The following problems are due in 2 weeks (Feb 9th) before class.

1. Prove that the convex hull of a set of points \( S \) has the smallest perimeter among all convex polygons that contain \( S \).

2. Prove that the diameter of a set of points \( S \), defined as the pair of points with largest distance, is realized at two vertices on the convex hull of \( S \).

3. In the divide-and-conquer algorithm for computing the convex hull of a set of points \( S \), we need to find the tangent of two convex polygons \( P, Q \) in the merge step. The algorithm starts from the line segment connecting the rightmost of the left polygon \( P \) and the leftmost of the right polygon \( Q \) and gradually moves to the tangent. Prove that the edge identified in this process does not intersect the interior of \( P, Q \).

4. Let \( S \) be a set of \( n \) possibly intersecting unit circles in the plane. We want to compute the convex hull of \( S \).
   
   (a) Show that the boundary of the convex hull of \( S \) consists of straight line segments and pieces of circles in \( S \).
   
   (b) Show that each circle can occur at most once on the boundary of the convex hull.
   
   (c) Let \( S' \) be the set of points that are the centers of the circles in \( S \). Show that a circle in \( S \) appears on the boundary of the convex hull if and only if the center of the circle lies on the convex hull of \( S' \).