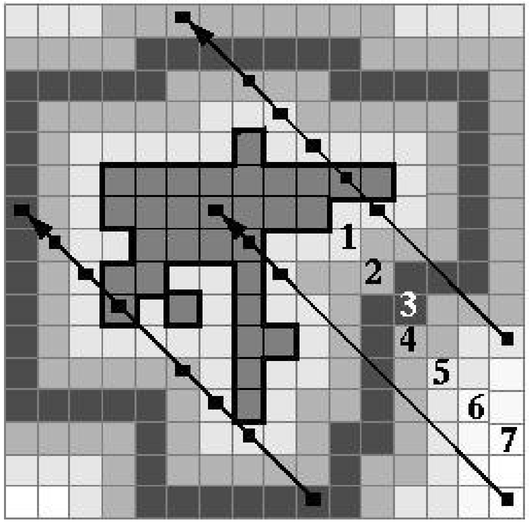
smooth sphere

Iso-Surface Rendering - Tips and Tricks (2)

- Ray stepsize must be chosen sufficiently small
  - choose stepsize of less or equal 1.0 voxel units (or we may get aliasing in the ray direction)
- But even for small stepsizes, we may never exactly hit the isosurface
  - isosurface goes through a cell when at least one vertex, but not all, has a density > isoValue
  - compute exact location of the iso-surface within a cell by solving a cubic function in *t*



space leaping



- A variety of acceleration methods are possible:
  - enclose the object in a bounding box and start rays at the bounding box intersection
  - store distance values in voxels outside the object → this enables quick *space leaping*
  - multi-resolution volume representation (octree)