

Iterative Reconstruction



Kernel-Centric Decomposition

We can consider each of these steps to be a SIMT kernel

Iterative 3D reconstruction with regularization:

- backprojection of volume into set's views → projection kernel
- correction factor computation → correction factor kernel
- backprojection of correction factors → *backprojection kernel*
- regularization → regularization kernel

projector with interpolation

- vector operations
- image processing filters

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Kernel-Centric Decomposition





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Kernel Scheduling

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Backprojection

(into voxel *i*)

Correction value

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 $\frac{k}{W}_{il}$

 W_{il}

 W_{ii}

 W_{ij}

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SIMT can only execute one kernel at a time

- this prohibits kernel overlap, even if mathematically correct
- we may merge kernels if targets are identical
 → this favors load balancing and the reduction of passes

First decompose the reconstruction pipeline into components

· develop an optimized kernel for each component

Iterative CT Example: SART/SIRT

Scanned pixel

Normalization

at pixel i

New (k+1) and previous (k)

values of voxel *j*

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Normalization at voxel *j*

- overlap (=hide) the loading of data (if needed) with execution of a prior kernel (or within kernel)
- optimize what platform to run the computations (CPU, GPU), but then consider transfer of data

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Projection (into pixel *i*)

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Terminology

We shall discuss all material in terms of 3D reconstruction

- the reduction to 2D slice reconstruction is straightforward
- Pixels: the basis elements (point samples) of the projection image (the photon measurements)
- Voxels: the basis elements (point samples) of the reconstruction volume (the attenuation densities or the tracer photon emissions)



Kernel-Centric Decomposition

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 $v_j = \sum_{p_i \in P_{set}} p_i w_{ij_{-}fdk} = \sum_{p_i \in P_{set}} B \cdot S$

FBP

S: scanner projections
I: identity projection/volume





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Backprojection: Options

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- voxel-driven: sample in projection space
- one write per thread



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CUDA Memory – Backprojection

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Backprojection: Options

- voxel-driven: sample in projection space
- pixel-driven, sample in volume space

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CUDA Configuration: 2D

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CUDA Configuration: 3D



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Transformation Matrix

A 3x4 matrix *M* transforms 3D voxel coordinates to 2D pixel coordinates on the detector

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Perform perspective divide if necessary (cone-beam)





accumulate sampled values on voxel

[Host]:

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Example: Feldkamp Cone-Beam Reconstruction

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360 projections (1024², general position), 512³ volume



FDK: Medical Datasets

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performance in seconds

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Expressed in Projections/Sec.

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360 projections, 512³ volume



Forward Projection

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Sample in volume space (pixel-driven / ray-driven)



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CUDA Memory – Forward Projection SPIE Medical Imaging

	Global Memory	Texture Memory
Access	Read/Write	Read only
Cached	No	Yes
Subject to coalescing	Yes	No
Interpolation	No support	Hardwired
Dimension	arbitrary	1D, 2D, 3D (supported after CUDA 2.0)
	-	-
	projections	volume
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Forward Projection: Memory

Ray-driven: sampling in volume space (trilinear interpolation) Volume can be represented as either

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- a single 3D texture (supported after CUDA 2.0)
- stacks of 2D textures
 - A 3rd interpolation between adjacent 2D slices



Projection Algorithm

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Raycasting methods [Krueger'03]

• [Host]:

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- generate volume bounding box (aligned with axis X/Y/Z)
- generate threads for each pixel (ray), trigger kernel on device
- [Device]: in each thread
 - obtain ray entry & exit points using volume bounding box info

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- get ray directions using entry & exit points
- cast rays, inside the loop:
 - sample in volume space
 - accumulate values
 - step forward equidistantly



Projection Accuracy

Investigated various schemes in terms of accuracy:



It was shown that the convenient grid-interpolated (trilinear) scheme is qualitatively competitive to the more involved ones listed here.

 see Xu / Mueller, "A comparative study of popular interpolation and integration methods for use in computed tomography," IEEE 2006 International Symposium on Biomedical Imaging (ISBI '06)

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Regularization

Overall goal: make the reconstruction conform to expectations

- reconstruction is not noisy
- reconstruction has sharp edges

Various techniques

- Total Variation Minimization (TVM)
- bilateral filter (BLF) ٠
- non-local means filter (NLM)

TVM

motivated by compressive sensing (sparseness) theory

BLF, NLM

• popular in image processing and computer vision

Motivation

Sync

forward projection

(pixel-driven)

backprojection

(voxel-driven)

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texture memory (r)

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Want to remove low-dose CT artifacts:

global memory (r/w)

add

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CT with low dose data

high-dose data CT

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Motivation



What we want to achieve - ideally:







20 projections SNR=10 CT + regularization

high-dose CT

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Total Variation Minimization (TVM)

Goal is to minimize the overall energy:

$$E_{TV} = \int_{\Omega} |\nabla I| + \frac{1}{2} \lambda (I - I_0)^2 dx dy$$

variation fidelity

Minimize using the steepest descent method

• for each voxel v_i do iteratively:

$$v_i^{k+1} = v_i^k - \beta \cdot \left(div \left(\frac{\nabla v_i^k}{\left| \nabla v_i^k \right|} \right) + \lambda (v_i^k - v_i^0) \right)$$



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original voxel value

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Relaxation Parameters (TVM)

Gradient step size β :

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• << 1, usually 0.2

Fidelity term λ :

- initially set to 0
- next iterations:

$$\lambda = \frac{1}{\sigma^2 |\Omega|} \int_{\Omega} div \left(\frac{\nabla I}{|\nabla I|}\right) (I - I_0) dx dy$$

• assuming:

$$\min_{I} \int_{\Omega} |\nabla I| \, dx \, dy \quad \text{subject to } \frac{1}{|\Omega|} \int_{\Omega} (I - I_0)^2 \, dx \, dy = \sigma^2$$

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Non-linear Neighborhood Filters

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• Generalization of discrete convolution



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Bilateral Filter (BLF)

Edge-preserving non-linear filter:







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original edge

bilateral filter

smoothed edge

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Non-Local Means Filter

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Replaces a pixel at x with the mean of the pixels y with similar Gaussian-weighted neighborhood:



(search) window W

patch with updated pixel x

Gaussian-weighted neighborhood patches with pixels y (only highly-weighted shown)

Bilateral Filter (BLF)

• Edge-preserving non-linear filter:



Non-Local Means Filter

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Replaces a pixel at x with the mean of the pixels y with similar Gaussian-weighted neighborhood:

$$NLM(x) = \frac{\sum_{y \in W} e^{-\frac{\sum_{i \in N} G_a(t) |img(x+t) - img(y+t)|^2}{h^2} img(y)}}{\sum_{y \in W} e^{-\frac{\sum_{i \in N} G_a(t) |img(x+t) - img(y+t)|^2}{h^2}}}$$

x, *y*, *t*: spatial variables W: window centered at x *N*: neighborhood centered at $x, y = G_a$: Gaussian kernel h: filtering weight controls the influence of dissimilar pixels

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NLM vs. TVM: Quality



NLM is as good (often better) than TVM



input

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NLM, h=15

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NLM vs. TVM: Speed

NLM is typically faster than TVM because it is non-iterative

- all parameters were manually set to yield similar visual quality
- CUDA GPU implementations (NVIDIA GTX 480)
- in seconds:

Image size	TV	NLM
256 ²	57	12
512 ²	80	42

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Bilateral vs. NLM

Faster than NLM, but quality is lower



Course Schedule		SPIE SPIE Medical Imaging
1:30 – 1:45:	Introduction (Klaus)	
1:45 – 2:00:	Parallel programming primer (Klaus)	
2:00 - 2:15:	GPU hardware (Ziyi)	
2:15 – 3:00:	CUDA API, threads (Ziyi)	
Coffee Break		
3:30 - 4:00:	CUDA memory optimization (Eric)	
4:00 - 4:15:	CUDA programming environment (Ziyi)	
4:15 - 4:45:	Parallelism in CT reconstruction (Klaus)	
4:45 – 5:25:	CT reconstruction examples (Eric)	
5:25 – 5:30:	Closing remarks (Klaus)	
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