1. Let $F(x, y)$ mean that $x$ is the father of $y$; $M(x, y)$, that $x$ is the mother of $y$; $H(x, y)$, that $x$ is the husband of $y$; $S(x, y)$, that $x$ is the sister of $y$; and $B(x, y)$, that $x$ is the brother of $y$. We can use these predicates to define other family relationships. For example, the sibling predicate, $Sb(x, y)$, can be defined by:

$$Sb(x, y) \iff S(x, y) \lor B(x, y).$$

Give definitions for the following predicates:

(a) A predicate $Ch(x, y)$ to mean that $x$ is a child of $y$.
(b) A predicate $A(x, y)$ to mean that $x$ is an aunt of $y$.
(c) A predicate $U(x, y)$ to mean that $x$ is an uncle of $y$.
(d) A predicate $C(x, y)$ to mean that $x$ is a cousin of $y$.

Note that an uncle is (i) the brother of one’s father or mother or (ii) the husband of one’s aunt; and an aunt is (i) the sister of one’s father or mother or (ii) the wife of one’s uncle. A cousin is a child of one’s uncle or aunt.

2. Give recursive definitions of functions as specified. You may use the standard arithmetic operators, the equality predicate, the empty list, and list functions for (i) inserting an element at the front of a list (\texttt{::} or \texttt{cons}), (ii) returning the first element of a non-empty list (\texttt{hd}), (iii) returning the result of removing the first element from a non-empty list (\texttt{tl}), and (iv) concatenating two lists (\texttt{@}).

(a) A function $\texttt{sum\_all}$ that takes as input a list of integers and returns as output the sum of all the integers in the input list.

For example, $\texttt{sum\_all([2,5,6,8,2])} = 23$.

(b) Write a function $\texttt{adjacent}$ that takes as input a list of integers and returns $\texttt{true}$ if the input list contains two adjacent occurrences of the same element, and $\texttt{false}$ otherwise.

For example, $\texttt{adjacent([2,3,2,3])} = \texttt{false}$ and $\texttt{adjacent([2,3,3,2])} = \texttt{true}$.

(c) Write a function $\texttt{updown}$ that takes as input an integer $k$ and returns the list $[0,\ldots,k,\ldots,0]$. If $k$ is negative, the empty list should be returned.

For example, $\texttt{updown(-1)} = \texttt{[]}$, $\texttt{updown(0)} = [0]$, and $\texttt{updown(2)} = [0,1,2,1,0]$. 

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