1. Exercise 1.2.1
   (b) \{5, 9, 13, 17, 21\} or \{5, 9, 13, 17\}, depending on how "between" is interpreted.
   (d) \{January, February, May, July\}
   (f) \{1, 2, 3, 4, 6, 8, 12, 24\}

2. Exercise 1.2.3 Let \(A\) be the set \(\{a, \emptyset\}\).
   (b) The statement \(\{a\} \in A\) is false.
   (d) The statement \(\{a\} \subseteq A\) is true.
   (f) The statement \(\emptyset \in A\) is true.
   (h) The statement \(\emptyset \in A\) is false.

3. Exercise 1.2.7
   (b) \(\{a, \emptyset\}\)
   (d) \(\{a, b, \{b\}\}\)

4. Exercise 1.2.8
   (b) The natural numbers 17, 29, 41, and 53 are elements of \(A \cap B\).

5. Exercise 1.2.10
   (b) The union of the collection is \(A_9\), which equals \(\{-8, -7, \ldots, 7, 8\}\).
   (d) The union of the collection is the set of all integers.
   (f) The intersection of the collection is \(A_1\), which equals \(\{0\}\).
   (h) The intersection of the collection is the empty set. (Note that the sets \(A_{-1}, A_{-3}, \text{etc.}\) are empty.)

6. Exercise 1.2.13
   (b) \((B - C) \cup (C - A)\)
7. **Exercise 1.2.19**

(b) \([x, y, x] \cup [y, x, y] = [x, x, y] \) and \([x, y, x] \cap [y, x, y] = [x, x, y]\)

(d) \([1, 2, 2, 3, 4, 4] \cup [2, 3, 3, 4, 5] = [1, 2, 2, 3, 3, 4, 4, 5]\)

and \([1, 2, 2, 3, 3, 4, 4] \cap [2, 3, 3, 4, 5] = [2, 3, 3, 4]\

(f) \([a, a, [b, b], [a, [b]]] \cup [a, a, [b], [b]] = [a, a, [b, b], [a, [b]]]\)

and \([a, a, [b, b], [a, [b]]] \cap [a, a, [b], [b]] = [a, a]\)

8. **Exercise 1.2.20**

(b) If \(B = [2, 3, 3, 4, 5]\) then \(B \cup [2, 2, 3, 4] = [2, 2, 2, 3, 3, 4, 4, 5]\) and \(B \cap [2, 2, 3, 4, 5] = [2, 3, 4, 5]\).

9. **Exercise 1.2.21**

If \(A\) and \(B\) are multisets over a set \(S\), we define the difference \(A - B\) by: \((A - B)(x) = \max(A(x) - B(x), 0)\), for all \(x \in S\).

10. **Exercise 1.2.22**

(b) We prove that \(A \cup B = B \cup A\), for all sets \(A\) and \(B\). Let \(A\) and \(B\) be arbitrary sets. We show that \(A \cup B\) and \(B \cup\) have the same elements:

\[
x \in A \cup B
\]

iff \(x \in A\) or \(x \in B\)

iff \(x \in B\) or \(x \in A\)

iff \(x \in B \cup A\)

11. **Exercise 1.2.26**

(b) Let \(A\) and \(B\) be arbitrary sets. We show that \(A \cup (B \cap A)\) and \(A\) have the same elements:

\[
x \in A \cup (B \cap A)
\]

iff \(x \in A\) or \(x \in (B \cap A)\)

iff \(x \in A\) or else both \(x \in B\) and \(x \in A\)

iff \(x \in A\)

The same set identity can also be proved by observing that by commutativity, \(A \cup (B \cap A) = A \cup (A \cap B)\), and by the absorption law, \(A \cup (A \cap B) = A\).
12. Exercise 1.2.27

We prove that \((A \cap B) \cup C = A \cap (B \cup C)\) if and only if \(C \subseteq A\).

(i) First assume that \(C \subseteq A\). By distributivity, \((A \cap B) \cup C = (A \cup C) \cap (B \cup C)\). Since \(C \subseteq A\), we have \(A \cup C = A\) and hence \((A \cup C) \cap (B \cup C) = A \cap (B \cup C)\). In sum, we have \((A \cap B) \cup C = A \cap (B \cup C)\).

(ii) Next suppose \((A \cap B) \cup C = A \cap (B \cup C)\). By the properties of set union and intersection, we have \(C \subseteq (A \cap B) \cup C\) and also \(A \cap (B \cup C) \subseteq A\). Thus by our assumption and the transitivity of the subset relation, we obtain \(C \subseteq A\).

13. Exercise 1.2.28

(d) Consider the identity \(A \oplus A = A\).

By definition of the symmetric difference of sets, \(A \oplus A = (A \cup A) - (A \cap A)\). By the idempotence laws (cf. Exercises 1.2.22(c) and 1.2.23(d)), \((A \cup A) = A\) and \((A \cap A) = A\). Thus, \(A \oplus A = A - A = \emptyset\). Consequently, if \(A\) is a nonempty set, then \(A \oplus A \neq A\).