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# A Denotational Model for Mobile Many-to-Many Data-flow Networks

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## Abstract

We present a fully abstract, denotational model for mobile, timed, nondeterministic data-flow networks whose components communicate in a many-to-many fashion. In this model components and networks of components are represented by sets of stream processing functions. Each stream processing function is required to be strongly guarded and generic. A stream processing function is strongly guarded if it is contractive with respect to the standard metric on streams. This property guarantees the existence of unique fix-points. The genericity property can be thought of as an invariant, or alternatively, a privacy requirement, that is satisfied by any mobile system. It guarantees that a function never accesses, depends on or forwards a port whose name it does not already know. Our model allows the description of a wide variety of networks — in particular, the description of unbounded nondeterministic networks. We demonstrate some features of our model by specifying a mobile telephone network.

## 1 Introduction

One of the most prominent theories for interactive computation is the theory of data-flow networks. In this theory, an interactive system is represented by a network of autonomous components which communicate solely by asynchronous transmission of messages via directed channels.

A very elegant model for static, deterministic data-flow networks whose components communicate in a point-to-point fashion, was given by Kahn in [Kah74]. Despite of its elegant foundation, this class of networks is, however, too restrictive for many practical applications. In this paper we extend Kahn's model in a number of ways.

Firstly, contrary to [Kah74], we model nondeterministic behavior. Like Park [Par83], Broy [Bro87] and Russell [Rus90], we represent nondeterministic data-flow networks by sets of stream processing functions. However, in contrast with [Par83] and [Bro87], our model is fully abstract. This is achieved, by considering only sets of functions which are closed with respect to the external observations. The closure idea was used by [Rus90] for the same purpose. However, contrary to [Rus90], we use a timed model and a different notion of observation. This allows us to describe a considerably greater class of networks. In

particular, we can describe all fair merge components discussed in [PS92]. In fact, we can describe any liveness property that can be expressed in standard property oriented specification languages for distributed systems [CM88], [Lam91], [BDD<sup>+</sup>93]. Moreover, since our model is fully abstract, we obviously avoid the expressiveness problem known as the Brock/Ackermann anomaly [BA81].

Secondly, contrary to [Kah74], and also contrary to [Par83], [Bro87], [Rus90], we handle many-to-many communication. This is achieved by building implicit fair merge components into the network operators — the operators used to build networks from basic components. Despite the fact that several components may have both receive and send access to the same channel, each component is described by a set of functions mapping input streams to output streams. The input streams contain the messages sent by the environment; the output streams contain the messages sent by the component. Thus, we model the shared-state interference caused by the many-to-many communication at a very abstract level — the interference is isolated and placed in the network operators.

Thirdly, contrary to [Kah74] and also contrary to [Par83], [Bro87], [Rus90], we describe dynamically reconfigurable or mobile networks — networks in which every component may change its communication partners on the basis of computation and interaction. The formal modeling of mobility has been a very popular research direction in recent years. However, most models published so far have been formalized mainly in operational terms. Examples of such models are the Actor Model [HBS73, AMST92], the  $\pi$ -Calculus [EN86, MPW92a, MPW92b], the Chemical Abstract Machine [BB90], the Rewriting Logic [Mes91] and the Higher Order CCS [Tho89]. On the contrary, our model gives a denotational formalization of mobility. As in the above models, this formalization is based on two assumptions. Firstly, ports are allowed to be passed between the network components. Secondly, the components preserve privacy: their behavior do not depend on ports they do not know. Although it is well understood how to express privacy operationally, there is less denotational understanding. Our solution is to require each stream processing function to be generic. This requirement can be thought of as an invariant satisfied by any mobile system. Informally speaking, the genericity property makes sure that a function accesses, depends on and forwards only ports it already knows. By “the ports it already knows” we basically mean any port which is in its initial interface, it has already received or it has already created itself. Any port created by the function itself is assigned a “new” name taken from a set that is “private” to the component in question.

Although we could have formulated our semantics in a cpo context, we decided to base it on the topological tradition of metric spaces [Niv82, dBZ82, AdBKR89]. Firstly, we wanted to understand the exact relationship between our approach and those based on metric spaces. Secondly, the use of metric spaces seems more natural since our approach is based on infinite streams, and since our strong guardedness constraint, guaranteeing the existence of a unique fix-point, corresponds straightforwardly to contractivity.

The rest of the paper is split into seven sections. Section 2 introduces basic notions like communication histories and stream processing functions. Section 3 formalizes the genericity constraint. Section 4 introduces mobile components. Section 5 describes the modeling of interference. Section 6 is devoted to the network operators. Section 7 gives an example. Section 8 relates our approach to other approaches known from the literature. Finally, there is an appendix reviewing some basic stuff on metric spaces and streams.

## 2 Basic Notions

We model an interactive system by a network of autonomous components communicating via directed channels in a time-synchronous and message-asynchronous way. Time-synchrony is achieved by using a global clock splitting the time axis into discrete, equidistant time units. Message-asynchrony is achieved by allowing arbitrary, but finitely many messages to be sent along a channel in each time unit.

### 2.1 Communication Histories

We model the communication histories of directed channels by infinite streams of finite streams of messages. Each finite stream represents the communication history within a time unit. The first finite stream contains the messages received within the first time unit, the second the messages received within the second time unit, and so on. Since time never halts, any complete communication history is infinite. Let  $M$  be the set of all messages. Then  $[M^*]$  is the set of all complete communication histories, and  $(M^*)^*$  is the set of all partial communication histories<sup>1</sup>.

In the introduction we anticipated that components may transmit *ports*. A port is a *channel name* together with an *access right*, which is either a receive right, represented by  $?$ , or a send right, represented by  $!$ . Hence, if  $N$  is the set of all channel names, then  $?N = \{?n \mid n \in N\}$  is the corresponding set of receive ports, and  $!N = \{!n \mid n \in N\}$  is the corresponding set of send ports. We also write  $?!N$  for  $?N \cup !N$ . We assume that  $?!N \subseteq M$ . Let  $D$  be the set of all messages not contained in the set of ports, i.e.,  $D = M \setminus ?!N$ .

Since components may exchange ports, each component can potentially access any channel in  $N$ . For that reason we model the complete input and output histories of a component by named stream tuples contained in  $N \rightarrow [M^*]$ . The partial ones are modeled by  $N \rightarrow (M^*)^*$ . In the sequel we refer to named stream tuples of these signatures as *named communication histories*. Thus, each named communication history assigns a communication history to each channel name in  $N$ .

### 2.2 Guarded Functions

A *deterministic component* is modeled by a stream processing function

$$f \in (N \rightarrow [M^*]) \rightarrow (N \rightarrow [M^*])$$

mapping complete named communication histories for its input channels to complete named communication histories for its output channels. Note that if no message is communicated along an input channel within a time unit then the empty stream occurs in the communication history for that channel. The lack of this information causes the fair merge anomaly [Kel78].

The functions process their inputs *incrementally* — at any point in time, their outputs do not depend on future inputs. Functions satisfying this constraint are called *weakly*

---

<sup>1</sup>For an arbitrary set  $S$ , by  $S^*$  we denote the set of finite streams over  $S$  and by  $[S]$  the set of infinite streams over  $S$ . See also the appendix.

*guarded*. If the outputs they produce in time unit  $t$  are not only independent of future inputs, i.e., the inputs received during time unit  $t + 1$  or later, but also of the inputs received during time unit  $t$ , then they are called *strongly guarded*. Intuitively, the strongly guarded functions introduce a delay of at least one time unit between input and output. The weakly guarded functions allow in addition zero-delay behavior.

Let  $\theta \downarrow_j$  represent the prefix of  $\theta$  of length  $j$ , i.e., the result of cutting  $\theta$  after the  $j$ 'th time unit. Then weak and strong guardedness can be formalized as below:

**Definition 1 (Guarded functions)** A function  $f \in (N \rightarrow [M^*]) \rightarrow (N \rightarrow [M^*])$  is weakly guarded if

$$\forall \theta, \varphi, j : \theta \downarrow_j = \varphi \downarrow_j \Rightarrow f(\theta) \downarrow_j = f(\varphi) \downarrow_j,$$

and strongly guarded if

$$\forall \theta, \varphi, j : \theta \downarrow_j = \varphi \downarrow_j \Rightarrow f(\theta) \downarrow_{j+1} = f(\varphi) \downarrow_{j+1}. \quad \square$$

We use the arrow  $\xrightarrow{w}$  to characterize sets of weakly guarded functions.

A weakly guarded function is *non-expansive* and a strongly guarded function is *contractive* with respect to the metric on streams (see appendix). As a consequence, by Banach's fix-point theorem, strong guardedness not only replaces the usual monotonicity and continuity constraints of domain theory but also guarantees unique fix-points of feedback loops.

### 3 Genericity

A stream processing function  $f \in (N \rightarrow [M^*]) \rightarrow (N \rightarrow [M^*])$  used to model a component is not only required to be strongly guarded but also to be generic. That it is *generic* means that it *accesses*, *depends on* and *forwards* only ports it already knows. Thus, the genericity property basically characterizes the way the function gains access to ports. In this section we formalize this additional property.

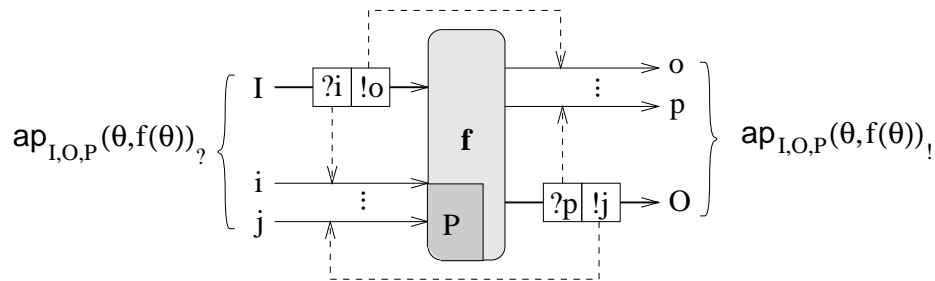


Figure 1: Generic Stream Processing Function

The behavior of a generic function can be described with respect to Figure 1, as follows. Initially,  $f$  receives from a designated set of input channels  $I$  and sends on a designated set of output channels  $O$ . These two sets name the *static* channels or the initial wiring. To make sure that channels *created* by the different components in a network have different



names, each mobile function is assigned a set of *private names*  $P$ . Obviously, this set should be disjoint from the static interface. Thus, we require that  $(I \cup O) \cap P = \emptyset$ .

During the computation, the sets of accessible channels gradually grow. For example, if the function receives a receive port  $?i$  ( $i \notin P$ ) then it may receive from the channel  $i$ , and if it receives a send port  $!o$  ( $o \notin P$ ) then it may send on the channel  $o$ . Similarly, whenever the function sends a send port  $!j$ , whose channel  $j \in P$  it has created itself, it may later receive what is sent along  $j$ , or whenever it sends a receive port  $?p$ , whose channel  $p \in P$  it has created itself, it may itself send messages along  $p$  which eventually are received by the components which received the receive port.

To formally characterize this behavior we introduce some additional notation. For a given point in time  $n$  and named input history  $\theta$ , by  $\text{ap}(\theta, f(\theta))(n)$  we denote the set of *active* input and output ports, and by  $\text{pp}(\theta, f(\theta))(n)$  we denote the set of *passive* input and output ports. At any point in time  $n$  these two sets are disjoint. For any channel  $p$ , by  $\tilde{p}$  we denote its complement, i.e.,  $!p = ?\tilde{p}$  and  $?\tilde{p} = !p$ . A port  $p$  in  $\text{pp}(\theta, f(\theta))(n)$  remains passive as long as its complement port  $\tilde{p}$  is unknown to the environment. After all, if  $\tilde{p}$  is unknown to the environment then there is no way the environment can receive what  $f$  sends along  $p$  if  $p$  is a send port, and there is no way the environment can influence what  $f$  receives on  $p$  if  $p$  is a receive port. Formally:

**Definition 2 (Active and passive ports)** For any  $n, I, O, P, \theta$  and  $\delta$ , let

$$\text{ap}_{I,O,P}(\theta, \delta)(n) \stackrel{\text{def}}{=} \text{ap}_n, \quad \text{pp}_{I,O,P}(\theta, \delta)(n) \stackrel{\text{def}}{=} \text{pp}_n,$$

where  $\text{ap}_n$  and  $\text{pp}_n$  are defined recursively as follows:

$$\begin{aligned} \text{ap}_1 &\stackrel{\text{def}}{=} ?I \cup !O, & \text{pp}_1 &\stackrel{\text{def}}{=} ?!P, \\ \text{ap}_{n+1} &\stackrel{\text{def}}{=} \text{ap}_n \cup r_n \cup g_n, & \text{pp}_{n+1} &\stackrel{\text{def}}{=} \text{pp}_n \setminus g_n, \end{aligned}$$

where

$$\begin{aligned} r_n &= \bigcup_{?i \in \text{ap}_n} \{p \mid p \in \overline{\text{pp}_n} \wedge p \in \theta(i)(n)\}, \\ g_n &= \bigcup_{!i \in \text{pp}_n} \{p \mid p \in \text{pp}_n \wedge \tilde{p} \in \delta(i)(n)\}. \end{aligned}$$

□

The sets  $r_n$  and  $g_n$  are the sets of received and generated ports, respectively.

**Theorem 1** For any  $n, I, O, P, \theta$  and  $\delta$

$$\begin{aligned} \text{pp}_{I,O,P}(\theta, \delta)(1) &\subseteq \text{ap}_{I,O,P}(\theta, \delta)(n) \cup \text{pp}_{I,O,P}(\theta, \delta)(n) \\ \text{pp}_{I,O,P}(\theta, \delta)(n) &\subseteq \text{pp}_{I,O,P}(\theta, \delta)(1) \end{aligned}$$

**Proof:** Follows trivially since anything that is removed from the set of passive ports is added to the set of active ports, and since nothing is ever added to the set of passive ports. □

With  $\text{dom}_{I,O,P}$  and  $\text{rng}_{I,O,P}$  we describe how  $f$  dynamically gain access to ports. The expression  $\text{dom}_{I,O,P}(\theta, f(\theta))(n)$  characterizes the input history that is actually considered by  $f$  in time unit  $n$ . Accordingly,  $\text{rng}_{I,O,P}(\theta, f(\theta))(n)$  is the output history that is actually produced by  $f$  in time unit  $n$ . In the definition of genericity below,  $\text{dom}_{I,O,P}$  and  $\text{rng}_{I,O,P}$  constrain  $f$  to maintain the privacy invariant described above. Since the function  $f$  runs in an open environment this constraint is of course not sufficient unless also the environment

sticks to the rules of the game. There are basically two ways in which the environment of  $f$  can break the rules of the game. Firstly, the environment can send  $f$  a port  $p \in !?P$  which the environment has not yet received from  $f$ , i.e., a port “it does not yet know” because it has not yet been output by  $f$ . Secondly, the environment can send along a channel  $c \in P$  without first having received  $!c$ .

There are two ways to deal with this problem. One alternative is to impose an environment assumption in all definitions characterizing exactly those input histories in which the environment sticks to the rules of the game. The other alternative, which is used in this paper, is to constrain the functions to ignore the input messages which do not respect the privacy restrictions. This is okay, because we are only interested in environments that can be understood as mobile components in accordance with the definition below. Such components will never break the rules of the game. For that reason,  $\text{dom}_{I,O,P}$  and  $\text{rng}_{I,O,P}$  have been defined in such a way that, in addition to their main task of characterizing the actual domain and range of a function, they also correct environment mistakes. Ports sent by the environment that should be unknown to the environment are filtered away. Moreover, messages sent by the environment along channels it does know are removed. Formally:<sup>2</sup>

**Definition 3 (Domain and range)** For any  $n, I, O, P, \theta$  and  $\delta$ , we define

$$\begin{aligned} \text{dom}_{I,O,P}(\theta, \delta)(i)(n) &\stackrel{\text{def}}{=} \begin{cases} (\overline{\text{pp}}_n \cup D) \odot \theta(i)(n) & \text{if } ?i \in \text{ap}_n \\ \epsilon & \text{otherwise} \end{cases} \\ \text{rng}_{I,O,P}(\theta, \delta)(i)(n) &\stackrel{\text{def}}{=} \begin{cases} (\text{pp}_n \cup \text{ap}_n \cup D) \odot \delta(i)(n) & \text{if } !i \in \text{ap}_n \\ \epsilon & \text{otherwise} \end{cases} \end{aligned}$$

where  $\text{ap}_n = \text{ap}_{I,O,P}(\theta, \delta)(n)$  and  $\text{pp}_n = \text{pp}_{I,O,P}(\theta, \delta)(n)$ .  $\square$

**Theorem 2** The functions  $\text{pp}$  and  $\text{ap}$  are strongly guarded, and the functions  $\text{dom}$  and  $\text{rng}$  are weakly guarded.

**Proof:**  $\text{pp}_{I,O,P}(\theta, \delta)(n)$  and  $\text{ap}_{I,O,P}(\theta, \delta)(n)$  depend only on  $\theta \downarrow_{n-1}$  and  $\delta \downarrow_{n-1}$ .  $\text{dom}_{I,O,P}(\theta, \delta)(n)$  and  $\text{rng}_{I,O,P}(\theta, \delta)(n)$  depend only on  $\theta \downarrow_n$  and  $\delta \downarrow_n$ .  $\square$

**Theorem 3** The functions  $\text{dom}$  and  $\text{rng}$  have the following properties:

$$\begin{aligned} \text{dom}_{I,O,P}(\theta, \delta) &= \text{dom}_{I,O,P}(\text{dom}_{I,O,P}(\theta, \delta), \delta) = \text{dom}_{I,O,P}(\theta, \text{rng}_{I,O,P}(\theta, \delta)), \\ \text{rng}_{I,O,P}(\theta, \delta) &= \text{rng}_{I,O,P}(\text{dom}_{I,O,P}(\theta, \delta), \delta) = \text{rng}_{I,O,P}(\theta, \text{rng}_{I,O,P}(\theta, \delta)). \end{aligned}$$

**Proof:** The proof is based on the inductive definitions of  $\text{ap}$  and  $\text{pp}$ .

Induction hypothesis:

$$\begin{aligned} \text{ap}_{I,O,P}(\theta, \delta)(n) &= \text{ap}_{I,O,P}(\text{dom}_{I,O,P}(\theta, \delta), \delta)(n) = \text{ap}_{I,O,P}(\theta, \text{rng}_{I,O,P}(\theta, \delta))(n), \\ \text{pp}_{I,O,P}(\theta, \delta)(n) &= \text{pp}_{I,O,P}(\text{dom}_{I,O,P}(\theta, \delta), \delta)(n) = \text{pp}_{I,O,P}(\theta, \text{rng}_{I,O,P}(\theta, \delta))(n). \end{aligned}$$

To simplify the notation we write:

$$\begin{aligned} \text{ap}_n &= \text{ap}_{I,O,P}(\theta, \delta)(n), \\ \text{ap}'_n &= \text{ap}_{I,O,P}(\text{dom}_{I,O,P}(\theta, \delta), \delta)(n), \quad \text{ap}''_n = \text{ap}_{I,O,P}(\theta, \text{rng}_{I,O,P}(\theta, \delta))(n), \\ \text{pp}_n &= \text{pp}_{I,O,P}(\theta, \delta)(n), \\ \text{pp}'_n &= \text{pp}_{I,O,P}(\text{dom}_{I,O,P}(\theta, \delta), \delta)(n), \quad \text{pp}''_n = \text{pp}_{I,O,P}(\theta, \text{rng}_{I,O,P}(\theta, \delta))(n). \end{aligned}$$

<sup>2</sup>The operator  $\in$  is overloaded to test for containment in a list. Moreover, for any set of ports  $S \subseteq ?!N$ ,  $\overline{S} \stackrel{\text{def}}{=} ?!N \setminus S$  and  $\widetilde{S} \stackrel{\text{def}}{=} \{\tilde{p} \mid p \in S\}$ . When convenient, we also view  $\theta(i)(n)$  as a set.

Base case:  $\text{ap}_1 = \text{ap}'_1 = \text{ap}''_1 = ?I \cup !O$  and  $\text{pp}_1 = \text{pp}'_1 = \text{pp}''_1 = ?!P$ .

Induction Step: By induction hypothesis  $\text{ap}_n = \text{ap}'_n = \text{ap}''_n$  and  $\text{pp}_n = \text{pp}'_n = \text{pp}''_n$ . By definition of  $\text{ap}$  and  $\text{pp}$ :

$$\begin{aligned} \text{ap}_{n+1} &= (\text{ap}_n \cup \bigcup_{?i \in \text{ap}_n} \{c \mid c \in \overline{\text{pp}_n} \wedge c \in \theta(i)(n)\} \cup \\ &\quad \bigcup_{!i \in \text{ap}_n} \{c \mid c \in \text{pp}_n \wedge \tilde{c} \in \delta(i)(n)\}), \\ \text{ap}'_{n+1} &= (\text{ap}'_n \cup \bigcup_{?i \in \text{ap}'_n} \{c \mid c \in \overline{\text{pp}'_n} \wedge c \in \text{dom}_{I,O,P}(\theta, \delta)(i)(n)\} \cup \\ &\quad \bigcup_{!i \in \text{ap}'_n} \{c \mid c \in \text{pp}'_n \wedge \tilde{c} \in \delta(i)(n)\}), \\ \text{ap}''_{n+1} &= (\text{ap}''_n \cup \bigcup_{?i \in \text{ap}''_n} \{c \mid c \in \overline{\text{pp}''_n} \wedge c \in \theta(i)(n)\} \cup \\ &\quad \bigcup_{!i \in \text{ap}''_n} \{c \mid c \in \text{pp}''_n \wedge \tilde{c} \in \text{rng}_{I,O,P}(\theta, \delta)(i)(n)\}), \\ \text{pp}_{n+1} &= \text{pp}_n \setminus \bigcup_{!i \in \text{ap}_n} \{c \mid c \in \text{pp}_n \wedge \tilde{c} \in \delta(i)(n)\}, \\ \text{pp}'_{n+1} &= \text{pp}'_n \setminus \bigcup_{!i \in \text{ap}'_n} \{c \mid c \in \text{pp}'_n \wedge \tilde{c} \in \delta(i)(n)\}, \\ \text{pp}''_{n+1} &= \text{pp}''_n \setminus \bigcup_{!i \in \text{ap}''_n} \{c \mid c \in \text{pp}''_n \wedge \tilde{c} \in \text{rng}_{I,O,P}(\theta, \delta)(i)(n)\}. \end{aligned}$$

By definition of  $\text{dom}$  and  $\text{rng}$ :

$$\begin{aligned} \text{dom}_{I,O,P}(\theta, \delta)(i)(n) &= (\overline{\text{pp}_n} \cup D) \odot \theta(i)(n) && \text{if } ?i \in \text{ap}_n = \text{ap}'_n = \text{ap}''_n, \\ \text{rng}_{I,O,P}(\theta, \delta)(i)(n) &= (\text{pp}_n \cup \text{ap}_n \cup D) \odot \delta(i)(n) && \text{if } !i \in \text{ap}_n = \text{ap}'_n = \text{ap}''_n. \end{aligned}$$

The first union in the definition of  $\text{ap}_{n+1}$ ,  $\text{ap}'_{n+1}$  and  $\text{ap}''_{n+1}$  is taken over  $?i \in \text{ap}_n = \text{ap}'_n = \text{ap}''_n$ . As a consequence

$$\text{dom}_{I,O,P}(\theta, \delta)(i)(n) = (\overline{\text{pp}_n} \cup D) \odot \theta(i)(n)$$

inside this union. It is enough to show that

$$c \in \theta(i)(n) \Leftrightarrow c \in (\overline{\text{pp}_n} \cup D) \odot \theta(i)(n)$$

under the assumption that  $c \notin \text{ap}_n$  and  $c \in \overline{\text{pp}_n}$ . This follows trivially since  $\tilde{c} \in \text{pp}_1 \Leftrightarrow c \in \text{pp}_1$ , Theorem 1 and the two assumptions imply that  $c \notin \overline{\text{pp}_n}$ .

The second union in the definition of  $\text{ap}_{n+1}$ ,  $\text{ap}'_{n+1}$  and  $\text{ap}''_{n+1}$  is taken over  $!i \in \text{ap}_n = \text{ap}'_n = \text{ap}''_n$ . As a consequence

$$\text{rng}_{I,O,P}(\theta, \delta)(i)(n) = (\text{pp}_n \cup \text{ap}_n \cup D) \odot \delta(i)(n)$$

inside this union. It is enough to show that

$$\tilde{c} \in \delta(i)(n) \Leftrightarrow \tilde{c} \in (\text{pp}_n \cup \text{ap}_n \cup D) \odot \delta(i)(n)$$

under the assumption that  $c \in \text{pp}_n$ . This follows trivially since  $\tilde{c} \in \text{pp}_1 \Leftrightarrow c \in \text{pp}_1$ , Theorem 1 and the assumption imply that  $\tilde{c} \in \text{pp}_n \cup \text{ap}_n$ . This proves that  $\text{ap}_{n+1} = \text{ap}'_{n+1} = \text{ap}''_{n+1}$ . That  $\text{pp}_{n+1} = \text{pp}'_{n+1} = \text{pp}''_{n+1}$  follows accordingly.

Finally, because of these equalities,  $\text{dom}_{I,O,P}(\theta, \delta)(i)(n)$  simplifies to  $\theta(i)(n)$  inside the definition of  $\text{dom}_{I,O,P}$  and  $\text{rng}_{I,O,P}(\theta, \delta)(i)(n)$  simplifies to  $\delta(i)(n)$  inside the definition of  $\text{rng}_{I,O,P}$ . This immediately proves the theorem.  $\square$

We can now characterize what it means for a function to be generic.

**Definition 4 (Generic functions)** A function  $f \in (N \rightarrow [M^*]) \rightarrow (N \rightarrow [M^*])$  is called generic with respect to the initial wiring  $(I, O)$  and the private names  $P$  iff:

$$\forall \theta : f(\theta) = f(\text{dom}_{I,O,P}(\theta, f(\theta))) = \text{rng}_{I,O,P}(\theta, f(\theta)). \quad \square$$

We use the  $Mob(I, O, P)$  to characterize the set of all strongly guarded functions that are generic with respect to  $(I, O, P)$ . In the sequel we refer to these functions as mobile.

## 4 Mobile Components

We model a nondeterministic component by a set of mobile functions  $F$ . Any pair  $(\theta, f(\theta))$ , where  $f \in F$ , is a possible *behavior* of the component. Intuitively, for any input history each mobile function  $f \in F$  represents one possible nondeterministic behavior. For any set of functions  $F$  we define  $\mathcal{O}(F)$  to be the set of all behaviors of  $F$ , i.e.,  $\mathcal{O}(F) = \{(x, f(x)) \mid f \in F\}$ .

Different sets of mobile functions may have the same set of behaviors. The reason is that for some sets of mobile functions we may find additional mobile functions which can be understood as combinations of the functions already in the set. For example, we may find a mobile function  $g$  which for one input history behaves as the function  $f \in F$  and for another input history behaves as the function  $f' \in F$ , and so on. This means, a model in which a nondeterministic component is represented by an arbitrary set of mobile functions, is too distinguishing, and consequently, not *fully abstract*. To achieve full abstraction we consider only closed sets, i.e., sets  $F$ , where each combination of functions in  $F$ , which gives a mobile function, is also in  $F$ .

**Definition 5 (Mobile components)** *A mobile component, with initial wiring  $(I, O)$  and private names  $P$ , where  $P \cap (I \cup O) = \emptyset$ , is modeled by a nonempty set of stream processing functions*

$$F \subseteq Mob(I, O, P)$$

that is closed in the sense that for any  $f \in Mob(I, O, P)$

$$(\forall \theta \in (N \rightarrow [M^*]) : \exists f' \in F : f(\theta) = f'(\theta)) \Rightarrow f \in F. \quad \square$$

It follows straightforwardly that if  $F_1$  and  $F_2$  are mobile components then  $F_1 = F_2$  iff  $\mathcal{O}(F_1) = \mathcal{O}(F_2)$ . Thus, our notion of a component is fully abstract with respect to the corresponding set of behaviors. Note the relationship to [Rus90]. That our semantics is fully abstract with respect to  $\mathcal{O}$  is of course trivial. Nevertheless, this notion of observation characterizes the expectations one has to a semantics dealing with time.

Note that if  $c \in N$  and  $(\theta, \delta) \in \mathcal{O}(F)$  then the communication history  $\theta(c)$  contains the history of all messages sent by the environment along the channel  $c$ . On the other hand, the communication history  $\delta(c)$  contains the history of all messages sent by  $F$  along the channel  $c$ . Thus, although we model many-to-many communication, each component can be understood as a pure relation between input and output histories where each input history contains only messages sent by the environment, and each output history contains only messages sent by the component.

## 5 Interference and Implicit Merge

Interference occurs when two mobile components send on the same channel. As should be clear from the discussion in the previous section, interference is not considered at the

component level. This allows us to describe each component in a very abstract and, in our opinion, intuitive way. Instead, interference is modeled by building implicit merge components into the network operators. Each such merge component  $MC$  takes two named communication histories as input and yields their merge as output. Since  $MC$  is hidden in the semantics, it should neither add nor reduce delay. Remember, we want to be able to express timing constraints. This means that its output history during time unit  $k$  should be a merge of the two finite streams characterizing the input histories in time unit  $k$ . Moreover,  $MC$  should not fix the interleaving. Thus, any interleaving of the messages received within a time unit should be allowed. This means that  $MC$  is nondeterministic.  $MC$  is now formally defined in two steps.

We first characterize what it means for a finite stream to be a merge of two finite streams. To do so, we introduce some operators. For any tuple  $t$  we use  $\pi_1(t)$  to denote the first component of  $t$ ; for any stream  $s$  we use  $\#s$  to denote the length of  $s$ . For a set of messages  $A$  and a stream of messages  $s$ ,  $A \odot s$  denotes the stream we obtain by removing any message in  $s$  that is not contained in  $A$ . This operator is overloaded to sets of pairs of messages  $A \times B$  and pairs of streams of messages  $(r, s)$  in a straightforward way: for each  $j$ ,  $(r(j), s(j))$  is filtered away iff it is not in  $A \times B$ .

**Definition 6 (The merge relation on finite streams)** *Let  $FM$  be the function such that:*

$$\begin{aligned}
 FM &\in M^* \times M^* \rightarrow \mathcal{P}(M^*) \\
 FM(s_1, s_2) &= \{s \mid \exists p \in \{1, 2\}^* : \#p = \#s \wedge \forall i \in \{1, 2\} : \\
 &\quad s_i = \pi_1((M \times \{i\}) \odot (s, p))\} \quad \square
 \end{aligned}$$

It is now straightforward to define the implicit merge component.

**Definition 7 (The merge component)** *The merge component is a set of weakly guarded functions defined as follows:*

$$\begin{aligned}
 MC &\subseteq (N \rightarrow [M^*]) \times (N \rightarrow [M^*]) \xrightarrow{w} (N \rightarrow [M^*]) \\
 MC &= \{f \in (N \rightarrow [M^*]) \times (N \rightarrow [M^*]) \xrightarrow{w} (N \rightarrow [M^*]) \mid \\
 &\quad \forall \varphi, \psi, i : N, n \in \mathbb{N} : f(\varphi, \psi)(i)(n) \in FM(\varphi(i)(n), \psi(i)(n))\} \quad \square
 \end{aligned}$$

Note that the functions contained in  $MC$  are only required to be weakly guarded. Nevertheless, due to the way this implicit component is used in the definitions of network operators, this does not lead to any problem.

**Theorem 4**  *$MC$  is a nonempty set.*

**Proof:** Trivial □

## 6 Network Operators

In this section we introduce three network operators, namely an operator for *parallel composition with mutual feedback*, a *feedback* operator and a *hiding* operator.

## 6.1 Many-to-Many Composition

The parallel composition with mutual feedback of two components  $F_1$  and  $F_2$  is characterized by the network in Figure 2.

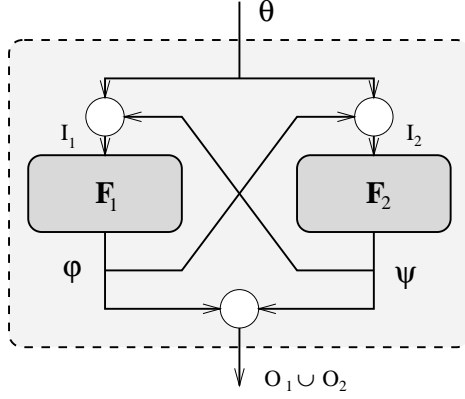


Figure 2: Many-to-many communication

**Definition 8 (Many-to-many composition)** *Given two mobile components*

$$F_1 \subseteq \text{Mob}(I_1, O_1, P_1), \quad F_2 \subseteq \text{Mob}(I_2, O_2, P_2),$$

where  $P_1 \cap (P_2 \cup I_2 \cup O_2) = P_2 \cap (P_1 \cup I_1 \cup O_1) = \emptyset$ . Let

$$I = I_1 \cup I_2, \quad O = O_1 \cup O_2, \quad P = P_1 \cup P_2,$$

then

$$F_1 \oplus F_2 \subseteq \text{Mob}(I, O, P)$$

$$F_1 \oplus F_2 = \{f \in \text{Mob}(I, O, P) \mid \forall \theta : \exists f_1 \in F_1, f_2 \in F_2, m_1, m_2, m_3 \in MC :$$

$$\begin{aligned} f(\theta) &= \text{rng}_{I,O,P}(\theta, \delta) \text{ where} \\ \delta &= m_3(\varphi, \psi), \quad \vartheta = \text{dom}_{I,O,P}(\theta, \delta) \\ \varphi &= f_1(m_1(\vartheta, \psi)), \quad \psi = f_2(m_2(\vartheta, \varphi)) \end{aligned}$$

□

Note the role of  $\text{dom}_{I,O,P}$  and  $\text{rng}_{I,O,P}$  in maintaining privacy. If  $F_1$  sends a private send port  $!p \in !P_1$  on a feedback channel (and it does not send  $?p$ ), then both the environment and  $F_2$  can send along  $p$ , but only  $F_1$  is allowed to receive on  $p$ . As a consequence, the output of  $F_2$  along  $p$  should not be observed by the environment. This is ensured by  $\text{rng}_{I,O,P}$ . Similarly, if  $F_1$  sends a private receive port  $?p \in ?P_1$  on a feedback channel (and it does not send  $!p$ ), then both the environment and  $F_2$  can receive on  $p$ , but only  $F_1$  is allowed to send along  $p$ . As a consequence, the input of  $F_2$  on  $p$  should contain only messages sent by  $F_1$ . This is ensured by  $\text{dom}_{I,O,P}$ .

**Theorem 5**  $F_1 \oplus F_2 \neq \emptyset$ .

**Proof:** Since  $F_1$  and  $F_2$  are mobile components and  $MC$  is not empty we may find functions  $f_1 \in F_1, f_2 \in F_2$  and  $m_1, m_2, m_3 \in MC$ . Based on these functions we construct a function  $f$  which is strongly guarded, generic and satisfies the recursive definition above.

Let  $g$  be defined as follows:

$$g \in ((N \rightarrow [M^*]) \times (N \rightarrow [M^*])) \times (N \rightarrow [M^*]) \rightarrow (N \rightarrow [M^*]) \times (N \rightarrow [M^*])$$

$$g((\varphi, \psi), \theta) = (f_1(m_1(\vartheta, \psi)), f_2(m_2(\vartheta, \varphi))), \quad \text{where } \vartheta = \text{dom}_{I,O,P}(\theta, \delta), \delta = m_3(\varphi, \psi).$$

Theorem 12 and the way  $g$  is defined in terms of strongly and weakly guarded functions imply that  $g$  is strongly guarded. Thus  $\mu g$  is well-defined, in which case Theorem 14 implies that  $\mu g$  is strongly guarded. That the function  $f$  defined below is also strongly guarded follows by a similar argument:

$$f \in (N \rightarrow [M^*]) \rightarrow (N \rightarrow [M^*])$$

$$f(\theta) = \text{rng}_{I,O,P}(\theta, \delta) \quad \text{where } \delta = m_3(\varphi, \psi), \quad (\varphi, \psi) = (\mu g)(\theta)$$

That  $f$  is generic is a consequence of the next two lemmas.

**Lemma 1**  $f(\theta) = f(\text{dom}_{I,O,P}(\theta, f(\theta)))$ .

**Proof:** The idea of the proof is to transform  $f(\theta)$  to  $f(\text{dom}_{I,O,P}(\theta, f(\theta)))$  by using the equalities from Theorem 3. By definition

$$f(\text{dom}_{I,O,P}(\theta, f(\theta))) = \text{rng}_{I,O,P}(\text{dom}_{I,O,P}(\theta, f(\theta)), \delta') \quad \text{where}$$

$$\delta' = m_3(\varphi', \psi'), \quad \vartheta' = \text{dom}_{I,O,P}(\text{dom}_{I,O,P}(\theta, f(\theta)), \delta'),$$

$$\varphi' = f_1(m_1(\vartheta', \psi')), \quad \psi' = f_2(m_2(\vartheta', \varphi')).$$

By Theorem 3 and definition of  $f$

$$\begin{aligned} \text{dom}_{I,O,P}(\theta, \delta) &= \text{dom}_{I,O,P}(\theta, \text{rng}_{I,O,P}(\theta, \delta)) &= \text{dom}_{I,O,P}(\theta, f(\theta)), \\ \text{dom}_{I,O,P}(\theta, \delta) &= \text{dom}_{I,O,P}(\text{dom}_{I,O,P}(\theta, \delta), \delta) &= \text{dom}_{I,O,P}(\text{dom}_{I,O,P}(\theta, f(\theta)), \delta), \\ \text{rng}_{I,O,P}(\theta, \delta) &= \text{rng}_{I,O,P}(\text{dom}_{I,O,P}(\theta, \delta), \delta) &= \text{rng}_{I,O,P}(\text{dom}_{I,O,P}(\theta, f(\theta)), \delta). \end{aligned}$$

Replacing  $\text{dom}_{I,O,P}(\theta, \delta)$  and  $\text{rng}_{I,O,P}(\theta, \delta)$  in the definition of  $f$  we obtain

$$f(\theta) = \text{rng}_{I,O,P}(\text{dom}_{I,O,P}(\theta, f(\theta)), \delta) \quad \text{where}$$

$$\delta = m_3(\varphi, \psi), \quad \vartheta = \text{dom}_{I,O,P}(\text{dom}_{I,O,P}(\theta, f(\theta)), \delta),$$

$$\varphi = f_1(m_1(\vartheta, \psi)), \quad \psi = f_2(m_2(\vartheta, \varphi)).$$

Now the fact that we have unique fix-points implies that  $\delta = \delta'$ ,  $\varphi = \varphi'$ ,  $\psi = \psi'$  and  $\vartheta = \vartheta'$  and consequently that  $f(\theta) = f(\text{dom}_{I,O,P}(\theta, f(\theta)))$ .  $\square$

**Lemma 2**  $f(\theta) = \text{rng}_{I,O,P}(\theta, f(\theta))$ .

**Proof:**

$$\begin{aligned} \text{rng}_{I,O,P}(\theta, f(\theta)) &= \\ \text{rng}_{I,O,P}(\theta, \text{rng}_{I,O,P}(\theta, \delta)) &= \quad \{\text{by definition of } f\} \\ \text{rng}_{I,O,P}(\theta, \delta) &= \quad \{\text{by Theorem 3}\} \\ f(\theta) &= \quad \{\text{by definition of } f\} \end{aligned} \quad \square$$

Finally, since  $\exists f_1, f_2, m_1, m_2, m_3 : \forall \theta : P$  implies  $\forall \theta : \exists f_1, f_2, m_1, m_2, m_3 : P$  it follows that  $f \in F_1 \oplus F_2$ .  $\square$

**Theorem 6**  $F_1 \oplus F_2$  is a mobile component.

**Proof:** That  $F_1 \oplus F_2 \neq \emptyset$  follows from Theorem 5. To see that  $F_1 \oplus F_2$  is closed, let  $f \in \text{Mob}(I, O, P)$ , and assume that

$$\forall \theta : \exists f' \in F_1 \oplus F_2 : f(\theta) = f'(\theta).$$

The definition of  $\oplus$  implies that for any  $\theta$  there are  $f' \in F_1 \oplus F_2$ ,  $f_1 \in F_1, f_2 \in F_2$  and  $m_1, m_2, m_3 \in MC$  such that:

$$f(\theta) = f'(\theta) = \text{rng}_{I,O,P}(\theta, \delta) \quad \text{where} \\ \varphi = f_1(m_1(\vartheta, \psi)), \quad \psi = f_2(m_2(\vartheta, \varphi)), \quad \vartheta = \text{dom}_{I,O,P}(\theta, \delta), \quad \delta = m_3(\varphi, \psi)$$

By the definition of  $\oplus$ , it follows that  $f \in F_1 \oplus F_2$ . □

## 6.2 Feedback

As we have already pointed out, if  $(\theta, \delta)$  is a behavior of a mobile component  $F$  then the communication history  $\theta(c)$  contains the messages sent by the environment along the channel  $c$ , and  $\delta(c)$  contains the messages sent by the component along the channel  $c$ . Clearly, the behavior of  $F$  has only indirect influence on its input via the environment. Thus, there is no direct interference. To allow the component's output to interfere with its input a simple feedback operator can be used. We then get the network pictured in Figure 3. Formally, the feedback operator can be defined as below:

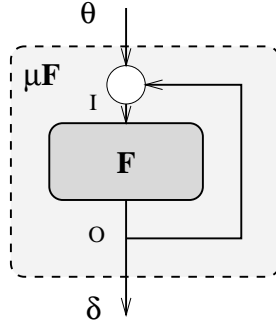


Figure 3: Feedback

**Definition 9 (Feedback)** *Given a mobile component  $F \subseteq \text{Mob}(I, O, P)$ . Then*

$$\mu F \subseteq \text{Mob}(I, O, P)$$

$$\mu F = \{f \in \text{Mob}(I, O, P) \mid \forall \theta : \exists g \in F, m \in MC :$$

$$f(\theta) = \text{rng}_{I,O,P}(\theta, \delta) \quad \text{where} \quad \delta = g(m(\text{dom}_{I,O,P}(\theta, \delta), \delta))\}.$$

□

Note the close relationship between Figure 3 and the definition. If  $I \cap O \neq \emptyset$  it is completely clear that interference will occur. However, interference can take place also if  $I \cap O = \emptyset$ . For example, this is the case if  $F$  receives a receive port  $?c$  to a channel in  $O$ .

**Theorem 7**  $\mu F \neq \emptyset$ .

**Proof:** Since  $F$  is a mobile component and  $MC$  is not empty we may find functions  $g \in F$  and  $m \in MC$ . Based on these functions we construct a function  $f$  which is strongly guarded, generic and satisfies the recursive definition above. Let  $h$  be defined as follows:



$$h \in (N \rightarrow [M^*]) \times (N \rightarrow [M^*]) \rightarrow (N \rightarrow [M^*])$$

$$h(\delta, \theta) = g(m(\text{dom}_{I,O,P}(\theta, \delta), \delta))$$

Theorem 12 and the way  $h$  is defined in terms of strongly and weakly guarded functions imply that  $h$  is strongly guarded. Thus  $\mu h$  is well-defined, in which case Theorem 14 implies that  $\mu h$  is strongly guarded. By a similar argument,  $f$  defined below is also strongly guarded:

$$f(\theta) = \text{rng}_{I,O,P}(\theta, \delta) \quad \text{where} \quad \delta = \mu h(\theta)$$

That  $f$  is generic is a consequence of the next two lemmas.

**Lemma 3**  $f(\theta) = f(\text{dom}_{I,O,P}(\theta, f(\theta)))$ .

**Proof:** The idea of the proof is to transform  $f(\theta)$  to  $f(\text{dom}_{I,O,P}(\theta, f(\theta)))$  by using the equalities from Theorem 3. By definition

$$\begin{aligned} f(\text{dom}_{I,O,P}(\theta, f(\theta))) &= \text{rng}_{I,O,P}(\text{dom}_{I,O,P}(\theta, f(\theta)), \delta') \quad \text{where} \\ \delta' &= g(m(\text{dom}_{I,O,P}(\text{dom}_{I,O,P}(\theta, f(\theta)), \delta'), \delta')). \end{aligned}$$

By Theorem 3 and definition of  $f$

$$\begin{aligned} \text{dom}_{I,O,P}(\theta, \delta) &= \text{dom}_{I,O,P}(\theta, \text{rng}_{I,O,P}(\theta, \delta)) = \text{dom}_{I,O,P}(\theta, f(\theta)) \\ \text{dom}_{I,O,P}(\theta, \delta) &= \text{dom}_{I,O,P}(\text{dom}_{I,O,P}(\theta, \delta), \delta) = \text{dom}_{I,O,P}(\text{dom}_{I,O,P}(\theta, f(\theta)), \delta) \\ \text{rng}_{I,O,P}(\theta, \delta) &= \text{rng}_{I,O,P}(\text{dom}_{I,O,P}(\theta, \delta), \delta) = \text{rng}_{I,O,P}(\text{dom}_{I,O,P}(\theta, f(\theta)), \delta). \end{aligned}$$

Replacing  $\text{dom}_{I,O,P}(\theta, \delta)$  and  $\text{rng}_{I,O,P}(\theta, \delta)$  in the definition of  $f$  we obtain

$$\begin{aligned} f(\theta) &= \text{rng}_{I,O,P}(\text{dom}_{I,O,P}(\theta, f(\theta)), \delta) \quad \text{where} \\ \delta &= g(m(\text{dom}_{I,O,P}(\text{dom}_{I,O,P}(\theta, f(\theta)), \delta), \delta)). \end{aligned}$$

Now the fact that we have unique fix-points implies that  $\delta = \delta'$  and consequently that  $f(\theta) = f(\text{dom}_{I,O,P}(\theta, f(\theta)))$ .  $\square$

**Lemma 4**  $f(\theta) = \text{rng}_{I,O,P}(\theta, f(\theta))$ .

**Proof:**

$$\begin{aligned} \text{rng}_{I,O,P}(\theta, f(\theta)) &= \\ \text{rng}_{I,O,P}(\theta, \text{rng}_{I,O,P}(\theta, \delta)) &= \quad \{\text{by definition of } f\} \\ \text{rng}_{I,O,P}(\theta, \delta) &= \quad \{\text{by Theorem 3}\} \\ f(\theta) &= \quad \{\text{by definition of } f\} \end{aligned} \quad \square$$

Finally, since  $\exists g, m : \forall \theta : P$  implies  $\forall \theta : \exists g, m : P$  it follows that  $f \in \mu F$ .  $\square$

**Theorem 8**  $\mu F$  is a mobile component.

**Proof:** That  $\mu F \neq \emptyset$  follows from Theorem 7. To see that  $\mu F$  is closed, let  $f \in \text{Mob}(I, O, P)$ , and assume that

$$\forall \theta : \exists f' \in \mu F : f(\theta) = f'(\theta).$$

The definition of feedback implies that for any  $\theta$  there are  $f' \in \mu F$ ,  $g \in F$  and  $m \in MC$  such that:

$$f(\theta) = f'(\theta) = \text{rng}_{I,O,P}(\theta, \delta) \quad \text{where} \quad \delta = g(m(\text{dom}_{I,O,P}(\theta, \delta), \delta))$$

By the definition of  $\mu$ , it follows that  $f \in \mu F$ .  $\square$

### 6.3 Hiding

The privacy of a non-static port is guaranteed by the genericity property. The privacy of a static port, i.e., a port from the initial wiring, can be ensured by using a hiding operator. If  $x$  is a port from the initial wiring of the component  $F$ , then  $\nu x : F$  is a component where  $x$  is added to the set of private channel names and deleted from the static interface. The domain and range of the functions modeling  $\nu x : F$  are changed accordingly. As a consequence, only components receiving  $!x$  or  $?x$  as a message can access this channel later on.

**Definition 10 (Hiding)** *Given a mobile component  $F \subseteq \text{Mob}(I', O', P')$  and a channel name  $x$  such that  $x \notin P'$ . Let:*

$$I = I' \setminus \{x\}, \quad O = O' \setminus \{x\}, \quad P = P' \cup \{x\}.$$

Then:

$$\nu x : F \subseteq \text{Mob}(I, O, P)$$

$$\nu x : F = \{f \in \text{Mob}(I, O, P) \mid \forall \theta : \exists g \in F :$$

$$f(\theta) = \text{rng}_{I,O,P}(\theta, \delta) \quad \text{where} \quad \delta = g(\text{dom}_{I,O,P}(\theta, \delta))\}$$

□

Note the role of  $\text{dom}_{I,O,P}$  and  $\text{rng}_{I,O,P}$  in maintaining privacy:  $\text{dom}_{I,O,P}$  makes sure that the behavior of  $\nu x : F$  is independent of what the environment sends along  $x$  before the environment has received the send port  $!x$ ;  $\text{rng}_{I,O,P}$  makes sure that  $\nu x : F$  does not send messages along  $x$  before it has sent the receive port  $?x$ .

**Theorem 9**  $\nu x : F \neq \emptyset$ .

**Proof:** Since  $F$  is a mobile component we may find a strongly guarded function  $g \in F$ . Based on  $g$  we construct a function  $f$  which is strongly guarded, generic and satisfies the recursive definition above.

Let  $h$  be defined as follows:

$$h(\delta, \theta) = g(\text{dom}_{I,O,P}(\theta, \delta))$$

Theorem 12 and the way  $h$  is defined in terms of strongly and weakly guarded functions imply that  $h$  is strongly guarded. Thus  $\mu h$  is well-defined, in which case Theorem 14 implies that  $\mu h$  is strongly guarded. By a similar argument,  $f$  defined below is also strongly guarded:

$$f(\theta) = \text{rng}_{I,O,P}(\theta, \delta) \quad \text{where} \quad \delta = \mu g(\theta)$$

That  $f$  is generic is a consequence of the next two lemmas.

**Lemma 5**  $f(\theta) = f(\text{dom}_{I,O,P}(\theta, f(\theta)))$ .

**Proof:** The idea of the proof is to transform  $f(\theta)$  to  $f(\text{dom}_{I,O,P}(\theta, f(\theta)))$  by using the equalities from Theorem 3. By definition

$$f(\text{dom}_{I,O,P}(\theta, f(\theta))) = \text{rng}_{I,O,P}(\text{dom}_{I,O,P}(\theta, f(\theta)), \delta') \quad \text{where}$$

$$\delta' = g(\text{dom}_{I,O,P}(\text{dom}_{I,O,P}(\theta, f(\theta)), \delta')).$$

By Theorem 3 and definition of  $f$

$$\begin{aligned}
\text{dom}_{I,O,P}(\theta, \delta) &= \text{dom}_{I,O,P}(\theta, \text{rng}_{I,O,P}(\theta, \delta)) &= \text{dom}_{I,O,P}(\theta, f(\theta)) \\
\text{dom}_{I,O,P}(\theta, \delta) &= \text{dom}_{I,O,P}(\text{dom}_{I,O,P}(\theta, \delta), \delta) &= \text{dom}_{I,O,P}(\text{dom}_{I,O,P}(\theta, f(\theta)), \delta) \\
\text{rng}_{I,O,P}(\theta, \delta) &= \text{rng}_{I,O,P}(\text{dom}_{I,O,P}(\theta, \delta), \delta) &= \text{rng}_{I,O,P}(\text{dom}_{I,O,P}(\theta, f(\theta)), \delta).
\end{aligned}$$

Replacing  $\text{dom}_{I,O,P}(\theta, \delta)$  and  $\text{rng}_{I,O,P}(\theta, \delta)$  in the definition of  $f$  we obtain

$$\begin{aligned}
f(\theta) &= \text{rng}_{I,O,P}(\text{dom}_{I,O,P}(\theta, f(\theta)), \delta) \text{ where} \\
\delta &= g(\text{dom}_{I,O,P}(\text{dom}_{I,O,P}(\theta, f(\theta)), \delta)).
\end{aligned}$$

Now the fact that we have unique fix-points implies that  $\delta = \delta'$  and consequently that  $f(\theta) = f(\text{dom}_{I,O,P}(\theta, f(\theta)))$ .  $\square$

**Lemma 6**  $f(\theta) = \text{rng}_{I,O,P}(\theta, f(\theta))$ .

**Proof:**

$$\begin{aligned}
\text{rng}_{I,O,P}(\theta, f(\theta)) &= \\
\text{rng}_{I,O,P}(\theta, \text{rng}_{I,O,P}(\theta, \delta)) &= \{\text{by definition of } f\} \\
\text{rng}_{I,O,P}(\theta, \delta) &= \{\text{by Theorem 3}\} \\
f(\theta) &= \{\text{by definition of } f\}
\end{aligned}
\quad \square$$

Finally, since  $\exists g : \forall \theta : P$  implies  $\forall \theta : \exists g : P$  it follows that  $f \in \nu x : F$ .  $\square$

**Theorem 10**  $\nu x : F$  is a mobile component.

**Proof:** That  $\nu x : F \neq \emptyset$  follows from Theorem 9. To see that  $\nu x : F$  is closed, let  $f \in \text{Mob}(I, O, P)$ , and assume that

$$\forall \theta : \exists f' \in \nu x : F : f(\theta) = f'(\theta).$$

The definition of hiding implies that for any  $\theta$  there are  $f' \in \nu x : F$  and  $g \in F$  such that:

$$f(\theta) = f'(\theta) = \text{rng}_{I,O,P}(\theta, \delta) \text{ where } \delta = g(\text{dom}_{I,O,P}(\theta, \delta))$$

By the definition of  $\nu$ , it follows that  $f \in \nu x : F$ .  $\square$

## 7 Mobile Telephones

Our denotational model is very expressive. In this section we show how we can deal with the mobile telephone example discussed in [Mil91]. A center is in permanent contact with two base stations; each in a different part of the country. A car with a mobile telephone moves about the country; it should always be in contact with a base. If it gets rather far from its current base contact, then a hand-over procedure is initiated, and as a result the car relinquishes contact with one base and assumes contact with another. The initial configuration of the network is illustrated by Figure 4.

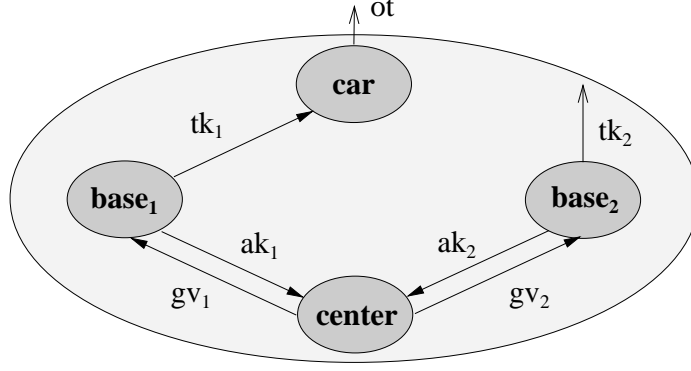


Figure 4: The initial system configuration

In this example there are no real-time constraints to be imposed. We therefore introduce a *time abstraction* operator: the untimed stream tuple  $\widehat{\theta}$  is obtained from the timed stream tuple  $\theta$  by removing time information. This is achieved by concatenating for each  $i \in N$  all the finite sequences in  $\theta(i)$ . The operation  $\{c \rightarrow m\} \& \varphi$  appends the message  $m$  to the head of the stream  $\varphi(c)$ . The other streams in  $\varphi$  remain unchanged. This operator is overloaded to streams in the obvious way.

Figure 4 shows the initial configuration of system. *car* is in contact with *base1*. However, there is no connection between *car* and *base2*. The system is specified as follows:

$$\begin{aligned} \text{system}(\triangleright ot) &\stackrel{\text{def}}{=} \nu tk_1, tk_2, gv_1, gv_2, ak_1, ak_2 : \\ &\text{car}(tk_1 \triangleright ot) \oplus \text{base}(gv_1 \triangleright tk_1, ak_1) \oplus \text{base}(gv_2 \triangleright tk_2, ak_2) \oplus \\ &\text{center}(tk_1, tk_2)(ak_1, ak_2 \triangleright gv_1, gv_2) \end{aligned}$$

The *car* is parametric upon a receive port  $tk$  and a send port  $ot$ . On  $tk$  it can either receive talk messages  $m$  or switch messages  $?x$ . Any talk message is forwarded along  $ot$ . The arrival of a receive port  $?c$  forces the component to receive on  $?c$  instead of on  $tk$ .

$$\begin{aligned} \text{car}(tk \triangleright ot) &\stackrel{\text{def}}{=} \{ f \in \text{Mob}(\{tk\}, \{ot\}, \emptyset) \mid \\ &\forall \theta : \widehat{f(\theta)} = g(tk)(\widehat{\theta}) \quad \textbf{where} \quad \forall \vartheta, m, c : \\ &g(tk)(\{tk \mapsto m\} \& \vartheta) = \{ot \mapsto m\} \& g(tk)(\vartheta) \\ &g(tk)(\{tk \mapsto ?c\} \& \vartheta) = g(c)(\vartheta) \quad \} \end{aligned}$$

A base can talk repeatedly with the *car*; but at any time it can receive on its give channel  $gv$  a new port which it should communicate to the *car* and then become idle itself. Whether it ignores the give channel or not is controlled with a prophecy stream. An idle base is reactivated upon receipt of an act message on its give channel.

$$\begin{aligned} \text{base}(gv \triangleright tk, ak) &\stackrel{\text{def}}{=} \{ f \in \text{Mob}(\{gv\}, \{tk, ak\}, \emptyset) \mid \\ &\forall \theta : \exists p \in [\{1, 2\}] : \widehat{f(\theta)} = h(p)(\widehat{\theta}) \quad \textbf{where} \quad \forall p, \vartheta, x : \\ &h(p)(\{gv \mapsto \text{act}\} \& \vartheta) = g(p)(\vartheta) \\ &g(1 \& p)(\vartheta) = \{tk \mapsto m\} \& g(p)(\vartheta) \\ &g(2 \& p)(\{gv \mapsto ?x\} \& \vartheta) = \{tk \mapsto ?x, ak \mapsto \text{ok}\} \& h(p)(\vartheta) \quad \} \end{aligned}$$

The center which initially knows that the car is in connection with  $\text{base}_1$ , can decide (according to information which we do not model) to transmit the receive port  $?tk_2$  to the car via  $\text{base}_1$ . Upon receipt of an acknowledgment on  $ak_1$  from  $\text{base}_1$  it alerts  $\text{base}_2$  of this fact.

$$\text{center}(tk_1, tk_2)(ak_1, ak_2 \triangleright gv_1, gv_2) \stackrel{\text{def}}{=} \{ f \in \text{Mob}(\{ak_1, ak_2\}, \{gv_1, gv_2\}, \emptyset) \mid$$

$$\forall \theta : \widehat{f(\theta)} = g(1)(\widehat{\theta}) \quad \text{where} \quad \forall i \in \{1, 2\}, \vartheta$$

$$g(i)(\vartheta) = \{gv_i \mapsto \text{act}\} \& \{gv_i \mapsto ?tk_{\neg i}\} \& h(i)(\vartheta)$$

$$h(i)(\{ak_i \mapsto \text{ok}\} \& \vartheta) = g(\neg i)(\vartheta) \}$$

It is here assumed that  $\neg 1 = 2$  and that  $\neg 2 = 1$ .

## 8 Discussion

Our main contribution is that we have extended a denotational model for timed, many-to-many, nondeterministic data-flow networks to handle *mobility*. Our model is fully compositional. It allows us to reason about mobility at a very abstract level. In fact, we believe our semantics is well-suited as a foundation for a method for the specification and development of mobile systems. The exact relationship between our model and other models like for instance the  $\pi$ -calculus [Mil91] and actor-based approaches [AMST92] is an interesting area for future research. For example, we believe that our model can be used to give a denotational semantics for the asynchronous  $\pi$ -calculus. We also believe that the actor languages can be smoothly integrated within our formalism.

Our semantics can easily be adapted to model other kinds of communication. For example, in [GS96b, GS96a] we show how two different kinds of point-to-point communication can be modeled in our approach.

Our approach is related to the work of Kok [Kok87, Kok89]. The major difference is that Kok does not deal with mobility. Moreover, his handling of nondeterminism differs from ours. In [Kok89], where he uses a metric on relations, he can basically handle only bounded nondeterminism. In [Kok87], which is not based on metric spaces, an automaton is used to generate the behavior of basic agents. This guarantees the existence of fix-points. We use sets of strongly guarded functions for the same purpose.

## References

- [AdBKR89] P. America, J. de Bakker, J. N. Kok, and J. Rutten. Denotational semantics of a parallel object-oriented language. *Information and Computation*, 83:152–205, 1989.
- [AMST92] G. Agha, I. A. Mason, S. F. Smith, and C. L. Talcott. Towards a theory of actor computation. In *Proc. CONCUR'92, Lecture Notes in Computer Science 630*, pages 565–579, 1992.

- [BA81] J. D. Brock and W. B. Ackermann. Scenarios: A model of non-determinate computation. In *Proc. Formalization of Programming Concepts, Lecture Notes in Computer Science 107*, pages 252–259, 1981.
- [BB90] G. Berry and G. Boudol. The chemical abstract machine. In *Proc. POPL'90*, pages 81–94, 1990.
- [BDD<sup>+</sup>93] M. Broy, F. Dederichs, C. Dendorfer, M. Fuchs, T. F. Gritzner, and R. Weber. The design of distributed systems — an introduction to Focus (revised version). Technical Report SFB 342/2/92 A, Technische Universität München, 1993.
- [Bro87] M. Broy. Semantics of finite and infinite networks of concurrent communicating agents. *Distributed Computing*, 2:13–31, 1987.
- [CM88] K. M. Chandy and J. Misra. *Parallel Program Design, A Foundation*. Addison-Wesley, 1988.
- [dBZ82] J. W. de Bakker and J. I. Zucker. Denotational semantics of concurrency. In *Proc. 14 ACM Symposium on Theory of Computing*, pages 153–158, 1982.
- [EN86] U. Engberg and M Nielsen. A calculus of communicating systems with label-passing. Technical Report DAIMI PB-208, University of Aarhus, 1986.
- [Eng77] R. Engelking. *General Topology*. PWN — Polish Scientific Publishers, 1977.
- [GS96a] R. Grosu and K. Stølen. A denotational model for mobile point-to-point dataflow networks with channel sharing. To appear as technical report, Technische Universität München, 1996.
- [GS96b] R. Grosu and K. Stølen. A model for mobile point-to-point dataflow networks without channel sharing. To appear in the Proc. of AMAST'96, 1996.
- [HBS73] C. Hewitt, P. Bishop, and R. Steiger. A universal modular actor formalism for artificial intelligence. In *Proc. IJCAI'73*, pages 235–245, 1973.
- [Kah74] G. Kahn. The semantics of a simple language for parallel programming. In *Proc. Information Processing 74*, pages 471–475. North-Holland, 1974.
- [Kel78] R. M. Keller. Denotational models for parallel programs with indeterminate operators. In *Proc. Formal Description of Programming Concepts*, pages 337–366. North-Holland, 1978.
- [Kok87] J. N. Kok. A fully abstract semantics for data flow nets. In *Proc. PARLE'87, Lecture Notes in Computer Science 259*, pages 351–368, 1987.
- [Kok89] J. N. Kok. An iterative metric fully abstract semantics for nondeterministic dataflow. In *Proc. MFCS'89, Lecture Notes in Computer Science 379*, pages 321–331, 1989.
- [Lam91] L. Lamport. The temporal logic of actions. Technical Report 79, Digital, SRC, Palo Alto, 1991.

- [Mes91] J. Meseguer. Conditional rewriting logic as a unified model of concurrency. Technical Report SRI-CSL-91-05, SRI, 1991.
- [Mil91] R. Milner. The polyadic  $\pi$ -calculus: A tutorial. Technical Report ECS-LFCS-91-180, University of Edinburgh, 1991.
- [MPS86] D. MacQueen, G. Plotkin, and R. Sethi. An ideal model for recursive polymorphic types. *Information and Control*, 71:95–130, 1986.
- [MPW92a] R. Milner, J. Parrow, and D. Walker. A calculus of mobile processes, part I. *Information and Computation*, 100:1–40, 1992.
- [MPW92b] R. Milner, J. Parrow, and D. Walker. A calculus of mobile processes, part II. *Information and Computation*, 100:41–77, 1992.
- [Niv82] M Nivat. Behaviours of processes and synchronized systems of processes. In *Proc. Theoretical Foundations of Programming Methodology*, pages 473–551, 1982.
- [Par83] D. Park. The “fairness” problem and nondeterministic computing networks. In *Proc. 4th Foundations of Computer Science, Mathematical Centre Tracts 159*, pages 133–161. Mathematisch Centrum Amsterdam, 1983.
- [PS92] P. Panangaden and V. Shanbhogue. The expressive power of indeterminate dataflow primitives. *Information and Computation*, 98:99–131, 1992.
- [Rus90] J. R. Russell. On oraclizable networks and Kahn’s principle. In *Proc. POPL’90*, pages 320–328, 1990.
- [Tho89] B. Thomsen. A calculus of higher order communicating systems. In *Proc. POPL’89*, 1989.

## A Metric Space Definitions

### A.1 Metric Space Basics

The fundamental concept in metric spaces is the concept of distance.

**Definition 11 (Metric Space)** *A metric space is a pair  $(D, d)$  consisting of a nonempty set  $D$  and a mapping  $d \in D \times D \rightarrow R$ , called a metric or distance, which has the following properties:*

- (1)  $\forall x, y \in D : d(x, y) = 0 \Leftrightarrow x = y$
- (2)  $\forall x, y \in D : d(x, y) = d(y, x)$
- (3)  $\forall x, y, z \in D : d(x, y) \leq d(x, z) + d(z, y)$  □

A very simple example of a metric is the discrete metric.

**Definition 12 (The discrete metric)** *The discrete metric  $(D, d)$  over a set  $D$  is defined as follows:*

$$d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases} \quad \square$$

Measuring the distance between the elements of a sequence  $(x_i)_{i \in \mathbb{N}}$  in  $D$  we obtain the familiar definitions for convergence and limits.

**Definition 13 (Convergence and limits)** *Let  $(D, d)$  be a metric space and let  $(x_i)_{i \in \mathbb{N}}$  be a sequence in  $D$ .*

(1) *We say that  $(x_i)_{i \in \mathbb{N}}$  is a Cauchy sequence whenever we have:*

$$\forall \epsilon > 0 : \exists N \in \mathbb{N} : \forall n, m > N : d(x_n, x_m) < \epsilon.$$

(2) *We say that  $(x_i)_{i \in \mathbb{N}}$  converges to  $x \in D$  denoted by  $x = \lim_{n \rightarrow \infty} x_i$  and call  $x$  the limit of  $(x_i)_{i \in \mathbb{N}}$  whenever we have:*

$$\forall \epsilon > 0 : \exists N \in \mathbb{N} : \forall n > N : d(x_n, x) < \epsilon.$$

(3) *The metric space  $(D, d)$  is called complete whenever each Cauchy sequence converges to an element of  $D$ .*

□

**Theorem 11** *The discrete metric is complete.*

**Proof:** Each Cauchy sequence is constant from a given  $N$ . □

A very important class of functions over metric spaces is the class of *Lipschitz functions*.

**Definition 14 (Lipschitz functions)** *Let  $(D_1, d_1)$  and  $(D_2, d_2)$  be metric spaces and let  $f \in D_1 \rightarrow D_2$  be a function. We call  $f$  Lipschitz function with constant  $c$  if there is a constant  $c \geq 0$  such that the following condition is satisfied:*

$$d(f(x), f(y)) \leq c \cdot d(x, y)$$

*For a function  $f$  with arity  $n$  the above condition generalizes to:*

$$d(f(x_1, \dots, x_n), f(y_1, \dots, y_n)) \leq c \cdot \max\{d(x_i, y_i) \mid i \in [1..n]\}$$

*If  $c = 1$  we call  $f$  non-expansive. If  $c < 1$  we call  $f$  contractive.*

□

**Theorem 12** *The composition of two Lipschitz functions  $f \in D_1 \rightarrow D_2$  and  $g \in D_2 \rightarrow D_3$  is a Lipschitz function with constant  $c_1 \cdot c_2$ .*

**Proof:**  $d(g(f(x_1)), g(f(x_2))) \leq c_2 \cdot d(f(x_1), f(x_2)) \leq c_2 \cdot c_1 \cdot d(x_1, x_2)$  □

**Lemma 1** *The composition of a contractive and a non-expansive function is contractive. The composition of two non-expansive functions is non-expansive. Identity is non-expansive.* □

The main tool for handling recursion in metric spaces is the Banach's fixed point theorem. It guarantees the existence of a unique fixed point for every contractive function.



**Theorem 13 (Banach’s fixed point theorem)** *Let  $(D, d)$  be a complete metric space and  $f \in D \rightarrow D$  a contractive function. Then there exists an  $x \in D$ , such that the following holds:*

- (1)  $x = f(x)$  *( $x$  is a fixed point of  $f$ )*
- (2)  $\forall y \in D : y = f(y) \Rightarrow y = x$  *( $x$  is unique)*
- (3)  $\forall z \in D : x = \lim_{n \rightarrow \infty} f^n(z)$  *where*
  - $f^0(z) = z$
  - $f^{n+1}(z) = f(f^n(z))$

**Proof:** See [Eng77]. □

Usually we want to use a parameterized version of this theorem.

**Definition 15 (Parameterized fixed point)** *Let  $f \in D \times D_1 \times \dots \times D_n \rightarrow D$  be a function of non-empty complete metric spaces that is contractive in its first argument. We define the parameterized fixed point function  $\mu f$  as follows:*

$$(\mu f) \in D_1 \times \dots \times D_n \rightarrow D$$

$$(\mu f)(y_1, \dots, y_n) = x$$

where  $x$  is the unique element of  $D$  such that  $x = f(x, y_1, \dots, y_n)$  as guaranteed by Banach’s fixed point theorem. □

**Theorem 14** *If  $f$  is contractive (non-expansive) so is  $\mu f$ .*

**Proof:** See for example [MPS86] pages 114–115. □

## A.2 Streams and Named Stream Tuples

A stream is a finite or infinite sequence of elements. For any set of elements  $E$ , we use  $E^*$  to denote the set of all finite streams over  $E$ , and  $[E]$  to denote the set of all infinite streams over  $E$ . For any infinite stream  $s$ , we use  $s \downarrow_j$  to denote the prefix of  $s$  containing exactly  $j$  elements. We use  $\epsilon$  to denote the empty stream.

We define the metric of streams generically with respect to an arbitrary discrete metric  $(E, \rho)$ .

**Definition 16 (The metric of streams)** *The metric of streams  $([E], d)$  over a discrete metric  $(E, \rho)$  is defined as follows:*

$$[E] = \prod_{i \in \mathbb{N}} E$$

$$d(s, t) = \inf\{2^{-j} \mid s \downarrow_j = t \downarrow_j\}$$

□

This metric is also known as the Baire metric [Eng77].

**Theorem 15** *The metric space of streams  $([E], d)$  is complete.*

**Proof:** See for example [Eng77]. □

A *named stream tuple* is a mapping  $\theta \in (I \rightarrow [E])$  from a set of names to infinite streams.  $\downarrow$  is overloaded to named stream tuples in a point-wise style, i.e.,  $\theta \downarrow_j$  denotes the result of applying  $\downarrow_j$  to each component of  $\theta$ .

**Definition 17 (The metric of named stream tuples)** *The metric of named stream tuples  $(I \rightarrow [E], d)$  with names in  $I$  and elements in  $(E, \rho)$  is defined as follows:*

*$I \rightarrow [E]$  is the set of functions from the countable set  $I$  to the metric  $[E]$ ,*

$$d(s, t) = \inf\{2^{-j} \mid s \downarrow_j = t \downarrow_j\}$$

□

**Theorem 16** *The metric space of named stream tuples  $(I \rightarrow [E], d)$  is complete.*

**Proof:** This metric is equivalent to the Cartesian product metric  $\prod_{i \in I} [E]$  which is complete because  $[E]$  is [Eng77]. □

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