Scheduling and Motion Planning for Wireless Sensors and Mobile Networks

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September 11th, 2017.
Research Interest and Projects

Computational Geometry, Algorithm Design and Analysis, Wireless Networks, Social Networks

- Geometric Methods for Network Analysis.
- Location and Trajectory Privacy.
- Social Influence and Contagions
- Scheduling Algorithms.
Scheduling Wireless Devices

Considerations:

- Efficiency – energy, storage, bandwidth.
- Performance – coverage, detection, connectivity.

Constraint dimensions:

- Spatial – visibility, proximity.
- Temporal – mobility.
Two Application Scenarios

How to schedule and allocate resources in spatial and temporal domains?

- Smart building: optimize energy usage, improve safety & security.
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How to schedule and allocate resources in spatial and temporal domains?

- Smart building: optimize energy usage, improve safety & security.
- Mobile networks: assign mobile nodes to collect data from sensors with storage capacities.
Coverage in Smart Building

Given $n$ camera nodes and $m$ target nodes, the set of targets covered by sensors $g_i$ is $P(g_i)$, how to schedule the camera nodes?

▶ If sensors are turned on all the time: Art Gallery Problem;
▶ Sensors are not turned on all the time: Duty Cycle Scheduling.
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- Targets are frequently covered;

- Energy usage is reduced.

- Generic coverage assumption.

Question: when and which set of camera nodes to turn on?

- Time is slotted.

- At each time slot at most $k$ cameras are turned on.

- Minimize the maximum or average dark duration.

- Or, meet specific target coverage frequency requirements.
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Scheduling for Minimizing Dark Duration

Given $n$ camera nodes and $m$ target nodes, the set of targets covered by sensors $g_i$ is $P(g_i)$, suppose at any slot only $k$ sensors are turned on, how to schedule sensors such that no target stays ‘in dark’ for too long.
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- Define **max dark duration** $T(p)$ for $p \in D$: $p$ is lighted up at least once every $T(p)$ slots.
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<thead>
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<th>$g_3$</th>
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<tr>
<td>$p_1$</td>
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  $$\min_{p \in D} \max_{T(p)}$$
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- **Min Max Dark Duration Scheduling:**

  $$\min \max_{p \in D} T(p)$$

- **Min Average Dark Duration Scheduling:**

  $$\min \sum_{p \in D} w(p) \cdot T(p)$$

where $w(p)$ is a weight parameter.
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Assume that $G$ is a **minimal cover**, i.e., removing any sensor renders targets not fully covered, then round robin on $G$ is optimal.
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Assume \( C^* < \lceil n/k \rceil \) realized by \( p^* \in D \).
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![Diagram](attachment:diagram.png)

Assume $C^* < \lceil n/k \rceil$ realized by $p^* \in D$.

Then at least one guard $g$ does not appear during the interval of length $C^*$ as shown below.
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→ the point that is only guarded by \( g \) has dark duration \( > C^* \).
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If $G$ is not a minimal cover, then round robin on a minimum cover (a subset of $G$) is optimal.
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Min Max problem is tailored towards worst case & sensitive to outliers.
Min Average Dark Duration Scheduling: Motivation

Example:

- Three guards $g_1, g_2, g_3$ and six targets $p_1, p_2, p_3, p_4, p_5, p_6$. 

- $P(g_1) = \{p_1\}$, $P(g_2) = \{p_2\}$, $P(g_3) = \{p_3, p_4, p_5, p_6\}$.

- Min Max: round robin on $g_1, g_2, g_3$, each target has a dark duration 3.

- Min Average: it is beneficial to repeat $g_3$ more often.

- E.g., repeating $g_1, g_3, g_2, g_3$ yields average dark duration $8/3 < 3$.

- Min average optimization: use the weights $w(\cdot)$ to allow more flexibility to adjust to varying guarding requirements.

- Targets with higher importance have higher weights.
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Algorithms for Min Average Dark Duration Scheduling

Challenge:

- Which sensor to repeat, and how many times?
- How to schedule them?
Special Case: Round Robin On a Permutation $\pi$

Find the optimal one $\pi^*$ that minimizes the min average dark duration.

- Take a target $p \in D$, consider all guards that cover $p$ we wish them to uniformly spread in $\pi$. 

\[
\begin{align*}
G(p) &= \{g_1, g_2, g_3, g_4, g_5, g_6\} \\
g_1 &\in \text{bin} 1,3 \\
g_3, g_4, g_6 &\in \text{bin} 3,4, 4,6 \\
g_5 &\in \text{bin} 6,5 \\
g_2 &\in \text{bin} 5,2 \\
G(p) &\subseteq V, |V|= 16 \\
m &= |G \setminus G(p)| \text{balls} \\
\ell &= |G(p)| \text{bins}
\end{align*}
\]
Assume $G(p)$ appears in the permutation with this order $g_1, g_3, g_4, g_6, g_5, g_2$. 
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![Diagram of guards and bins with order and relationships indicated.]
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How about a random permutation?
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- Throw $m$ balls randomly into $\ell$ bins, how many balls do we have in the largest bin?
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- Throw $m$ balls randomly into $\ell$ bins, how many balls do we have in the largest bin?
- Compared to the optimal (uniform), the ratio is $\alpha = O(1)$ if $k \geq \log n / \log \log n$, and $\alpha = O(\log n / \log \log n)$ otherwise.
More General: Best Periodic Schedule

The guards in one full cycle $T$ of a periodic schedule: each guard $g$ appears $\tau(g)$ times. Total # guards in one cycle is $\sum_g \tau(g) = Tk$. 
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- Define $f^*(g) = \tau^*(g)/(kT^*)$, frequency.
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Claim: $T^*(p) \geq T^*/X^*(p)$ – the best case is when all guards that cover $p$ are uniformly spread in $T^*$. 

Objective function $A^* = \sum_p w(p) T^*(p) \geq B^* = \sum_p w(p)/k \sum_{g \text{ covers } p} f^*(g)$ – we can minimize the right hand side!
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$B$ is a convex function of $f(g)$ – we can minimize the right hand side!
Algorithm

Three steps:

- Any optimal schedule can be turned into a periodic schedule with a factor 2 approximation.
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- Run a convex optimization algorithm to find $f(g)$

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subject to $\sum_g f(g) = 1$, $f(g) \geq 0$. 

- Turn $f(g)$ into a nearby rational number and find $\tau(g)$, $\tau(g)/kT \approx f(g)$. 
- Repeating $g_i \tau(g_i)$ times, and choose a random permutation on them.

This algorithm gives $(2 + \varepsilon)\alpha$ approximation in expectation, $\alpha = O(1)$ if $k \geq \log n / \log \log n$, and $\alpha = O(\log n / \log \log n)$ otherwise.
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Min Energy Scheduling with Target Coverage Frequency:
Suppose target \( j \) needs to be covered every \( f_j \) slots, how to schedule sensors to meet the requirement such that at each slot only \( k \) sensors are turned on? Minimize \( k \).
Min Energy Scheduling with Target Coverage Frequency:
Suppose target $j$ needs to be covered every $f_j$ slots, how to schedule sensors to meet the requirement such that at each slot only $k$ sensors are turned on? Minimize $k$.

Bottom target must be covered every slot while each of the top target must be covered every $m$ slots.
Min Energy Scheduling with Target Coverage Frequency

Use set multi-cover & randomization. Details skipped.

- $O(\log n + \log m)$ approximation.
- Geometric setting: better approximation.
Part II: Path Planning for Mobile Nodes

Problem: Given a set of \( n \) sensor nodes \( \{p_1, p_2, \cdots, p_n\} \), schedule mobile nodes to serve them (data collection, battery recharging). Suppose the mobile node travels with unit speed.
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Additional constraints: data rate/node capacity, time-window.
Data Collection Problem

Each node has data rate $r_i$ and capacity $c_i$, if the capacity is reached additional data is lost. Schedule the path for $k$ mobile nodes to maximize data collected.
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- Even for $k = 1$ and sensors with unit data rates and capacities on a line, the optimal solution is not TSP anymore.
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- Theorem: The optimal solution is a zig-zag tour on an interval.

- Run Dynamic Programming in $O(n^2)$ time.
## Data Collection Problem

<table>
<thead>
<tr>
<th>With Sensors</th>
<th>Single mule</th>
<th>$k$-mule</th>
<th>No Data Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>on a Line</td>
<td>in $P$</td>
<td>$\frac{1}{3}$</td>
<td>exact</td>
</tr>
<tr>
<td>on a Tree</td>
<td>pseudo-poly</td>
<td>$\frac{1}{3}(1 - \frac{1}{e^{2+\varepsilon}})$</td>
<td>12</td>
</tr>
<tr>
<td>General Metric</td>
<td>$1/6 - \varepsilon$</td>
<td>$\frac{1}{3}(1 - \frac{1}{e^{2+\varepsilon}})$</td>
<td></td>
</tr>
<tr>
<td>Euclidean</td>
<td>$1/3 - \varepsilon$</td>
<td>$\frac{1}{3}(1 - \frac{1}{e^{1-\varepsilon}})$</td>
<td></td>
</tr>
<tr>
<td>Diff Capacities</td>
<td>$O\left(\frac{1}{m}\right)$</td>
<td>$O(m)$</td>
<td></td>
</tr>
</tbody>
</table>

**Table:** $m \leq \log\left(\frac{c_{\text{max}}}{c_{\text{min}}}\right)$ where $c_{\text{max}}$ is the largest capacity and $c_{\text{min}}$ is the smallest capacity. For the results in the first four rows, we assume that the sensor capacities are all the same. $\varepsilon$ is any positive constant.
Example: Single Mule, Capacity $c$, Euclidean Setting

Algorithm:

- Find a path of length $c/2$ w/ max $\# R$ of nodes.
- Travelling back and forth along it gives a tour.
- No data loss, data rate=$R$. 

Proof: $\text{OPT} \leq 3R$. 

- Find in the $\text{OPT}$ tour the interval of length $c/2$ with max data collection rate (which is at least $R$).
- The interval has at most $R$ distinct nodes.
- Total data collected from any sensor is $c$ (on the first visit) and $c/2$ after each $c/2$ interval. Data rate is at most $3R$. 

Example: Single Mule, Capacity $c$, Euclidean Setting

Algorithm:

- Find a path of length $c/2$ with max $\# R$ of nodes.
- Travelling back and forth along it gives a tour.
- No data loss, data rate $= R$.

Proof: $\text{OPT } R^* \leq 3R$.

- Find in the OPT tour the interval of length $c/2$ with max data collection rate (which is at least $R^*$).

![Diagram of OPT path with interval $c/2$]
Example: Single Mule, Capacity \( c \), Euclidean Setting

Algorithm:

- Find a path of length \( c/2 \) w/ max \# \( R \) of nodes.
- Travelling back and forth along it gives a tour.
- No data loss, data rate=\( R \).

Proof: \( \text{OPT} \ R^* \leq 3R \).

- Find in the OPT tour the interval of length \( c/2 \) with max data collection rate (which is at least \( R^* \)).

![Diagram showing a path with an interval of length \( c/2 \) and labeled \( \text{OPT} \).]

- The interval has at most \( R \) distinct nodes.
- Total data collected from any sensor is \( c \) (on the first visit) and \( c/2 \) after each \( c/2 \) interval. Data rate is at most \( 3R \).
Conclusion

- Classical problems revisited.
- Worst-case performance guarantees.
- Optimization and scheduling are a crucial part of the machine intelligence era.
Acknowledgement


▶ http://www3.cs.stonybrook.edu/~jgao/

▶ Questions and comments?