Link Analysis

Stony Brook University CSE545, Spring 2019

The Web, circa 1998

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The Web , circa 1998

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Click he	re for advertising information - reach millions every month!
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The Web , circa 1998





Match keywords, language (*information retrieval*) Explore directory





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Time-consuming; Not open-ended

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Enter PageRank

The Anatomy of a Large-Scale Hypertextual Web Search Engine

Sergey Brin and Lawrence Page

Computer Science Department, Stanford University, Stanford, CA 94305, USA sergey@cs.stanford.edu and page@cs.stanford.edu

Abstract

In this paper, we present Google, a prototype of a large-scale search engine which makes heavy

use of the structure and produce much 1 text and hyperlink c

The PageRank Citation Ranking: Bringing Order to the Web

January 29, 1998

...

Abstract

The importance of a Web page is an inherently subjective matter, which depends on the readers interests, knowledge and attitudes. But there is still much that can be said objectively

Key Idea: Consider the citations of the website.









J. Leskovec, A. Rajaraman, J. Ullman: Mining of Massive Datasets, http://www.mmds.org

View 1: Flow Model:

in-links (citations) as votes

but, citations from important pages should count more.

=> Use recursion to figure out if each page is important.

Innovation 1: What pages would a "random Web surfer" end up at?

View 1: Flow Model:



How to compute?

View 1: Flow Model:

A
$$r_{A}/1$$
 B
C $r_{C}/2$ D $r_{D} = r_{A}/1 + r_{B}/4 + r_{C}/2$

How to compute?

View 1: Flow Model:



How to compute?

View 1: Flow Model:



A System of Equations:

 $r_A = \frac{r_B}{2} + \frac{r_C}{1}$

How to compute?

View 1: Flow Model:

A System of Equations:



How to compute?



How to compute?

$$r_{A} = \frac{r_{B}}{2} + \frac{r_{C}}{1}$$

$$r_{B} = \frac{r_{A}}{3} + \frac{r_{D}}{2}$$

$$r_{C} = \frac{r_{A}}{3} + \frac{r_{D}}{2}$$

$$r_{D} = \frac{r_{A}}{3} + \frac{r_{B}}{2}$$



$$1 = r_A + r_B + r_C + r_D$$



to \ from	Α	B	С	D
A	0	1/2	1	0
В	1/3	0	0	1/2
С	1/3	0	0	1/2
D	1/3	1/2	0	0

View 2: Matrix Formulation

$$1 = r_A + r_B + r_C + r_D$$

$$r_{A} = \frac{r_{B}}{2} + \frac{r_{C}}{1}$$

$$r_{B} = \frac{r_{A}}{3} + \frac{r_{D}}{2}$$

$$r_{C} = \frac{r_{A}}{3} + \frac{r_{D}}{2}$$

$$r_{D} = \frac{r_{A}}{3} + \frac{r_{B}}{2}$$

to \ from	Α	B	С	D
A	0	1/2	1	0
В	1/3	0	0	1/2
С	1/3	0	0	1/2
D	1/3	1/2	0	0



View 2: Matrix Formulation 1 - m = 1

 $1 = r_A + r_B + r_C + r_D$



to \ from	Α	B	С	D
A	0	1/2	1	0
В	1/3	0	0	1/2
С	1/3	0	0	1/2
D	1/3	1/2	0	0

B

Innovation: What pages would a "random Web surfer" end up at?
To Start, all are equally likely at ¼

View 2: Matrix Formulation

 $1 = r_A + r_B + r_C + r_D$



to \ from	Α	B	С	D
A	0	1/2	1	0
В	1/3	0	0	1/2
С	1/3	0	0	1/2
D	1/3	1/2	0	0

B

Innovation: What pages would a "random Web surfer" end up at?
To Start, all are equally likely at ¼: ends up at D

View 2: Matrix Formulation $1 = r_A + r_B + r_C + r_D$

 $r_{A} = \frac{r_{B}}{2} + \frac{r_{C}}{1}$ $r_{B} = \frac{r_{A}}{3} + \frac{r_{D}}{2}$ $r_{C} = \frac{r_{A}}{3} + \frac{r_{D}}{2}$ $r_{D} = \frac{r_{A}}{3} + \frac{r_{B}}{2}$

to \ from	Α	B	С	D
A	0	1/2	1	0
В	1/3	0	0	1/2
С	1/3	0	0	1/2
D	1/3	1/2	0	0

Innovation: What pages would a "random Web surfer" end up at?
To Start, all are equally likely at ¼: ends up at D
C and B are then equally likely: ->D->B=¼*½; ->D->C=¼*½

View 2: Matrix Formulation

 $1 = r_A + r_B + r_C + r_D$



to \ from	Α	B	С	D
A	0	1/2	1	0
В	1/3	0	0	1/2
С	1/3	0	0	1/2
D	1/3	1/2	0	0

Innovation: What pages would a "random Web surfer" end up at?
To Start, all are equally likely at ¼: ends up at D
C and B are then equally likely: ->D->B=¼*½; ->D->C=¼*½
Ends up at C: then A is only option: ->D->C->A = ¼*½*1

View 2: Matrix Formulation

$$1 = r_A + r_B + r_C + r_D$$



to \ from	Α	B	С	D
A	0	1/2	1	0
В	1/3	0	0	1/2
С	1/3	0	0	1/2
D	1/3	1/2	0	0

View 2: Matrix Formulation $1 = r_A + r_B + r_C + r_D$



to \ from	A	В	С	D
A	0	1/2	1	0
В	1/3	0	0	1/2
С	1/3	0	0	1/2
D	1/3	1/2	0	0

R

View 2: Matrix Formulation $1 = r_A + r_B + r_C + r_D$



to \ from	Α	B	С	D
A	0	1/2	1	0
В	1/3	0	0	1/2
С	1/3	0	0	1/2
D	1/3	1/2	0	0



View 2: Matrix Formulation $1 = r_A + r_B + r_C + r_D$



to \ from	Α	B	С	D
A	0	1/2	1	0
В	1/3	0	0	1/2
С	1/3	0	0	1/2
D	1/3	1/2	0	0

R

Innovation: What pages would a "random Web surfer" end up at? To start: N=4 nodes, so $r = [\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}]$

View 2: Matrix Formulation

 $1 = r_A + r_B + r_C + r_D$



to \ from	Α	B	С	D
A	0	1/2	1	0
В	1/3	0	0	1/2
С	1/3	0	0	1/2
D	1/3	1/2	0	0

Innovation: What pages would a "random Web surfer" end up at? To start: N=4 nodes, so r = [%, %, %, %, %]after 1st iteration: $M \cdot r = [3/8, 5/24, 5/24, 5/24]$ В **View 2: Matrix Formulation** $1 = r_A + r_B + r_C + r_D$ $r_{A} = \frac{r_{B}}{2} + \frac{r_{C}}{1}$ $r_{B} = \frac{r_{A}}{3} + \frac{r_{D}}{2}$ $r_{C} = \frac{r_{A}}{3} + \frac{r_{D}}{2}$ $r_{D} = \frac{r_{A}}{3} + \frac{r_{B}}{2}$ to \ from B С D Α 1/2 Α 0 1 0 1/31/2 B 0 0 С 1/30 0 1/2 D 1/31/2 0 0 Transition Matrix, M

Innovation: What pages would a "random Web surfer" end up at? To start: N=4 nodes, so r = [%, %, %, %, %]after 1st iteration: $M \cdot r = [3/8, 5/24, 5/24, 5/24]$ after 2nd iteration: $M(M \cdot r) = M^2 \cdot r = [15/48, 11/48, ...]$ В **View 2: Matrix Formulation** $1 = r_A + r_B + r_C + r_D$ $r_{A} = \frac{r_{B}}{2} + \frac{r_{C}}{1}$ $r_{B} = \frac{r_{A}}{3} + \frac{r_{D}}{2}$ $r_{C} = \frac{r_{A}}{3} + \frac{r_{D}}{2}$ $r_{D} = \frac{r_{A}}{3} + \frac{r_{B}}{2}$ to \ from B С Α D 1/2 Α $\mathbf{0}$ 1 $\mathbf{0}$ 1/31/2 B 0 0 С 1/30 0 1/2 D 1/31/2 0 0





As err_norm gets smaller we are moving toward: $r = M \cdot r$

View 3: Eigenvectors:

Power iteration algorithm

As err_norm gets smaller we are moving toward: $r = M \cdot r$



As err_norm gets smaller we are moving toward: $r = M \cdot r$



View 4: Markov Process

Where is surfer at time t+1? $p(t+1) = M \cdot p(t)$

Suppose: p(t+1) = p(t), then p(t) is a *stationary distribution* of a *random walk*.

Thus, r is a stationary distribution. Probability of being at given node.

View 4: Markov Process

Where is surfer at time t+1? $p(t+1) = M \cdot p(t)$

Suppose: p(t+1) = p(t), then p(t) is a *stationary distribution* of a *random walk*.

Thus, r is a stationary distribution. Probability of being at given node.

aka 1st order Markov Process

- Rich probabilistic theory. One finding:
 - Stationary distributions have a unique distribution if:
 - No "dead-ends": a node can't propagate its rank
 - No *"spider traps"*: set of nodes with no way out.

View 4: Markov Process - Problems for vanilla PI



to \ from	Α	В	С	D
А	0	0	1	0
В	1/3	0	0	1
С	1/3	0	0	0
D	1/3	0	0	0

What would *r* converge to?

aka 1st order Markov Process

- Rich probabilistic theory. One finding:
 - Stationary distributions have a unique distribution if:
 - No "*dead-ends*": a node can't propagate its rank
 - No *"spider traps"*: set of nodes with no way out.

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А	0	0	1	0
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А	0	0	1	0
В	1/3	0	0	1
С	1/3	0	0	0
D	1/3	1	0	0

What would *r* converge to?

aka 1st order Markov Process

- Rich probabilistic theory. One finding:
 - Stationary distributions have a unique distribution if:

same node doesn't repeat at regular intervals columns sum to 1 non-zero chance of going to any other node

The "Google" PageRank Formulation

- 1. Follow a random link (probability, $\beta = \sim .85$)
- 2. Teleport to a random node (probability, $1-\beta$)





The "Google" PageRank Formulation

- 1. Follow a random link (probability, $\beta = \sim.85$)
- 2. Teleport to a random node (probability, $1-\beta$)

	to \ from	Α	В	С	D
Y	А	0	0	1	0
	В	1⁄3	0	0	1
)	С	1⁄3	0	0	0
	D	1⁄3	1	0	0



The "Google" PageRank Formulation

- 1. Follow a random link (probability, $\beta = \sim.85$)
- 2. Teleport to a random node (probability, $1-\beta$)

	to \ from	А	В	С	D
Y	A	0	0+.15*1⁄4	1	0+.15*1⁄4
	В	1⁄3	0+.15*1⁄4	0	.85*1+.15*1⁄4
)	С	1⁄3	0+.15*1⁄4	0	0+.15*1⁄4
	D	1⁄3	.85*1 +.15* ¹ ⁄4	0	0+.15*1⁄4



The "Google" PageRank Formulation

- 1. Follow a random link (probability, $\beta = \sim.85$)
- 2. Teleport to a random node (probability, $1-\beta$)

to \ from	Α	В	С	D
А	0+.15*1⁄4	0+.15*1⁄4	85*1+. <mark>15</mark> *¼	0+.15*1⁄4
В	.85* ¹ ⁄ ₃ +.15* ¹ ⁄ ₄	0+.15*1⁄4	0+.15*1⁄4	.85*1+. <mark>15</mark> *¼
С	.85* ¹ ⁄ ₃ +.15* ¹ ⁄ ₄	0+.15*1⁄4	0+.15*1⁄4	0+.15*1⁄4
D	.85* ¹ ⁄ ₃ +.15* ¹ ⁄ ₄	.85*1+.15*1⁄4	0+.15*1⁄4	0+.15*1⁄4



The "Google" PageRank Formulation

- 1. Follow a random link (probability, $\beta = \sim.85$)
- 2. Teleport to a random node (probability, $1-\beta$)



to \ from	Α	В	С	D
Α	0	0	1	0
В	1⁄3	0	0	1
С	1⁄3	0	0	0
D	1⁄3	0	0	0

The "Google" PageRank Formulation

- 1. Follow a random link (probability, $\beta = \sim.85$)
- 2. Teleport to a random node (probability, $1-\beta$)



to \ from	Α	В	С	D
Α	0	1⁄4	1	0
В	1⁄3	1⁄4	0	1
С	1⁄3	1⁄4	0	0
D	1⁄3	1⁄4	0	0

The "Google" PageRank Formulation

- 1. Follow a random link (probability, $\beta = \sim.85$)
- 2. Teleport to a random node (probability, $1-\beta$)



to \ from	Α	В	С	D
Α	0	.85*1⁄4+.15*1⁄4	1	0
B	1⁄3	.85*1⁄4+.15*1⁄4	0	1
С	1⁄3	.85*1⁄4+.15*1⁄4	0	0
D	1⁄3	.85*1⁄4+.15*1⁄4	0	0

The "Google" PageRank Formulation

Add teleportation: At each step, two choices

- 1. Follow a random link (probability, $\beta = \sim.85$)
- 2. Teleport to a random node (probability, $1-\beta$)

(Teleport from a dead-end has probability 1)



to \ from	Α	В	С	D
A	0+.15*1⁄4	1*1⁄4	85*1+. <mark>15</mark> *¼	0+.15*1⁄4
В	.85*1⁄3+.15*1⁄4	1*1⁄4	0+.15*1⁄4	.85*1+. <mark>15</mark> *¼
С	.85*1⁄3+.15*1⁄4	1*1⁄4	0+.15*1⁄4	0+.15*1⁄4
D	.85*1⁄3+.15*1⁄4	1*1⁄4	0+.15*1⁄4	0+.15*1⁄4

Teleportation, as Flow Model:

$$r_{j} = \sum_{i \to j} \beta \frac{r_{i}}{d_{i}} + (1 - \beta) \frac{1}{N}$$
(Brin and Page, 1998)



to \ from	Α	В	С	D
A	0+.15*1⁄4	1*1⁄4	85*1+. <mark>15</mark> *¼	0+.15*1⁄4
В	.85* ¹ ⁄ ₃ +.15* ¹ ⁄ ₄	1*1⁄4	0+.15*1⁄4	.85*1+. <mark>15</mark> *¼
С	.85* ¹ ⁄ ₃ +.15* ¹ ⁄ ₄	1*1⁄4	0+.15*1⁄4	0+.15*1⁄4
D	.85*1⁄3+.15*1⁄4	1*1⁄4	0+.15*1⁄4	0+.15*1⁄4



Teleportation, as Flow Model:

$$r_j = \sum_{i \to j} \beta \frac{r_i}{d_i} + (1 - \beta) \frac{1}{N}$$
(Brin and Page, 1998)

Teleportation, as Matrix Model: $M' = \beta M + (1 - \beta) \left[\frac{1}{N}\right]_{N \times N}$



to \ from	А	В	С	D
A	0+.15*1⁄4	1*1⁄4	85*1+. <mark>15</mark> *¼	0+.15*1⁄4
В	.85* ¹ ⁄ ₃ +.15* ¹ ⁄ ₄	1*1⁄4	0+.15*1⁄4	.85*1+. <mark>15</mark> *¼
С	.85* ¹ ⁄ ₃ +.15* ¹ ⁄ ₄	1*1⁄4	0+.15*1⁄4	0+.15*1⁄4
D	.85*1⁄3+.15*1⁄4	1*1⁄4	0+.15*1⁄4	0+.15*1⁄4

Teleportation, as Flow Model:

$$r_j = \sum_{i \to j} \beta \frac{r_i}{d_i} + (1 - \beta) \frac{1}{N}$$
(Brin and Page, 1998)

Teleportation, as Matrix Model: $M' = \beta M + (1 - \beta) \left[\frac{1}{N}\right]_{N \times N}$

to \ from	A	В	С	D
A	0+.15*1⁄4	.85*1⁄4+.15*1⁄4	85*1+.15*1⁄4	0+.15*1⁄4
В	.85*1⁄3+.15*1⁄4	.85*1⁄4+.15*1⁄4	0+.15*1⁄4	.85*1+.15*¼
С	.85*1⁄3+.15*1⁄4	.85*1⁄4+.15*1⁄4	0+.15*1⁄4	0+.15*1⁄4
D	.85*1⁄3+.15*1⁄4	.85*1⁄4+.15*1⁄4	0+.15*1⁄4	0+.15*1⁄4



Teleportation, as Flow Model:

$$r_j = \sum_{i \to j} \beta \frac{r_i}{d_i} + (1 - \beta) \frac{1}{N}$$
(Brin and Page, 1998)

Teleportation, as Matrix Model: $M' = \beta M + (1 - \beta) \left[\frac{1}{N}\right]_{M}$

Steps:

- 1. Compute M
- 2. Add 1/N to all dead-ends.
- 3. Convert *M* to *M*'
- 4. Run Power Iterations.

to \ from	Α	В	С	D
Α	0+.15*1⁄4	1*1⁄4	85*1+.15*¼	0+.15*1⁄4
В	.85*1⁄3+.15*1⁄4	1*1⁄4	0+.15*1⁄4	.85*1+.15*¼
С	.85*1⁄3+.15*1⁄4	1*1⁄4	0+.15*1⁄4	0+.15*1⁄4
D	.85*1⁄3+.15*1⁄4	1*1⁄4	0+.15*1⁄4	0+.15*1⁄4





Summary

- Flow View: Link Voting
- Matrix View: Linear Algebra
 Eigenvectors View
- Markov Process View
- How to remove:
 - Dead Ends
 - Spider Traps

In practice, sparse matrix, implement teleportation functionally rather than update M'