Streaming Algorithms

CSE 545 - Spring 2017

Big Data Analytics -- The Class

We will learn:

- to analyze different types of data:
  - high dimensional
  - graphs
  - infinite/never-ending
  - labeled
- to use different models of computation:
  - MapReduce
  - streams and online algorithms
  - single machine in-memory
  - *Spark*

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Motivation

One often does not know when a set of data will end.

- Can not store
- Not practical to access repeatedly
- Rapidly arriving
- Does not make sense to ever “insert” into a database

Can not fit on disk but would like to generalize / summarize the data?

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Examples: Google search queries

Satellite imagery data

Text Messages, Status updates

Click Streams

Stream Queries

Standing Queries: Stored and permanently executing.

Ad-Hoc:

One-time questions
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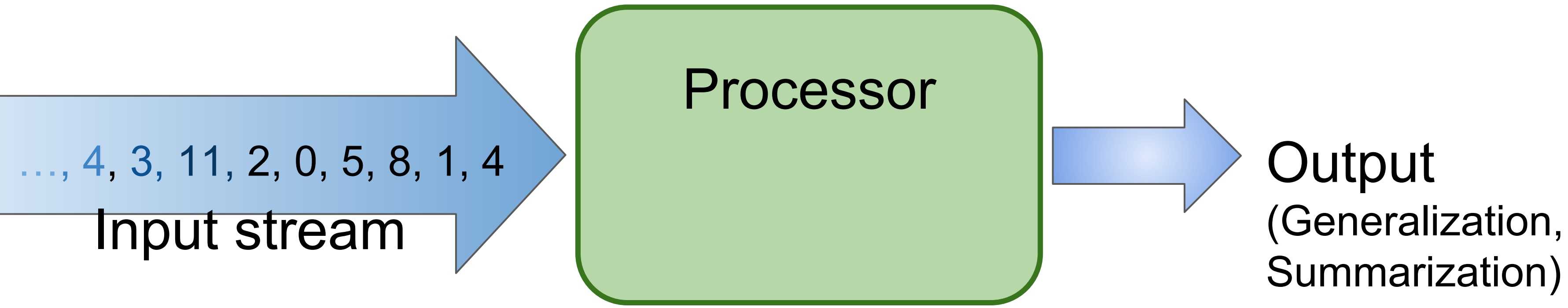
E.g. How would you handle:

What is the mean of values seen so far?

We will cover the following algorithms:

- Sampling
- Filtering Data
- Count Distinct Elements
- Counting Moments

General Stream Processing Model



A stream of records
(also often referred to as “elements” or “tuples”)

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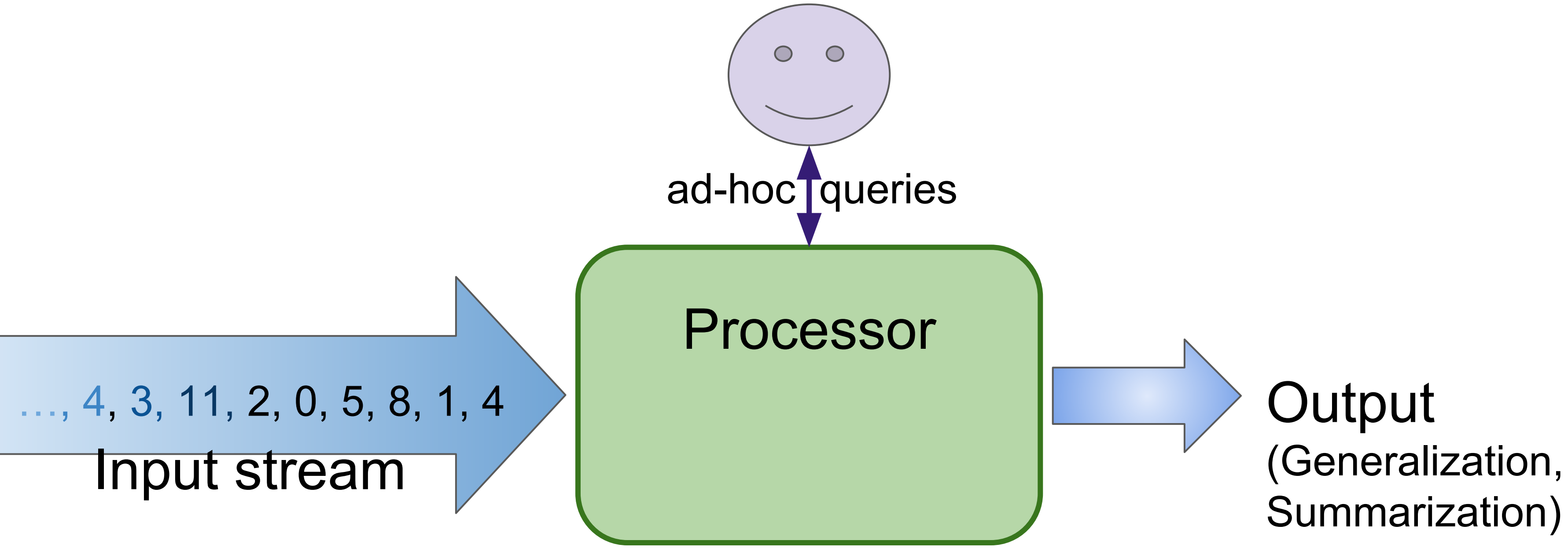

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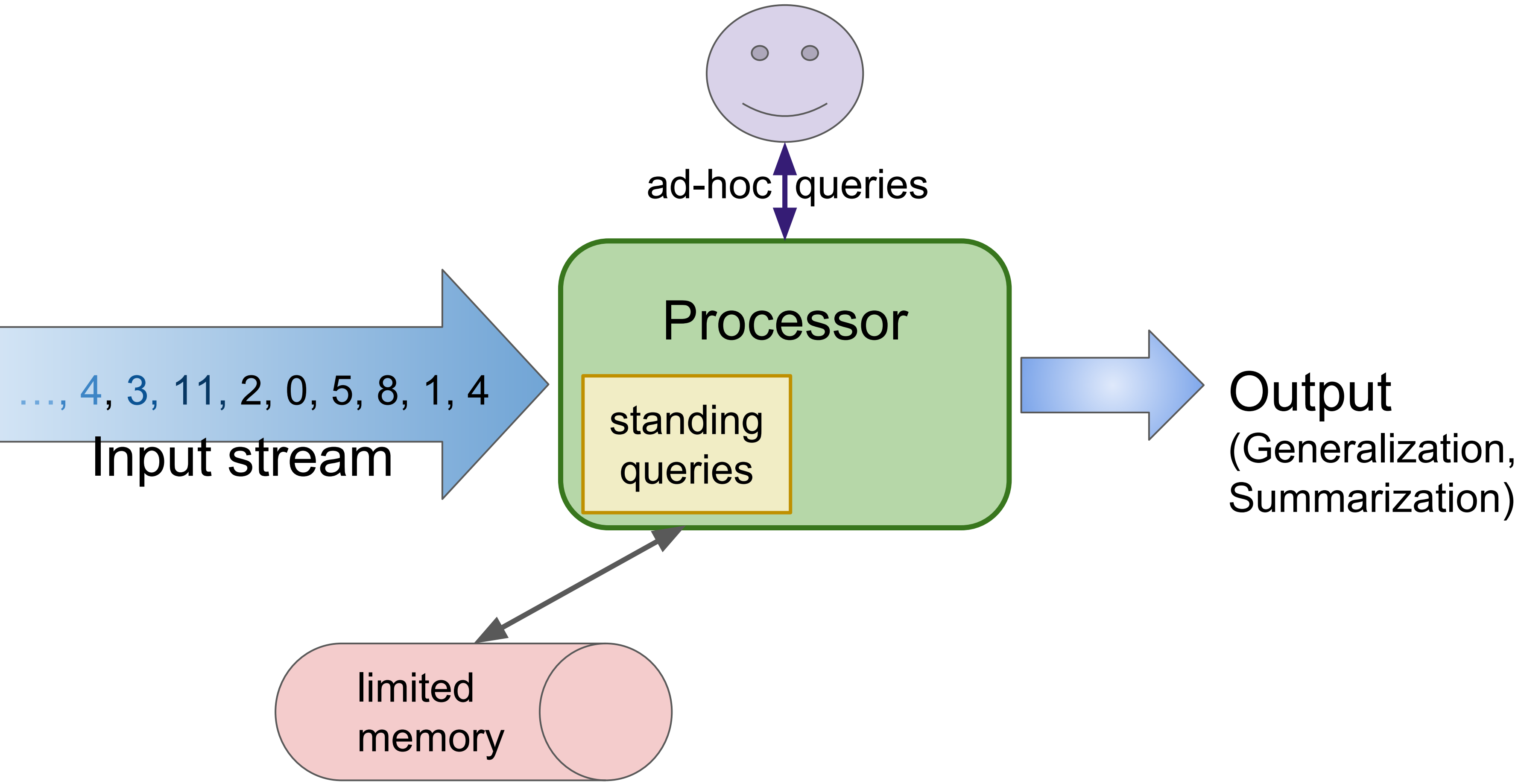

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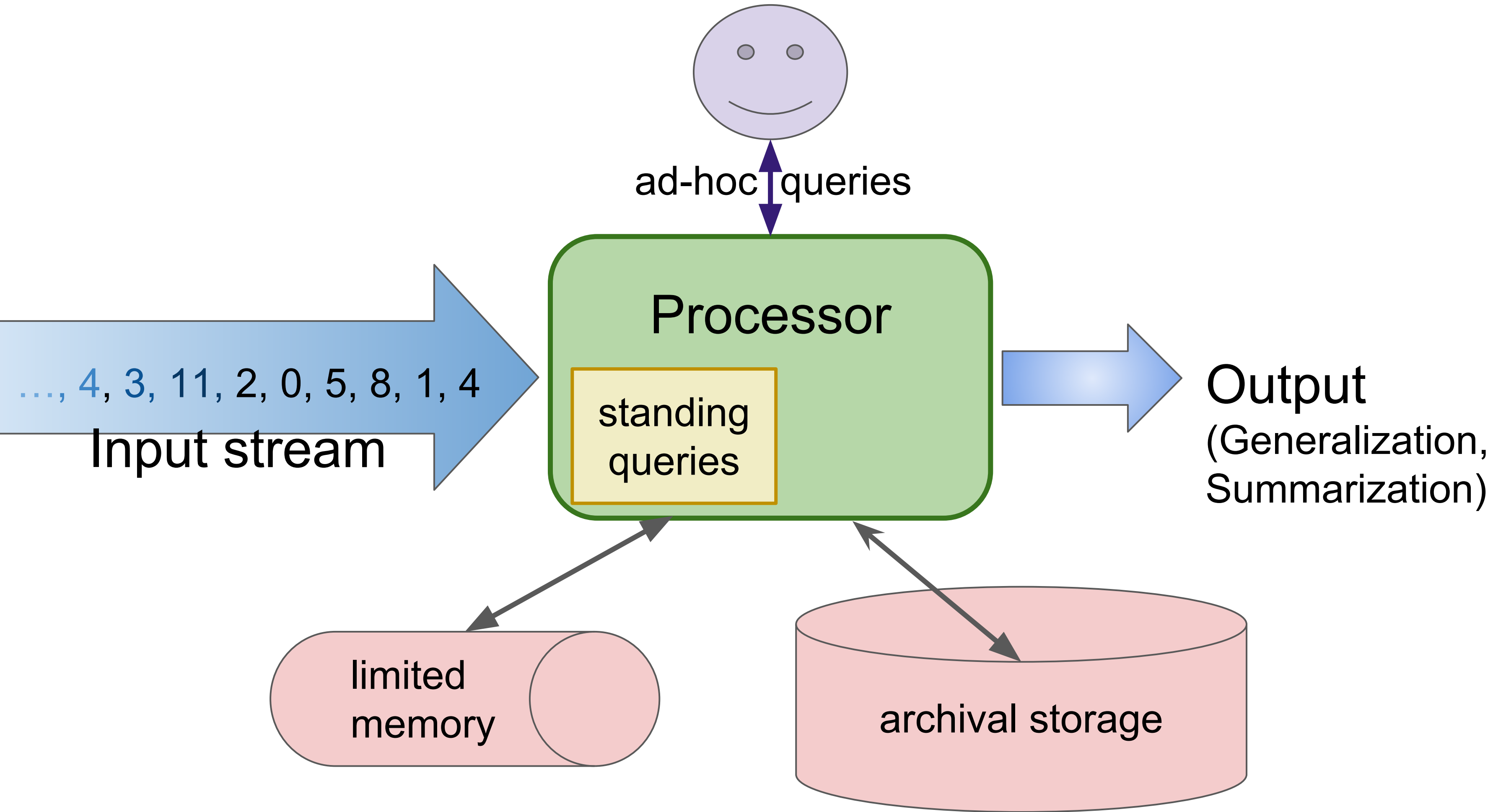

Sampling and Filtering Data

Sampling: Create a random sample for statistical analysis.

Basic Idea: generate random number; if < sample% keep

Problem: records/rows usually are not units-of-analysis for statistical analyses

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- Assume provided some key as unit-of analysis to sample over
    - E.g. ip_address, user_id, document_id, ...etc.…

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  - E.g. ip_address, user_id, document_id, ...etc.…
- Want 1/20th of all “keys” (e.g. users)
  - Hash to 20 buckets; bucket 1 is “in”; others are “out”
  - Note: do not need to store anything (except hash functions); may be part of standing query

Sampling and Filtering Data

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  - **Given:**
    - |S| keys to filter; will be mapped to |B| bits
    - hashes = $h_1, h_2, \ldots, h_k$ independent hash functions

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        - |S| keys to filter; will be mapped to |B| bits
        - hashes = $h_{1,}$ h_2, …, h_k independent hash functions
    - <u>**Algorithm**</u>

```
set all B to 0
for each i in hashes, for each s in S:
set B[$h_i$(s)] = 1
… #usually embedded in other code
while key x arrives next in stream
    if B[$h_i$(x)] == 1 for all $i$ in hashes:
        do as if x is in S
    else: do as if x not in S
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What is the probability of a false-positive?

What fraction of |B| are 1s?

Like throwing |S| * k darts at n targets.
1 dart: 1/n;
d darts: $(1 - 1/n)^d$ = prob of 0
= $e^{-d/n}$ faction are **0s**

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$$\mathbf{(1 - e^{-(|S|*k)/n})^k}$$

Note: Can expand S as stream continues as long as |B| has room (e.g. adding verified email addresses)

Counting Moments

Moments:

- Suppose m_i is the count of distinct element i in the data
- The kth moment of the stream is $\sum_{i \in \mathrm{Set}} m_i^k$

- 0th moment: count of distinct elements
- 1st moment: length of stream
- 2nd moment: sum of squares
  (measures *uneveness;* related to variance)

Counting Moments

0th moment
One Solution: Just keep a set (hashmap, dictionary, heap)

Problem: Can’t maintain that many in memory; disk storage is too slow

- 0th moment: count of distinct elements
- 1st moment: length of stream
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  (measures *uneveness;* related to variance)

Counting Moments

0th moment

Streaming Solution: Flajolet-Martin Algorithm

```
Pick a hash, $h$, to map each of n elements to $\log_2 n$ bits
R = 0 #potential max number of zeros at tail
for each stream element, e:
    r(e) = num of trailing 0s from $h$(e)
    R = r(e) if r(e) > R
estimated_distinct_elements = $2^R$
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Unstable in practice.

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Problem:
Unstable in practice.

Solution:
1. partition into groups
2. Take mean in group
3. Take median of means

- 0th moment: count of distinct elements
- 1st moment: length of stream
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  (measures *uneveness;* related to variance)

Counting Moments

1st moment
Streaming Solution: Simply keep a counter

- 0th moment: count of distinct elements
- **1st moment: length of stream**
- 2nd moment: sum of squares (measures *uneveness* related to variance)