A Probabilistic Constructive Approach To Optimization Problems

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Probabilistic Constructive & MIS
Probabilistic Constructive & MIS
Probabilistic Constructive & MIS

\[ \text{OF} \]
\[ \begin{array}{c|c}
    6 & 0 \\
\end{array} \]
Probabilistic Constructive & MIS
Probabilistic Constructive & MIS

Solution:

- E
- G
- K
- C
- I
- H
- B
- J

OF
- 6
- 0
- 3
Probabilistic Constructive & MIS

Solution: B J H E
Probabilistic Constructive & MIS
Talk Organization

- Motivation
- Global Picture
- Generic Approach
- Applications
  - Graph Coloring, MIS, Sequence Covering, Scheduling
- Experimental Results
- Conclusion
## Algorithm Classification

<table>
<thead>
<tr>
<th></th>
<th>Probabilistic</th>
<th>Deterministic</th>
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</table>
| Iterative Improvement | • Metropolis  
• Simulated Annealing  
• Genetic Algorithms  
• Tabu Search |
| Constructive | • Kernighan-Lin  
• Fiduccia-Mattheyses  
• Sanchis  
• Krishnamurthy  
|• Branch & Bound  
• Divide & Conquer  
• Dynamic Programming  
• Force Directed |
Related Work - Deterministic Constructive

Branch & Bound
(Lawler & Wood)
Related Work – Deterministic Constructive

**Generic**
- Maximally Constrained, Minimally Constraining
- Sorting Algorithms: Insertion, Selection, Quicksort, Radix, Bucket
- Basic Graph Algorithms: BFS, DFS, Topological Search, SCC
- Complex Graph Algorithms: Dijkstra, Bellman–Ford, Floyd-Warshall

**CAD**
- Force-directed Scheduling
- Retiming for Critical Path
- List Scheduling
- DAGON: Technology Mapping
- Op-amp Design
- LC Oscillators
- Branch-&-Bound Clock Selection
Related Work - Deterministic Iterative Improvement
Related Work - Probabilistic Iterative Improvement

Simulated Annealing

- Cost function
- Solution space
- Local Search
Related Work - Probabilistic Iterative Improvement

**Generic**
- Simulated Annealing
- Rejectionless SA
- Mean Field Annealing
- Simulated Evolution
- Tabu Search
- Genetic Algorithms

**CAD**
- TimberWolf (Sechen '85)
- Roy & Sechen ('93), Chatterjee & Hartley ('90)
- Van den Bout & Miller ('90)
- Saab & Rao ('90, '92)
- Tao ('91), Lim & Chee ('91)
- Inayoshi & Manderick ('92)

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Generic Probabilistic Constructive Approach

- Solve difficult small part of the problem well
- Find probabilistically many difficult small parts of the problem
- Find the best small part
- Continue to build solution
- Reduce the instance size
Main Idea – Candidate Part

Solution Space

Atomic portion of the solution
Main Idea – Probabilistic Search

- Two Types
  - Constructive
  - Iterative Improvement
Main Idea – Probabilistic Search

- Two Types
  - Constructive
  - Iterative Improvement

Solution Space
Main Idea – Probabilistic Search

- Two Types
  - Constructive
  - Iterative Improvement

Solution Space
Main Idea – Objective Function

- Scope
- Accuracy

Solution Space
Main Idea – Comprehensive Objective Function

- Scope is the entire instance
- Analysis of overlap between potential CPs
Main Idea – Candidate List

Solution Space

Candidate List

.34
.95
.57
.92
.89
.95
.92
.89
Main Idea – Stopping Criteria

- Minimum Size
- Adaptive
  - Search for long period w/o any improvement
  - Restriction to the number of overlapping CP
Main Idea – Best Candidate Selection

- Select the CP with best COF
  - Advantage: Very fast approach

- Conduct Comprehensive Analysis of CPs in CL
  - Correlation of CP Pairs, Triplets, ….
Main Idea – Solution Integration
Main Idea – Solution Integration
Main Idea – Overall Control Strategy

- Probabilistic → different solutions/ run-times
- Variety of control strategies
  - Multi-starts
  - Learning Examples
Additional Augmentation Mechanisms: Delayed Binding
Applications

- Generic Optimization Problems
  - GC
  - MIS
- CAD-related Optimization Problems
  - Sequence covering
  - Scheduling
Probabilistic Constructive - MIS

Two approaches

- Inclusion
  - Selection of nodes to **include** in MIS

- Exclusion
  - Selection of nodes to **exclude** from MIS
Probabilistic Constructive – MIS Inclusion

**Candidate Part (CP)**
- Subset of nodes with no edges between them (k=4)

**Probabilistic Search**
- Remove single node from CP, include another

**Candidate List (CL)**
- No node can exist in more than 1/5 of the CPs
- $K_{\text{min}}$, except if OFs of CPs in CL are consistent -> $2^* K_{\text{min}}$
Probabilistic Constructive – MIS Inclusion

Objective Function (OF)
- \( n_r \): Number of nodes in remainder of graph
- \( e \): Total number of edges - incident edges

\[
\text{OF} = \alpha_1 n_r + \alpha_2 e
\]

Comprehensive Objective Function (COF)
- \( n_i \): Number of neighbors of node \( i \) in the CP

\[
\text{COF} = \text{OF} + \alpha_3 \sum n_i^2
\]

Stopping Criteria
- No CPs with improved OF after \( kn_r \) attempts (k = 5)
Probabilistic Constructive – MIS Inclusion

- **Best Candidate Selection (BCS)**
  \[ BCS = COF + \sum_{i=1}^{\vert CP \vert} 1/a_i \]

- **Solution Integration**
  - Integrate BCS into solution
  - Remove all nodes in CP, neighbors of nodes, & incident edges

- **Overall Control Strategy**
  - N/10 multi starts
  - N: number of nodes in the original instance
Probabilistic Constructive – MIS Exclusion

- **Probabilistic Search**
  - Select node from CP to possibly include in MIS & replace with node to be excluded
  - Remove single node from CP, include another

- **Objective Function (OF)**
  - \( e \): number of edges remaining in the resulting graph
  - \[ \text{OF} = \alpha e \]
Experimental Results

- Generic Optimization Problems
  - GC
- MIS
- CAD-related Optimization Problems
  - Sequence covering
  - Scheduling

Experimental Results - MIS

- DIMACS Instances for Maximum Clique
  - Maximum Clique ➔ MIS in $G_C$
# Experimental Results - MIS

<table>
<thead>
<tr>
<th>Name</th>
<th>V</th>
<th>E in Clique</th>
<th>E in MIS</th>
<th>γ</th>
<th>CPU</th>
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</table>
Conclusion

- Probabilistic – Fast
- Constructive – Flexible
- Generic CAD Optimization Problems
- Easy Implementation/ Generic Approach
- New Algorithmic Paradigm