A brief overview of geographical routing
Geographical routing may get stuck

- Geographical routing may stuck at a node whose neighbors are all further away from the destination than itself.

Send packets to the neighbor closest to the destination
Face Routing

• Keep left hand on the wall, walk until hit the straight line connecting source to destination.
• Then switch to the next face.
Relative Neighborhood Graph and Gabriel Graph

- Relative Neighborhood Graph (RNG) contains an edge $uv$ if the lune is empty of other points.
- Gabriel Graph (GG) contains an edge $uv$ if the disk with $uv$ as diameter is empty of other points.
- Both can be constructed in a distributed way.
An example of GG and RNG
Two problems remain in geographical routing

- Both RNG and GG remove some edges → a short path may not exist!

- The shortest path on RNG or GG might be much longer than the shortest path on the original network.

- Even if the planar subgraph contains a short path, can greedy routing and face routing find a short one?
Tackle problem I:
Find a planar spanner
Find a good subgraph

- Goal: a planar spanner such that the shortest path is at most $\alpha$ times the shortest path in the unit disk graph.
  - Euclidean spanner: The shortest path length is measured in total Euclidean length.
  - Hop spanner: The shortest path length is measured in hop count.
- $\alpha$: spanning ratio.
  - Euclidean spanning ratio $\geq \sqrt{2}$
  - Hop spanning ratio $\geq 2$.
- Let’s first focus on Euclidean spanner.
Delaunay triangulation is an Euclidean spanner

- DT is a 2.42-spanner of the Euclidean distance.
- For any two nodes uv, the Euclidean length of the shortest path in DT is at most 2.42 times $|uv|$.
Restricted Delaunay graph

- Keep all the Delaunay edges no longer than 1.
- Claim: RDG is a 2.42-spanner (in total Euclidean length) of the UDG.
- Proof sketch: If an edge in UDG is deleted in RDG, then it’s replaced by a path with length at most 2.42 longer.
Construction of RDG

• Easy to compute a superset of RDG: Each node computes a local Delaunay of its 1-hop neighbors.
  – A global Delaunay edge is always a local Delaunay edge, due to the empty-circle property.
  – A local Delaunay may not be a global Delaunay edges.

• What if the superset has crossing edges?
Crossing Lemma

- **Crossing lemma**: if two edges cross in a UDG, then one node has edges to the three other nodes in UDG.

\[
\begin{align*}
|u_w| & \leq |w_p| + |u_p| \\
|v_x| & \leq |v_p| + |x_p| \\
\implies |w_u| + |v_x| & \leq |w_x| + |u_x| \leq 2
\end{align*}
\]

Also, \(|w_v| + |u_x| \leq |w_x| + |u_x| \leq 2\)

There must be 2 edges on the quad adjacent to the same node.
Detect crossings between local delaunay edges

- By the crossing Lemma: if two edges cross in a UDG, one of them has 3 nodes in its neighborhood and can tell which one is not Delaunay.

- Neighbors exchange their local DTs to resolve inconsistency.
  - A node tells its 1-hop neighbors the non-Delaunay edges in its local graph.
  - A node receiving a “forbidden” edge will delete it from its local graph.

- Completely distributed and local.
RDG construction

- 1-hop information exchange is sufficient.
  - Planar graph;
  - All the short Delaunay edges are included.
  - We may have some planar non-Delaunay edges but that does not hurt spanning property.

![Diagram showing a's local Delaunay and b's local Delaunay with some edges marked differently.]
More on RDG construction

• RDG can be constructed without the full location information.

• Only local angle information suffices.

• Key operation: If two edges in the unit-disk graph cross, remove the one that is not in the Delaunay triangulation.

• How to tell that an edge is not in the Delaunay triangulation?
Removing non-Delaunay edges

If two edges AB, CD cross, there are only three cases:

(i)

(ii)

(iii)
Removing non-Delaunay edges

If two edges AB, CD cross, there are only three cases:

With angle info, the shape is fixed! Node C can tell which edge is not Delaunay.
Removing non-Delaunay edges

Case (i) : Use the “empty-circle” test of Delaunay triangulation

\[ |AC| > 1 \geq |CD| \]
\[ |BC| > 1 \geq |CD| \]

(i)

Conclusion: The edge AB is not a Delaunay edge.
Find a hop spanner

- Restricted Delaunay graph is not a hop spanner.
  - Take \( n \) nodes uniformly in a segment of length 1. The hop count can be as large as \( n-1 \).
- Reduce the density of the sensors.
  - Use clustering to reduce density.
  - Compute RDG on the subset to get a hop spanner.
  - Clustering also reduce interference and enables efficient resource reuse such as bandwidth.
Reduce node density

- Find a subset of nodes, called clusterheads
  - Each node is directly connected to at least 1 clusterhead.
  - No two clusterheads are connected.
- Use a greedy algorithm. Pick a node as a clusterhead, remove all the 1-hop neighbors, continue.
- Constant density: $\leq 6$ clusterheads in any unit disk.
  - The angle spanned by two clusterheads is at least $\pi/3$. 

\[ \pi/3 \]
Connect clusterheads by gateways

- For two clusterheads, if their clients have an edge, then we pick one pair as **gateway** nodes.

- Notice that clusterheads $x, y$ are within 3 hops to have a pair of gateways.

- There are constant clusterheads and gateways inside any unit disk.
Path on clusterheads and gateways

- For two nodes $u, v$ that are $k$ hops away, there is a path through clusterheads and gateways with at most $3k+2$ hops.

- Construct RDG on clusterheads and gateways, which have constant bounded density.

Shortest path

$u = u_1, u_2, u_3, u_4, \ldots, u_{k+1} = v$
A Routing Graph Sample

Select clusterheads

Clusterheads select gateways

RDG on clusterheads & gateways
**Restricted Delaunay graph**

- **Claim:** (RDG on clusterheads and gateways + edges from clients to clusterheads) is a constant hop spanner of the original UDG.

- **Proof sketch:**
  - The shortest path $P$ in the unit disk graph has $k$ hops.
  - Through clusterheads and gateways $\exists$ a path $Q$ with $\leq 3k+2$ hops.
  - $Q$’s total Euclidean length is $\leq 3k+2$.
  - The shortest path on the RDG, $H$, has Euclidean length $\leq 2.42 \times (3k+2)$.
  - By constant density property a region with width 1 and length $2.42 \times (3k+2)$ has $O(k)$ nodes inside. So # hops of $H$ is $O(k)$.
  - This concludes the hop spanner property.
Restricted Delaunay graph

RNG

RDG
Restricted Delaunay graph

RNG

RDG

Clusterhead
Gateway
Tackle problem II: Improve face routing to find a short path & Geographic routing in practice
Papers

Geographic routing in practice:

Virtual coordinates:
Overview of geographical routing

• Routing with geographical location information.
  – Greedy forwarding.
  – If stuck, do face routing on a planar sub-graph.
Overview of last lecture

• How to find a planar subgraph?
  – Use distributed construction: relative neighborhood graph, Gabriel graph, etc.
  – A planar subgraph that contains a short path: restricted Delaunay graph: short Delaunay edges.

• Big problem: how is the performance of geo-routing?
  – Can we always find a short path?
Bad news: Lower bound of localized routing

- Any deterministic or randomized localized routing algorithm takes a path of length $\Omega(k^2)$, if the optimal path has length $k$.

- The adversary decides where the chain $w_t$ is. Since we store no information on nodes, in the worst case we have to visit about $\Omega(k)$ chains and pay a cost of $\Omega(k^2)$.
Good news: greedy forwarding is optimal

- If greedy routing gets to the destination, then the path length is at most $O(k^2)$, if the optimal path has length $k$.

- $|uv|$ is at most $k$. On the greedy path, every other node is not visible, so they are of distance at least 1 away. By a packing lemma, there are at most $O(k^2)$ nodes inside a disk of radius $k$.

How is face routing? How is greedy + face routing?