Exercise 3.6

Let \( M = (Q, \Sigma, q_0, T, \Delta) \) be a nondeterministic finite automaton and suppose \( \{q'_s, q'_t\} \cap Q = \emptyset \). Then the automaton \( M' = (Q \cup \{q'_s, q'_t\}, \Sigma, q'_s, \{q'_t\}, \Delta \cup \{(q'_s, \epsilon, q_0)\} \cup \{(q, \epsilon, q'_t) : q \in T\}) \) recognizes the same language as \( M \) and has a unique accept state with no transitions ending at the start state and no transitions starting at the accept state.

Exercise 3.7

Let \( M = (Q, \Sigma, q_0, T, \Delta) \) be a nondeterministic automaton with \( Q = \{q_0, q_1, q_2\} \), \( \Sigma = \{0, 1\} \), \( T = \{q_1\} \), and \( \Delta = \{(q_0, 0, q_1), (q_0, 0, q_1)\} \). The automaton \( M \) recognizes the language \( \{0\} \), whereas \( M' = (Q, \Sigma, q_0, Q \setminus T, \Delta) \) recognizes \( \{\epsilon, 0\} \). Thus, in this case \( L(M') \) is not the complement of \( L(M) \).

Exercise 4.1

The set of regular languages is closed under reversal. Let \( L \) be a regular language and \( M = (Q, \Sigma, q_0, T, \delta) \) be a DFA that recognizes \( L \), and suppose \( q'_0 \not\in Q \). Then the automaton \( M' = (Q \cup \{q'_0\}, \Sigma, q'_0, \{q_0\}, \{(q'_0, \epsilon, q) : q \in T\} \cup \{(q, a, p) : \delta(p, a) = q\}) \) recognizes the set \( \{x^R : x \in L\} \).

Exercise 4.2.b

Let \( L_1 \) be the nonregular language \( \{0^n1^n : n \geq 0\} \) from Example 4.4. We may use Exercise 4.1 to show that the language \( L_2 = \{1^n0^n : n \geq 0\} \) is also nonregular. The intersection of the two languages, \( L_1 \cap L_2 = \{\epsilon\} \) is regular.

On the other hand, if \( L_1 \) and \( L_2 \) are nonregular languages with \( L_1 = L_2 \), then obviously the intersection, \( L_1 \cap L_2 = L_1 \), is also nonregular.

Exercise 4.13

a. The set of all binary strings with an equal number of occurrences of 01 and 10 is regular. At first sight it would seem necessary to count the
number of occurrences of 01 and 10, respectively. However, observe that between any two occurrences of 01 in a binary string there has to be an occurrence of 10, and vice versa. For example, the string 011101 contains pairs 01 at the beginning and at the end, and an occurrence of 10 in between. Thus, it is sufficient to keep track of the parity in the number of pairs 01 and 10, respectively. A suitable finite automaton can be designed easily.

b. Example 4.5 and closure of regular languages under complementation can be used to prove that the set of binary nonpalindromes is not regular.

c. The language \( L = \{a^{2^n} : n \geq 0\} \) is nonregular, as its elements are pairwise distinguishable (with respect to \( L \)): if \( k < l \) then \( a^{2^k} a^{2^k} = a^{2^k+2^k} = a^{2^{k+1}} \in L \), whereas \( a^{2^l} a^{2^k} = a^{2^{k+2^l}} \not\in L \).