The Scientific Method

Develop General Theories
General theories must be consistent with most or all available data and with other current theories.

Make Observations
What do I see in nature? This can be from one's own experiences, thoughts or reading.

Think of Interesting Questions
Why does that pattern occur?

Refine, Alter, Expand or Reject Hypotheses

Formulate Hypotheses
What are the general causes of the phenomenon I am wondering about?

Gather Data to Test Predictions
Relevant data can come from the literature, new observations or formal experiments. Thorough testing requires replication to verify results.

Develop Testable Predictions
If my hypothesis is correct, then I expect a, b, c, ...

(Garland, 2015)
Hypothesis Testing

Hypothesis -- something one asserts to be true.

Classical Approach:

$H_0$: null hypothesis -- some “default” value (usually that one’s hypothesis is false)

$H_1$: the alternative -- usually that one’s “hypothesis” is true
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**Goal:** Use probability to determine if we can “reject the null”(\( H_0 \)) in favor of \( H_1 \).

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Example: Hypothesize a coin is biased.

\( H_0: \text{the coin is not biased} \) (i.e. flipping \( n \) times results in a Binomial\( (n, 0.5) \))
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More formally: Let $X$ be a random variable and let $R$ be the range of $X$. $R_{\text{reject}} \subset R$ is the rejection region. If $X \in R_{\text{reject}}$ then we reject the null.

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In the example, if $n = 1000$, then then $R_{\text{reject}} = [0, 469] \cup [531, 1000]$

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Example: Communities with higher population have different amounts of violent crimes (per capita) than those with lower population.

Assignment 1, Programming Problem “C) 9.”
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\[ t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_{x_1} + s_{x_2}} \cdot \sqrt{\frac{1}{n}}} \]

\( t \) statistic for 2 iid (independent, identically distributed) samples
Hypothesis Testing

Important logical question:

Does failure to reject the null mean the null is true?