CSE 215 Midterms (80 points)
14\textsuperscript{th} October: 2:30-3:50pm (80 minutes)
Closed Book (1 Sheet of notes allowed).

\textbf{Name:}
1. Quantified Statements (20 points)

Consider the following statements.

- All CSE215 students are smart.
- Some smart students have a GPA of less than 3.5.
- There are students in CSE215 that have a GPA of 4.
- All students with a GPA of 3.8 or more are enrolled in CSE215.

(a) Convert each of the above statements to a quantified statement. Use $S$ as the set of all students, $s(x)$ to denote that $x$ is smart, and $c(x)$ to denote that $x$ is in CSE215. You may use the function $g(x)$ to represent $x$’s GPA.

(b) John is enrolled in CSE215. What can you say (if anything) about his GPA? Explain your answer briefly.

(c) Jane is not enrolled in CSE215. What can you say (if anything) about her GPA? Explain your answer briefly.
2. Proofs - I (20 points)

Prove that $\sqrt{2} + \sqrt{3}$ is irrational. Hint: You may assume that $\sqrt{6}$ is irrational.
3. **Proofs - II (20 points)**

Given any integer $n > 3$, prove that one of $n$, $n + 2$ or $n + 4$ must be composite. Hint: Use case analysis based on the quotient-remainder theorem.
4. **Proofs - III (20 points)**

Prove or disprove the following two statements.

(a) If $a$ is rational and $b$ is irrational, then $(a + b)$ is irrational.

(b) If $a$ is rational and $b$ is irrational, then $a \cdot b$ is irrational.
Scratch Work