Modular Refinement of Hierarchic Reactive Machines¹

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Scalable formal analysis of reactive programs demands integration of modular reasoning techniques with existing analysis tools. Modular reasoning principles such as abstraction, compositional refinement, and assume-guarantee reasoning are well understood for architectural hierarchy that describes the communication structure between component processes, and have been shown to be useful. In this paper, we develop the theory of modular reasoning for *behavior hierarchy* that describes control structure using hierarchic modes. From STATECHARTS to UML, behavior hierarchy has been an integral component of many software design languages, but only syntactically. We present the *hierarchic reactive modules* language that retains powerful features such as nested modes, mode reuse, exceptions, group transitions, history, and conjunctive modes, and yet has a *semantic* notion of mode hierarchy. We present an observational trace semantics for modes that provides the basis for mode refinement. We show the refinement to be compositional with respect to the mode constructors, and develop an assume-guarantee reasoning principle.

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1. INTRODUCTION

The complexity and subtlety of programming reactive systems, such as telecommunications and avionics software, demands increased design automation and effective debugging tools. Recent advances in formal verification have led to powerful

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design tools for hardware (see [Clarke and Kurshan 1996] for a survey), and subsequently, have brought a lot of hope of their application to reactive programming. The most successful verification technique has been *model checking* [Clarke and Emerson 1981]. In model checking, the system is described by a state-machine model, and is analyzed by an algorithm that explores the reachable state-space of the model. The state-of-the-art model checkers (e.g. SPIN [Holzmann 1997] and SMV [McMillan 1993]) employ a variety of heuristics for efficient search, but are typically unable to analyze models with more than hundred state variables. Consequently, application of formal verification requires augmenting model checking with *modular reasoning* that allows decomposition of the analysis problem into smaller subproblems, or abstracting a component into a simpler one. Typically, such simplification is done manually, and requires considerable expertise. Much of today's research in formal verification aims to develop techniques to automate modular reasoning and abstraction techniques.

To be able to exploit the design structure effectively for modular reasoning, the modeling language must support syntactically as well as semantically modular constructs. While modern programming languages offer a rich set of modular constructs (e.g. procedures, objects), the standard model checkers assume the model to be a state-transition graph (or a Kripke structure) with no structure. Our first attempt to enrich the modeling language resulted in the definition of *reactive* modules [Alur and Henzinger 1999]. In this language, an atomic module is a statemachine whose variables are explicitly partitioned into input, output, and private variables. The operations of parallel composition, instantiation, and variable hiding allow building complex modules from atomic ones. The denotational semantics of each module consists of its input and output variables, together with the set of its traces, which captures the observable interaction of a module with its environment. The notion of *refinement* between two modules is based on inclusion of traces, and provides the basis for abstraction. The refinement relation is compositional with respect to the module operations. Thus, to show that the composite module $P_1 || P_2$ refines the module $Q_1 || Q_2$, it suffices to establish that P_1 refines Q_1 and P_2 refines Q_2 . While the compositional proof rule decomposes the verification task of proving implementation between compound modules into subtasks, it may not always be applicable. In particular, P_1 may not implement Q_1 for all environments, but only if the environment behaves like P_2 , and vice versa. For such cases, we must employ the assume-guarantee proof rule which asserts that in order to prove that $P_1 || P_2$ implements $Q_1 || Q_2$, it suffices to prove (1) $P_1 || Q_2$ implements Q_1 , and (2) $Q_1 || P_2$ implements Q_2 . The language of reactive modules, along with the assume-guarantee refinement checker, is supported by the model checker MOCHA [Alur et al. 2001], and the utility of the assume-guarantee reasoning has been demonstrated in analysis of a video-graphics image processor [Henzinger et al. 1998] and the network protocol PPP [Alur and Wang 2001].

While the reactive modules language supports *architectural hierarchy*, it offers little structure to express the behavior of individual modules. In this paper, we present the language of *hierarchic reactive modules* that supports both *architectural and behavioral hierarchy*, along with its compositional semantics and assumeguarantee proof calculus. The notion of behavior hierarchy was popularized by the introduction of STATECHARTS [Harel 1987], and exists in many related modeling

formalisms such as MODECHARTS [Jahanian and Mok 1987] and RSML [Leveson et al. 1994]. It is a central component of various object-oriented software development methodologies developed in recent years, such as ROOM [Selic et al. 1994], and the Unified Modeling Language (UML [Booch et al. 1997]).

The central component of the behavioral description in our language is a *mode*. A mode consists of global variables used to share data with its environment, local variables, well-defined entry and exit points, and submodes that are connected with each other by transitions. The transitions are labeled with guarded commands that access the variables according to the the natural scoping rules. Note that the transitions can connect to a mode only at its entry/exit points, as in ROOM, but unlike STATECHARTS. This choice is important in viewing the mode as a black box whose internal structure is not visible from outside. The mode has a *default exit* point, and transitions leaving the default exit are applicable at all control points within the mode and its submodes. The default exit retains the history, and the state upon exit is automatically restored by transitions entering the default entry point. Thus, a transition from default exit to entry models a group transition applicable to all control points inside. While defining the operational semantics of modes, we follow the standard paradigm in which transitions are executed repeatedly as long as at least one is enabled (has a true guard). Since the control can be simultaneously in multiple nested modes, the order in which the transitions are tried for execution is important. Unlike STATECHARTS, but as in ROOM, the operational semantics tries the transitions inside out, that is, we give priority to the internal transitions over the group transitions of the enclosing mode. This choice is also crucial for the clean denotational semantics. Our language allows mode instantiation and thus, reuse.

Our denotational semantics of a mode consists of its global variables, entry/exit points, and traces over global variables that capture a mode's behavior. The key step leading to such semantics involves a *closure* construction that adds transitions connecting the default points. This construction makes the transfer of control between a mode and its environment explicit. Consequently, the behavior of a mode can be viewed as a game in which the environment transfers control to the mode at one of its entry points, and the mode transfers the control back to the environment at one of its exit points. The *macro-transition* from an entry point to an exit point, thus, consists of multiple transitions, and can be constructed from the macro-transitions of the submodes together with the transition relation of the mode. The macro-transitions are then used to associate a set of executions and a corresponding set of traces with a mode. We show that the traces of a mode can be constructed from the traces of its submodes.

The denotational trace semantics naturally leads to a notion of refinement among modes based on inclusion of traces, and provides the basis for mode abstraction and substitution. We show that the constructors are compositional with respect to this refinement relation, and this leads to compositional proof rules for refinement. In particular, to establish that a mode M with a submode N refines a mode M' with submode N', it suffices to prove that (1) mode N refines N', and (2) mode M with N substituted by a "free" mode that allows most general update, refines mode M'with N' made free. Thus, compositional rule allows us to decouple the reasoning about a submode from the reasoning about its context. We also present a circular assume-guarantee proof rule in which the specification context M' can be assumed

while establishing the first sub-goal, and the specification submode N' can be used while establishing the second sub-goal.

Hierarchic languages such as STATECHARTS allow the notion of *conjunctive (par-allel) states.* We argue that conjunction can be defined cleanly as a constructor over submodes such that a step of the constructed mode consists of a sequence of micro steps, one micro step for each submode. We establish that such a conjunctive constructor is compositional with respect to refinement and that its trace semantics is essentially the same as the trace semantics of the parallel composition constructor over modules. This suggests a scheme for mixing modes and modules interchangeably in system descriptions.

Related work

This paper develops compositional semantics for hierarchical reactive systems. As discussed earlier, it builds upon the compositional language of *reactive modules* [Alur and Henzinger 1999]. The notion of compositional refinement based on observable behaviors is central to many other formal models such as CCS [Milner 1980], I/O automata [Lynch and Tuttle 1987], TLA [Lamport 1994]. Languages such as reactive modules, I/O automata and TLA are concerned only with concurrent composition, and thus, only with the architectural hierarchy. In behavior hierarchy, modes at the same level are composed sequentially, and modes at different levels can be active concurrently. Consequently, a language with behavior hierarchy cannot be given a natural and compositional semantics in models that support only concurrent composition.

Process algebras such as CCS allow both sequential and concurrent composition, and thus, both architectural and behavioral hierarchies. Compared to process algebras, our formalism differs in the following ways. First, it allows explicit modeling of hierarchy and group transitions that is similar to many visual software design languages. Second, while communication in process algebras uses events (or actions), we use shared variables. Third, the notion of equivalence for process algebras such as CCS is based on bisimulation, while our semantics uses the weaker notion of traces. Another focus of our compositionality is assume-guarantee reasoning. Since assume-guarantee reasoning allows modeling of assumptions about the interface of a component, it promises to be more relevant in decomposing proofs. Due to the inherent circularity, assume-guarantee reasoning [Stark 1985; Abadi and Lamport 1995; Alur and Henzinger 1999; McMillan 1997] is valid only when the interaction of a module with its environment is non-blocking, and is not valid for frameworks such as CCS.

The concept of behavioral hierarchy in state-machine-based formalisms can be traced to STATECHARTS [Harel 1987]. Such hierarchic specifications have many powerful primitives such as exceptions, group transitions, and history, which lead to complex semantics [Harel et al. 1987; Pnueli and Shalev 1991; Harel and Naamad 1996; Huber et al. 1996; Grosu et al. 1998; Lüttgen et al. 2000]. There have been several attempts to define a rigorous semantics of STATECHARTS alone. Typically the semantics is defined operationally by considering the global state of the system. In fact, multiple papers offer a *compositional* semantics that is congruent with the language constructs [Uselton and Smolka 1994]. However, the notion of *semantic equivalence* used in all these papers is structural isomorphism of underlying state-

transition graphs (or bisimilarity but with most of the structure visible), rather than conventional observational equivalences. Also there have been attempts to exploit the hierarchic states for efficient reachability analysis [Chan et al. 1998; Alur and Yannakakis 1998; Behrmann et al. 1999]. However, there is no *observational* semantics that allows defining a refinement preorder on hierarchic states.

Thus, the main contribution of this paper is a modular semantics for hierarchical specifications with a supporting trace-based refinement calculus. We hope that it will provide insights for incorporating hierarchical constructs in modeling languages so that the hierarchy is semantic, and will also provide a basis for restricting or adapting popular languages such as STATECHARTS so that modularity principles such as assume-guarantee reasoning can be applied to their formal analyses.

Outline

The rest of the paper is organized as follows. In Section 2 we review reactive modules: their syntax, semantics and refinement rules. In Section 3 we follow a similar pattern for modes. We first introduce their syntax and then we define their semantics and refinement rules. Section 4 is devoted to conjunctive modes. First we define conjunction as a particular mode constructor. Then we show that conjunction is compositional with respect to refinement and relate it to module composition. Finally in Section 5 we draw some conclusions. During the entire paper, we use the specification of a small village telephone system, inspired from [Bhargavan et al. 1998], as a working example to illustrate definitions.

Notation

Given a set V of typed variables, a *state* over V is a function mapping variables to their values. The set of states over V is denoted Q_V . Given a state s over V and a subset W of V, s.W denotes the state over W obtained by restricting s to the variables in W. The projection operator extends to sequences of states also. Given two sets V and W of variables, an *action* from V to W is a binary relation between the states over V and the states over W. An action α from V to W is said to be *enabled* at a state s over V if $(s,t) \in \alpha$ for some state t. An action α from V to W is said to be *non-blocking* if it is enabled at every state over V. The domain and range of an action can be expanded implicitly in a natural way: if α is an action from V to W, s is a state over $V' \supseteq V$ and t is a state over $W' \supseteq W$, then define $(s,t) \in \alpha$ if $(s.V,t.W) \in \alpha$ and t.v = s.v for $v \in (V' \cap W') \setminus W$. This implicit coercion is quite common and allows one to assume that the variables not explicitly occurring in a transition remain unchanged.

2. MODULES

2.1 Syntax

A module is defined by the set of its variables, rules for initializing the variables, and rules for updating the variables.

The variables of a module P are partitioned into three classes: *private* variables that cannot be read or written by other modules, *interface* variables that are written only by P, but can be read by other modules, and *external* variables that can only be read by P, and are written by other modules. Thus, interface and external

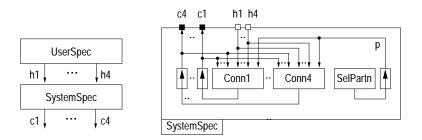


Fig. 1. Architecture diagrams for the VTS

variables are used for communication, and are called *observable* variables. The private and interface variables are written by the module, and are called *controlled* variables.

The execution of a module proceeds in a sequence of rounds. The initialization specifies initial controlled states. The subsequent rounds are update rounds determined by the transition relation which specifies how to change the controlled state as a function of the current state. In each round, the external variables are assigned arbitrary values.

Definition 1. (Modules) A module P consists of

Variables. A finite set V of typed variables that is partitioned into private variables V_p , interface variables V_i and external variables V_e . The variables in $V_c = V_p \cup V_i$ are called *controlled variables*, and the variables in $V_o = V_i \cup V_e$ are called *observable variables*.

Initial states. A non-empty subset I of states over V_c .

Update relation. A non-blocking action U from V to V_c .

Example 1. (Village telephone system) Consider a simple village telephone system that is able to establish a point-to-point connection between any two telephone lines that are disconnected and off hook. For simplicity, assume that the system has only four lines. The block (architecture) diagram for the system and and the users is shown in Figure 1, left.

To define and analyze the behavior of the village telephone system, we associate each block in the diagram to a reactive module with the interface shown in the diagram. A high level definition of the environment is the UserSpec module given below.

type hookType is {on, off}

module UserSpec is

interface h1, h2, h3, h4 : hookType; init [] true -> h1 := on; h2 := on; h3 := on; h4 := on; update [] h1 = on -> h1 := off; [] h1 = off -> h1 := on; [] h2 = on -> h2 := off;

[] h2 = off -> h2 := on; [] h3 = on -> h3 := off; [] h3 = off -> h3 := on; [] h4 = on -> h4 := off; [] h4 = off -> h4 := on; [] true -> ;

The module has four variables, all of which are interface variables. In this example, the initialization and the update relation is presented using guarded commands. In each update round it may toggle one of the four lines between on and off. The choice is nondeterministic. Due to the last clause, it is also allowed to idle in a round, and thus, to arbitrarily delay toggling. \Box

As the above example shows, reactive modules offer little support for factoring out common (sequential) behavior. The introduction of modes in Section 3 allows more structuring.

2.2 Semantics

For a module P, we use the notation $P.V_p$ to refer to the private variables of P, P.U to refer to the update action of P, etc. The semantics of a module is captured by defining its executions and traces:

Definition 2. (Traces) An execution of a module P is a sequence $s_0s_1 \cdots s_n$ of states over P.V such that $s_0.(P.V_c) \in P.I$ and $(s_i, s_{i+1}.(P.V_c)) \in P.U$ for $0 \le i < n$. If σ is an execution of P, then the corresponding sequence $\sigma.V_o$ of observable states is called a *trace* of P. The set of all traces of P is called the *trace-language* of P, and is denoted L_P . \Box

The *denotational semantics* of the module P is captured by its interface variables, external variables, and trace language. The requirement that I is non-empty and U is non-blocking ensures that the module can always take a step, and the trace-set L_P is infinite.

Remark 1. (Awaits dependencies and sub-rounds) In our definition, the initial values of the controlled variables cannot depend on each other and in each update step, the new values of the controlled variables cannot depend on the new values of the external and controlled variables. Thus, modules are like Moore machines. The definition of reactive modules [Alur and Henzinger 1999] allows specification of a partial order — called *awaits dependencies*, over variables such that if a variable x is greater than a variable y in this ordering, then the initial value of x can refer to the initial value of y and the update rule for x can refer to the updated value of y. This allows more complex forms of interaction between a module and its environment by splitting each update round into a fixed number of sub-rounds. In this paper, we want to focus on the hierarchical specification of the update relation, so we have chosen a simpler form of interaction with no awaits dependencies for the sake of clarity of presentation.

Remark 2. (Fairness) By labeling subsets of the update relation as strongly or weakly fair one obtains fair modules [Alur and Henzinger 1999]. Fairness is however not the main focus of this paper and it is therefore not further pursued. \Box

2.3 Hierarchy

Operators on modules include instantiation, hiding of interface variables, and parallel composition. We first discuss the parallel composition operator that allows to combine two modules into a single one.

Definition 3. (Parallel Composition) The modules P and Q are composable if $P.V_c \cap Q.V_c$ is empty. The composition R of composable modules P and Q, denoted P||Q, is the module with:

Variables. The sets of interface, external and private variables:

 $R.V_i = P.V_i \cup Q.V_i, \quad R.V_e = (P.V_e \cup Q.V_e) \setminus R.V_i, \quad R.V_p = P.V_p \cup Q.V_p$

Initial states. The set $R.I = P.I \times Q.I$.

Update relation. The relation R.U such that:

 $(s,t) \in R.U \quad \text{iff} \quad (s.(P.V),t.(P.V_c)) \in P.U \ \land \ (s.(Q.V),t.(Q.V_c)) \in Q.U \quad \Box$

Note that the denotational semantics of P || Q can be completely constructed from the denotational semantics of P and Q. This is because a sequence σ belongs to the trace language $L_{P||Q}$ iff its corresponding projections belong to the trace languages L_P and L_Q [Alur and Henzinger 1999].

The hiding operation makes an interface variable a private one, and thus, allows us to construct module abstractions of varying degrees of detail.

Definition 4. (Hiding) Given a module P and an interface variable $x \in P.V_i$, the module hide x in P has the set $P.V_p \cup \{x\}$ of private variables, the set $P.V_i \setminus \{x\}$ of interface variables, the set $P.V_e$ of external variables, the set P.I of initial states, and the action P.U as the update relation.

Starting with atomic modules, the operations of parallel composition and hiding allow us to describe complex modules in a hierarchical way.

Example 2. (Village telephone system) A possible hierarchic decomposition of the module SystemSpec is obtained by introducing for each telephone line *i* a connection module Conn*i*. An additional module SelPartn is used to guide the selection of the communication partner. This defines the architecture shown by the block diagram in Figure 1, right. The blocks marked with a thick arrow represent registers. They separate the current state from the next state. The specification of the module SelPartn is trivial. It nondeterministically chooses one of the selection modes.

type matchingType is {"1-2/3-4","1-3/2-4","1-4/2-3"}

module SelPartn is

```
interface p:matchingType;
```

```
init update true -> p := nondet;
```

The specification of the connection module Conn1 is as follows. If the module is disconnected and its line is off hook, then it chooses a partner connection module as specified by the value of p, provided this partner module is also disconnected and its line is off hook. In this case it becomes connected. If the current communication

ACM Transactions on Programming Languages and Systems, Vol. TBD, No. TDB, Month Year.

9

partner goes on hook then the module goes in the drooping state. Finally, if its own line goes on hook, then it becomes disconnected.

```
type connectionType is {disconnected, 1, 2, 3, 4, drooping}
module Conn1 is
  interface c1 : connType;
  external c2,c3,c4:connType; h1,h2,h3,h4:hookType; p:matchingType;
init
  [] true -> c1 := disconnected;
update
  [] h1 = on \rightarrow c1 := disconnected;
  [] c1 = 2 \& h2 = on -> c1 := drooping;
  [] c1 = 3 & h3 = on -> c1 := drooping;
  [] c1 = 4 & h4 = on -> c1 := drooping;
  [] c1 = disconnected & h1 = off & c2 = disconnected & h2 = off &
     p = "1-2/3-4" \rightarrow c1 := 2;
  [] c1 = disconnected & h1 = off & c3 = disconnected & h3 = off &
     p = "1-3/2-4" -> c1 := 3;
  [] c1 = disconnected & h1 = off & c4 = disconnected & h4 = off &
     p = "1-4/2-3" \rightarrow c1 := 4;
```

The other connection modules are specified in a similar way. Composing the above modules gives the specification of the entire system, as shown below.

```
module SystemSpec is hide p in ( Conn1 ||..|| Conn4 || SelPartn )
module Spec is UserSpec || SystemSpec
```

2.4 Refinement

The notion of refinement between successive levels of abstraction is formalized by the definition of the implementation preorder:

Definition 5. (Implementation) A module P implements a module Q, written $P \preceq Q$, if P and Q have identical interface variables, identical external variables, and $L_P \subseteq L_Q$.

Intuitively, $P \leq Q$ holds if every observable behavior of P is also a possible behavior of Q, and thus, if the implementation P is more constrained than the specification Q. We have required both the modules to have identical observable variables, but this can be generalized in a straightforward way to allow the implementation to have more interface and external variables such that the external variables of the specification are a subset of the external and interface variables of the implementation. Hence, the implementation is allowed to constrain some of the external variables of the specification.

A key property of the implementation relation is *compositionality* which ensures that the refinement preorder is congruent with respect to the module operations [Alur and Henzinger 1999].

PROPOSITION 1. (Compositionality) If $P \preceq Q$ then $P || R \preceq Q || R$.

By applying the compositionality rule twice and using the transitivity of refinement it follows that, in order to prove that a complex compound module $P_1||P_2$ (with a large state space) implements a simpler compound module $Q_1||Q_2$ (with a small state space), it suffices to prove (1) P_1 implements Q_1 and (2) P_2 implements Q_2 . We call this the *compositional proof rule* for reactive modules. It is valid, because parallel composition and implementation behave like language intersection and language containment, respectively.

While the compositional proof rule decomposes the verification task of proving implementation between compound modules into subtasks, it may not always be applicable. In particular, P_1 may not implement Q_1 for all environments, but only if the environment behaves like P_2 , and vice versa. For such cases, an assumeguarantee proof rule is needed [Stark 1985; Grümberg and Long 1994; Abadi and Lamport 1995; Alur and Henzinger 1999]. The assume-guarantee proof rule for reactive modules asserts that in order to prove that $P_1 || P_2$ implements $Q_1 || Q_2$, it suffices to prove (1) $P_1 || Q_2$ implements Q_1 , and (2) $Q_1 || P_2$ implements Q_2 [Alur and Henzinger 1999]. Both proof obligations (1) and (2) typically involve smaller state spaces than the original proof obligation, because the complex compound module $P_1 || P_2$ usually has the largest state space involved. The assume-guarantee proof rule is circular; unlike the compositional proof rule, it does not simply follow from the fact that parallel composition and implementation behave like language intersection and language containment. Rather the proof of the validity of the assume-guarantee proof rule proceeds by induction on the length of traces. For this, it is crucial that every trace of a module can be extended.

PROPOSITION 2. (Assume-Guarantee) If $P_1 ||Q_2 \leq Q_1 ||Q_2$ and $Q_1 ||P_2 \leq Q_1 ||Q_2$, then $P_1 ||P_2 \leq Q_1 ||Q_2$.

The language of reactive modules, along with the assume-guarantee refinement checker, is supported by the model checker MOCHA [Alur et al. 2001], and the utility of the assume-guarantee reasoning has been demonstrated in analysis of a video-graphics image processor [Henzinger et al. 1998] and the network protocol PPP [Alur and Wang 2001].

3. MODES

The architectural hierarchy of a system can be formally captured by modules. In this section, we introduce modes to describe the behavior of atomic modules in a structured and hierarchical manner.

3.1 Syntax

3.1.1 Hierarchy. A mode has a refined control structure given by a hierarchical state machine. It basically consists of a set of submode instances SM connected by transitions T such that at each moment of time only one of the submode instances is active. A subset $I_m \subseteq SM$ designates the initial submode instances (visually marked with an arrowhead). A submode instance has an associated mode definition. Different instances can be associated with the same definition, but we require that

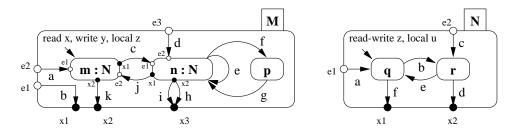


Fig. 2. Scoping rules and transition types

the modes form an *acyclic* graph with respect to this association². For example, the mode M in Figure 2 contains two submode instances, m and n pointing to the mode N; m is the initial submode instance.

By distinguishing between modes and instances we may control the degree of *sharing* of submodes. For example, the submode instances m and n in Figure 2 share the same mode N. Note that a mode resembles an *or* state in STATECHARTS.

3.1.2 Variables. A mode may have global as well as local variables. The set of global variables V_g is used to share data with the mode's environment. The variables in V_g are classified into a set V_r of read variables and a set V_w of write variables. Hence, $V_g = V_r \cup V_w$. The set of local variables V_l of a mode is accessible only by its transitions and submodes. The variables in $V_c = V_w \cup V_l$ are called controlled variables. Each mode has associated a set of initial states I_s over V_c (each controlled variable has to be initialized when first declared).

The scoping rules for variables are as in standard structured programming languages. For example, the mode M in Figure 2 has the global read variable \mathbf{x} , the global write variable \mathbf{y} and the local read-write variable \mathbf{z} . Similarly, the mode N has the global read-write variable \mathbf{z} and the local read-write variable \mathbf{u} .

The transitions of a mode may refer only to the declared global and local variables of that mode and only according to the declared read/write permission. For example, the transitions a,b,c,d,e,f,g,h,i,j and k of the mode M may refer only to the variables x, y and z. Moreover, they may read only x and z and write y and z.

The global and local variables of a mode may be shared between submode instances if the associated submodes declare them as global (the set of global variables of a submode has to be included in the set of global and local variables of its parent mode). For example, the value of the variable z in Figure 2 is shared between the submode instances m and n. However, the value of the local variable u is not shared between m and n.

3.1.3 *Entry/exit points.* To obtain a modular language, we require the modes to have well defined *control points* classified into entry points (marked as white bullets) and exit points (marked as black bullets). For example, the mode M in Figure 2 has the entry points e1,e2, e3 and the exit points x1,x2,x3. Similarly, the mode N has the entry points e1,e2 and the exit points x1,x2.

²Removing this restriction one obtains recursive state machines.

ACM Transactions on Programming Languages and Systems, Vol. TBD, No. TDB, Month Year.

The transitions connect the control points of a mode and of its submode instances to each other. For example, in Figure 2 the transition a connects the entry point e2 of the mode M with the entry point m.e1 of the submode instance m.

According to the points they connect, the transitions of a mode may be classified into *entry*, *internal* and *exit* transitions. For example, in Figure 2, a,d are *entry* transitions, h, i, k are *exit* transitions, b is an *entry/exit* transition and c,e,f,g,j are *internal* transitions. Exit transitions implicitly reinitialize (forget) the value of the local variables.

3.1.4 Preemption. To model preemption each mode (instance) has a special, default exit point dx, represented visually as the border of the mode. A transition starting at dx is called a *preempting* or *group transition* of the corresponding mode. It may be taken whenever the control is inside the mode and no internal transition is enabled. For example, in Figure 2 left, the transition **f** is a group transition for the submode **n**.

To achieve the preempting behavior we add for each internal exit point a *default* exit transition (from this point to dx) that is enabled when all other transitions starting in this point are disabled. These transitions are not explicitly drawn. They are implicit in the semantics of a mode.

For example, if the current control point is q inside the submode instance n and neither the transition b nor the transition f is enabled, then the control is transferred to the default exit point dx. If one of e or f is enabled and taken then it acts as a preemption for n.

Thus, the inner transitions have a higher priority than the group transitions, that is, we use *weak preemption* (like the weak kill in Unix, versus the strong kill -9). This priority scheme facilitates a modular semantics.

3.1.5 *History*. To allow history retention, we use a special default entry point *de*, represented visually also as the border of the mode. A transition entering the default entry point of a mode restores the values of all local variables along with the position of the control (if the mode was most recently left along one of its explicit exit points then all local variables are reinitialized and control is passed to one of the initial submodes).

For example, both transitions **e** and **g** in Figure 2, enter the default entry point *de* of **n**. The transition **e** is called a *self* group transition. A self group transition like **e** or more generally a self loop like **f**,**p**,**g** may be understood as an interrupt handling routine. While a self loop may be arbitrarily complex, a self transition may do simple things like counting the number of occurrences of an event.

To achieve the above behavior we semantically add *default entry* transitions from the default entry point de of a mode m to its internal points. The default exit transitions save the current point in a local history variable m.h and the default entry transitions restore the current control point from this variable. The initial value of m.h is the default entry point of an initial submode. A mode enriched with default entry and exit transitions is said to be *closed*.

Remark 3. (History free modes) The closure construction is not necessary for modes that have at least one transition enabled at each control point, including de. We call these modes history free. Their set of initial submodes has to be empty. \Box

Modular Refinement of Hierarchic Reactive Machines · 13

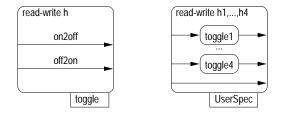


Fig. 3. UserSpec for VTS

Remark 4. (Mode instantiation) A mode can be viewed as an encapsulation operator over its submodes. Thus, modes are constructed from leaf-modes using encapsulation repeatedly in a non-recursive manner. Mode instantiation allows reuse and sharing by permitting both to refer to the same mode and to rename a (subset of) entry points, exit points, read variables, and write variables. With mode instantiation, the mode structure is a directed acyclic graph and it can be exploited in an efficient way for model checking [Alur and Yannakakis 1998; Alur et al. 2000]. To simplify the formal definitions in the following we assume a tree like structure obtained by replacing each instance by its corresponding mode. Moreover, we assume that there are no name conflicts regarding local variables and entry/exit points across modes.

Now we are ready to present a formal definition of modes.

Definition 6. (Mode) A mode consists of

Control points. A finite set E of entry points, and a finite set X of exit points. We also assume an additional default entry point de, and a default exit point dx, and define $dE = E \cup \{de\}$, and $dX = X \cup \{dx\}$.

Variables. A finite set V_r of read variables, a finite set V_w of write variables, and a finite set V_l of local variables. The variables $V_g = V_r \cup V_w$ and $V_c = V_w \cup V_l$ are called global and controlled variables, respectively. We assume that the sets V_g and V_l are disjoint (but the sets V_r and V_w need not be).

Submodes. A finite set SM of submodes. If N is a submode in SM, then it is required that $N.V_r \subseteq V_r \cup V_l$ and $N.V_w \subseteq V_w \cup V_l$.

Transitions. A finite set T of transitions of the form (e, α, x) , where e is in $dE \cup SM.dX$, x is in $dX \cup SM.dE$, and α is an action from $V_r \cup V_l$ to V_w if $x \in X$ and from $V_r \cup V_l$ to $V_w \cup V_l$ otherwise. We require that for each $e \in E$, the union $\cup \alpha$ such that $(e, \alpha, x) \in T$ for some x, is a non-blocking action.

Initial states. A non-empty subset I_s of states over V_c .

Initial submodes. A possibly empty subset $I_m \subseteq SM$ of initial submodes. If I_m is empty, we require that for each $e \in \{de\} \cup SM.dX$, the union $\cup \alpha$ such that $(e, \alpha, x) \in T$ for some x, is a non-blocking action (the mode is history free). Collectively we refer to I_s and I_m by I.

The interface of a mode is the tuple (V_r, V_w, E, X) . A leaf mode is a mode with no submodes and no local variables. A most general mode $G(V_r, V_w, E, X)$ is a mode that has the interface (V_r, V_w, E, X) and imposes no restriction on the update relation (it acts similarly to the environment).

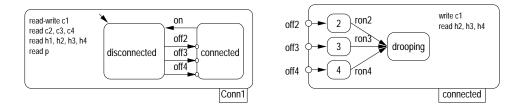


Fig. 4. The mode Conn1

Example 3. (Village telephone system) By using modes, the specification of the module UserSpec may be given by the history free modes UserSpec and toggle as shown in Figure 3. The modes toggle1 to toggle4 are obtained from mode toggle by renaming variable h with h1 to h4 respectively. The unmarked transition connecting de to dx is the identity transition expressing idling. The initial state and the other transitions are defined as follows.

read-write h1,h2,h3,h4: hookType := on on2off $\stackrel{\text{def}}{=}$ h = on -> h := off off2on $\stackrel{\text{def}}{=}$ h = off -> h := on

The connection module Conn1 may be restated as a hierarchic mode as shown in Figure 4 where initially c1 = disconnected. The transitions are defined as follows.

on $\stackrel{\text{def}}{=}$ h1 = on -> c1 := disconnected		
off2	$\stackrel{def}{=}$	h1 = off & h2 = off & c2 = disconnected & p = "1-2/3-4" -> c1 := 2
off3	$\stackrel{def}{=}$	h1 = off & h3 = off & c3 = disconnected & p = " $1-3/2-4$ " -> c1 := 3
off4	$\stackrel{def}{=}$	h1 = off & h4 = off & c4 = disconnected & p = " $1-4/2-3$ " -> c1 := 4
ron2	$\stackrel{def}{=}$	h2 = on -> c1 := drooping
ron3	$\stackrel{def}{=}$	h3 = on -> c1 := drooping
ron4	$\stackrel{def}{=}$	h4 = on -> c1 := drooping

Note that by distinguishing between control and data, mode diagrams are often more comprehensible than module specifications given by guarded commands. This may have an important impact if the control structure is quite involved and this is the reason why hierarchic state transition diagrams are so popular in software engineering methods.

When defining the behavior of a mode in the next section, we regard a mode as a black box, i.e., its submodes and (micro)transitions are hidden. However, in the compositionality proofs for modes, it is sometimes necessary to observe the behavior of a specific submode. Making explicit which submodes are to be observed motivates the introduction of generic modes.

Definition 7. (Generic mode) A generic mode (or mode context) $M[M_1, \ldots, M_k]$ consists of a mode M along with a set of visible submodes M_1, \ldots, M_k .

The visible submodes can also be viewed as the formal parameters of the generic mode. They can be substituted by other, compatible submodes. To simplify the presentation we consider only one visible submode. However, all results apply to the general case.

Definition 8. (Compatible modes) A mode M and a mode N are said to be compatible if $M.V_r = N.V_r$, $M.V_w = N.V_w$, M.E = N.E and M.X = N.X. \Box

If M[P] is a generic mode, and the submode P is compatible with a mode Q then M[Q] is the generic mode obtained by substituting P by Q; $\overline{M[P]}$ is the non-generic mode obtained by hiding P. Hence $\overline{M[P]} = M$. However, $M \neq M[Q]$ because they contain different submodes and $M \neq M[P]$ because P is hidden in M and visible in M[P].

Remark 5. (Choice of the language) Our goal is to show how behavior hierarchy can be handled semantically; a design language to be used by practicing software engineers would require many enhancements (such as parametric modes, rich set of data types and expressions). We had to make many choices to make the definition of hierarchy concrete. We discuss these choices by comparing them with the popular language STATECHARTS. As in STATECHARTS, multiple nested modes at different levels of hierarchy can be active simultaneously. Modes at the same level of hierarchy are composed only sequentially (that is, only one mode is active at any point in time). STATECHARTS, on the other hand, allows both sequential and concurrent composition of modes at the same level, and later we will illustrate how concurrent modes can be modeled in our language. In STATECHARTS, communication is by instantaneous broadcast of events. Events issued by one mode are available to all modes, and consequently, there can be no truly modular semantics of STATE-CHARTS. We have chosen shared-variables based communication, and the standard scoping rules are essential to our modular semantics. Entry and exit points in our language are inspired by the modeling language supported by UML-RT. Semantically, the critical entry/exit points are the default ones. These default points are used in the closure construction, to be discussed in the next section, which allows us to express transfer of control between a mode and its environment in a modular fashion. Transitions starting from default exit points allow modeling of exceptions and group transitions. This powerful feature is present in STATECHARTS as well as UML-RT, and our modular treatment of this feature is an important contribution. We allow both interleaving and synchronous semantics for top-level modes (that is, modules), and when a mode is chosen, it executes transitions until the control reaches one of its exit points (or no more enabled transitions are available). Alternative choices are possible. However, the choice for assigning higher priority to the inside transitions than the outside group transitions is necessary for modularity. \Box

3.2 Operational Semantics

We introduce some additional notation to formally define the set of executions of a mode. For a (generic) mode M we use O to denote the set $dE \cup dX$ of observable control points and C to denote the set $dE \cup dX \cup SM.dE \cup SM.dX$ of all control

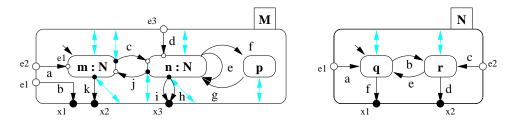


Fig. 5. Closed behavior diagrams

points. Pairs of the form (c, s), where c is a control point and s is a state are called *configurations*. For notational convenience, we view the set T of transitions also as a binary relation over configurations: if $(e, \alpha, x) \in T$ and $(s, t) \in \alpha$, we write $((e, s), (x, t)) \in T$.

3.2.1 The priority among transitions. In Figure 5, the execution of a mode, say n, starts when the environment transfers the control to one of its entry points e1 or e2. The execution of n terminates either by transferring the control back to the environment along the exit points x1 or x2 or by "getting stuck" in q or r as all transitions starting from these leaf modes are disabled.

In this case the control is implicitly transferred to M along the default exit point n.dx. Then, if the transitions e and f are enabled, one of them is nondeterministically chosen and the execution continues with n and respectively with p. If both transitions are disabled the execution of M terminates by passing the control implicitly to its environment at the default exit M.dx. Thus, the transitions within a mode have a higher priority compared to the group transitions of the enclosing modes.

3.2.2Default exit transitions. In any mode, some transition leaving an entry point is guaranteed to be enabled, so execution can get stuck only at an exit point of a submode. In Figure 5 these points are explicitly drawn as black bullets. To make the transfer of control explicit, we add *default exit transitions* as follows. From an exit point x of a submode of M, we add a transition to the default exit point dxthat is enabled if and only if all the explicit outgoing transitions from x are disabled. If the actions are given by guarded commands, and if g_1, \ldots, g_n are the guards of the explicit transitions, the guard of the default transition is $\neg(g_1 \lor \ldots \lor g_n)$. For example, in Figure 5, the default exit transitions starting in q and r have the guards $\neg(g_b \lor g_f)$ and $\neg(g_e \lor g_d)$ respectively, where g_b, g_d, g_e, g_f are the guards of the transitions b,d,e,f, respectively. Similarly, the default exit transition starting in **n.dx** has the guard $\neg(g_e \lor g_f)$ and the default exit transition starting in **p** has the guard $\neg g_q$. Each default exit transition saves the local state which is restored upon the subsequent entry to the default entry point. To remember the location of control, we add a new local variable h to a mode M and an action body to each default exit transition (from an exit point x to dx) that saves x in this history variable h.

3.2.3 Default entry transitions. The transitions entering the default entry point of a mode M restore the local state. Again, we introduce explicit default entry ACM Transactions on Programming Languages and Systems, Vol. TBD, No. TDB, Month Year.

transitions to restore the location of control. For each default exit transition from an exit x of a submode of M, there is a *default entry transition* from *de* to x that is taken when the value of the local history variable h coincides with x. If x was a default exit point n.dx of a submode n then, as shown in Figure 5, the default entry transition is directed to n.de. The reason is that in this case, the control was blocked somewhere inside of n and default entry transitions originating in n.de will restore this control.

The closure of the mode M of Figure 2 is shown in Figure 5, where each gray bidirectional arrow represents two unidirectional arrows. The closure construction is defined formally below.

Definition 9. (Closure) Let $M = (E, X, V_r, V_w, V_l, SM, T, I)$ be a mode. The closure c(M) of M is defined to be: (1) M if SM is empty, (2) $(E, X, V_r, V_w, V_l, c(SM), T, I)$ if M is history free, and (3) $(E, X, V_r, V_w, V_l \cup \{h\}, c(SM), dT, dI)$ otherwise. c(SM) is the set of closed submodes where $c(SM) = \{c(m) \mid m \in SM\}$. dI is the set of initial states and modes extending I_s with an initial value for h in $I_m.de. dT$ is a set of transitions obtained from T by adding reinitialization of local variables to the exit transitions and by adding, for each exit $x \in SM.dX$, the transitions (x, α_x, dx) and (de, β_x, \tilde{x}) , where

- —for $x \in SM.X$, $\tilde{x} = x$, and for x = N.dx, $\tilde{x} = N.de$,
- —for states s and t, $(s,t) \in \alpha_x$ iff $t \cdot h = x$, $t \cdot y = s \cdot y$ for $y \neq h$, and for every transition (x, α, x') in T, α is disabled at s,
- —for states $s, (s, s) \in \beta_x$ iff s.h = x.

Now we proceed to define the operational semantics. Intuitively, a round of the machine associated to a mode starts when the environment passes the updated state along a mode's entry point and ends when the state is passed to the environment along a mode's exit point. All the internal steps (the *micro steps*) are hidden. We call a round also a *macro step*. Note that the macro step of a mode is obtained by alternating its closed transitions and the macro steps of the submodes.

Definition 10. (Macro transitions of modes) The set V_p of private variables of a mode $M = (E, X, V_r, V_w, V_l, SM, T, I)$ is defined to be the set $V_l \cup SM.V_p$. The set mT of macro-transitions consists of transitions of the form (e, α, x) with $e \in dE$, $x \in dX$, and α is the action from from $V_r \cup V_p$ to $V_w \cup V_p$, defined as follows. Given the macro-transitions of the submodes of M, a micro-execution of M is a sequence of the form $(e_0, s_0) \to (e_1, s_1) \to \cdots \to (e_n, s_n)$ of control points $e_i \in C$ and states s_i over $V_g \cup V_p$ such that

- -for even *i*, the transition $((e_i, s_i), (e_{i+1}, s_{i+1}))$ is in the closure dT of T,
- —for odd *i*, the transition $((e_i, s_i), (e_{i+1}, s_{i+1}))$ is in *SM*.*mT*.

Given such an execution for an entry point e_0 and an exit point e_n of M, the macro-transition relation mT contains $((e_0, s_0), (e_n, s_n))$.

In the above definition of micro-executions of a mode, the states s_i are valuations to the variables $V_g \cup V_p$, but only a subset of these influence each step. The other remain unchanged. The *operational semantics of a mode* M consists of its control points, global variables, private variables, and its macro-transitions.

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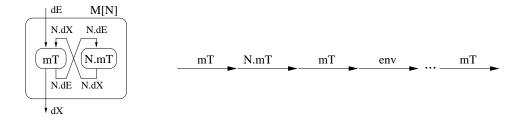


Fig. 6. The traces of M[N]

Remark 6. (Consistency of modes) To ensure consistency we assume the closed transition relation contains no cycles. In the finite state case, checking for cycles is easy. In the infinite state case, one can check for sufficient conditions. For example, one can check for the occurrence of a predefined "wait" mode within each cycle. This mode contains two explicit points e and x and two identity transitions: one from e to dx and one from de to x.

A top-level mode is a mode M with default entry/exit points only. Such a mode can be viewed as a module with private variables V_p , interface variables V_w , external variables $V_r \setminus V_w$, initialization specified by the initial states and update specified by macro-transitions from de to dx. For example, mode **Conn1** is a top-level mode.

The operational semantics of a mode context M[N] is defined the same way as that of a mode, except that the submode N and the transfer of control between the mode M and the submode N is visible. Thus, a macro step of M[N] starts either at an entry point of M or at an exit point of N, and terminates at an exit point of M or at an entry point of N.

Definition 11. (Macro transitions of generic modes) For a generic mode M[N], the macro-transition relation mT contains the pair ((e, s), (e', s')) if $e \in M.dE \cup$ $N.dX, e' \in M.dX \cup N.dE, s$ and s' are states over $M.V_g \cup M.V_p$, and there is a micro-execution of M from (e, s) to (e', s').

The operational semantics of a generic mode M[N], consists of the visible control points $M.O \cup N.O$, global variables $M.V_g$, private variables $M.V_p$, macro-transition relation of M[N], and the operational semantics of the submode N.

3.2.4 Trace Semantics. The execution of a mode may be best understood as a game, i.e., as an alternation of moves, between the mode and its environment. In a mode move, the mode gets the state from the environment along its entry points. It then keeps executing until it gives the state back to the environment along one of its exit points. In an environment move, the environment gets the state along one of the mode's exit points. Then it may update any variable except the mode's private ones. Finally, it gives the state back to the mode along one of its entry points. An execution of a mode is obtained by repeating the mode and environment moves, and a trace is obtained from an execution by retaining only the global states.

Definition 12. (Denotational semantics of modes) An execution of a mode M is a sequence

 $(e_0, s_0) \rightarrow (x_0, t_0) \rightarrow (e_1, s_1) \rightarrow (x_1, t_1) \rightarrow \cdots \rightarrow (x_n, t_n)$

of control points $e_i \in dE$, $x_i \in dX$ with e_0 in E, $s_0.V_c$ in I and states s_i and t_i over $V_g \cup V_p$ such that for all i, $((e_i, s_i), (x_i, t_i)) \in mT$ and $s_{i+1}.V_p = t_i.V_p$. Given such an execution, the corresponding *trace* of M is obtained by projecting each state to the set V_g of global variables. The set of traces of M is denoted L_M . The *denotational semantics* of a mode M consists of its observable points O, global variables V_g , and the set L_M of traces. \Box

Note that, for a top level mode, the environment is another reactive module. For a lower level mode, the environment may be a regular or a group transition.

The execution of a generic mode M[N] can be defined similarly as alternation of moves of three kinds. The context mode M gets the state at an entry point or at an exit point of its submode N. It keeps executing until it gives the state back to the environment along one of its exit points or to the submode N at one of the entry points of N. The environment gets the state along one of the exit points of M. It possibly updates the global variables of M, and returns the state to the context M along one of the entry points of M. The submode N gets the state at one of its entry points. It executes one of its macro transitions, and returns the state to M at one of the exit points of N. To obtain a trace from an execution, we retain, at each control point, the values of the variables global at that point. For a mode context M[N], for a control point $c \in M.dX \cup M.dE$, let $c.V_g$ be $M.V_g$, and for a control point $c \in N.dX \cup N.dE$, let $c.V_g$ be $N.V_g$.

Definition 13. (Denotational semantics of generic modes) An executions of a generic mode M[N] is a sequence

$$(e_0, s_0) \rightarrow (x_0, t_0) \rightarrow (e_1, s_1) \rightarrow (x_1, t_1) \rightarrow \cdots \rightarrow (x_n, t_n)$$

of control points $e_i \in M.dE \cup N.dX$, $x_i \in M.dX \cup N.dE$, and states s_i and t_i over $M.V_g \cup M.V_p$ such that

- —the execution starts at $e_0 \in M.dE$ with $s_0.(M.V_c) \in M.I$,
- —for all i, $((e_i, s_i), (x_i, t_i))$ is a macro-transition of M[N],
- —for all i, if $x_i \in N.dE$ then $e_{i+1} \in N.dX$, and
- —for all *i*, the pair $((x_i, t_i), (e_{i+1}, s_{i+1}))$ is in N.mT if $x_i \in N.dE$ and $s_{i+1}.V_p = t_i.V_p$ otherwise.

Given such an execution, a trace of M[N] is obtained by projecting each state associated with a point c to the set $c.V_g$. The set of traces of M[N] is denoted as before by $L_{M[N]}$. The denotational semantics of a generic mode M[N] consists of the observable points and global variables of M, the observable points and global variables of N and the set of traces $L_{M[N]}$. \Box

In order to show that our trace semantics is compositional, we need to be able to define the semantics of a mode M only in terms of the trace semantics of its submodes. This is the same as being able to compute the set of traces of the generic mode M[N] in terms of the traces of N. Recall that an execution of a generic mode is obtained by alternating between its macro-transitions, its visible submodes macrotransitions and environment transitions (see Figure 6). To formalize the notion of compositionality, we need to define a projection operation for trace-like sequences.

Definition 14. (Trace extraction) Given a sequence $\sigma = (e_0, s_0)(e_1, s_1) \dots (e_n, s_n)$ of control points and states, and a mode N, the restriction $\sigma \uparrow N$ is the sequence obtained from σ by replacing each s_i by $s_i . (N.V_g)$ and by deleting pairs (e_i, s_i) if $e_i \notin N.dE \cup N.dX$. Similarly, $\sigma \uparrow N$ is the sequence obtained from σ by replacing each s_i by $s_i . (N.V)$ and by deleting pairs (e_i, s_i) if $e_i \notin N.dE \cup N.dX$. \Box

Recall that $G(V_r, V_w, E, X)$ denotes the most general mode with the given set of read/write variables and control points. For a generic mode M[N], we will use M[G] to denote the generic mode obtained by replacing N with the most general mode $G(N.V_r, N.V_w, N.E, N.X)$. The next lemma captures the essence of compositionality of trace semantics for the encapsulation.

LEMMA 1. (Trace construction) Let M[N] be a generic mode, and let τ be a sequence of the form $(c_0, s_0) \rightarrow (c_1, t_1) \rightarrow (c_2, s_2) \rightarrow (c_3, s_3) \rightarrow \cdots \rightarrow (c_n, s_n)$ such that each c_i is in $M.dE \cup N.dE \cup M.dX \cup N.dX$, and each s_i is a state over $c_i.V_g$. Then τ is a trace of M[N] iff τ is a trace of M[G] and $\tau \uparrow N$ is a trace of N.

PROOF. Consider a sequence τ of the form $(c_0, s_0) \to (c_1, t_1) \to (c_2, s_2) \to (c_3, s_3) \to \cdots \to (c_n, s_n)$ such that each c_i is in $M.dE \cup N.dE \cup M.dX \cup N.dX$, and each s_i is a state over $c_i.V_q$.

Suppose τ is a trace of M[N], and let α be the corresponding execution. Then $\alpha \uparrow N$ is an execution of N, and hence, $\tau \uparrow N$ is a trace of N. From α , if we project out the private variables of N, then, by definition of the most general mode, we get an execution of M[G], and thus, τ is a trace of M[G].

Suppose τ is a trace of M[G] and $\tau \uparrow N$ is a trace of N. Let α be an execution of M[G] corresponding to τ and β be an execution of N corresponding to $\tau \uparrow N$. Then α and β must have the form

$$\begin{aligned} (c_0, u_0) &\to (c_1, u_1) \to (c_2, u_2) \to (c_3, u_3) \to \dots \to (c_n, u_n) \\ (c_i, v_i) &\to (c_{i+1}, v_{i+1}) \to (c_j, v_j) \to (c_{j+1}, v_{j+1}) \dots \to (c_m, v_m) \end{aligned}$$

and agree on the global variables, i.e., for each c_i in β , $u_i.(N.V_g) = v_i.(N.V_g)$. Construct a new sequence γ from α by replacing $u_i.(N.V_p)$ with $v_i.(N.V_p)$ at points c_i in β and repeating the last values $v_i.(N.V_p)$ at the intermediate points. By construction, the N-transitions of γ are in N.mT. Moreover, since the environment cannot observe the private variables of N, the M-transitions of γ are in mT. Hence γ is an execution of M[N]. As a consequence, τ is a trace of M[N]. \Box

The above lemma is used to prove the following theorem which says that to compute the set of traces of a mode M, only the set of traces of its submode, and not the internal structure of the submode, is needed.

THEOREM 1. Trace construction For a generic mode M[N], the set of traces of M can be computed from the set of traces of the submode N and the set of traces of M[G].

PROOF. The set of traces of M[N] is computed as in Lemma 1. Then, the traces in L_M are the sequences $\sigma \Uparrow M$ where $\sigma \in L_{M[N]}$.

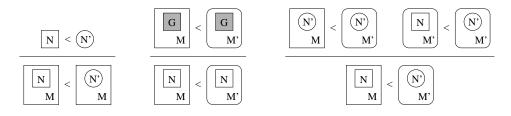


Fig. 7. Compositional and assume/guarantee rules

3.3 Refinement

The trace semantics leads to a natural notion of refinement between modes: a mode M refines a mode N if they are compatible and every trace of M is a possible trace of N.

Definition 15. (Refinement) A mode M refines a mode N, written as $M \leq N$, if M is compatible with N and $L_M \subseteq L_N$. A generic mode M[N] refines a generic mode M'[N'], if M is compatible with M', N is compatible with N', and $L_{M[N]} \subseteq$ $L_{M'[N']}$.

A most general mode $G(V_r, V_w, E, X)$ is most general for a given data and control interface: verify that every mode mode M refines $G(M.V_r, M.V_w, M.E, M.X)$. The refinement operator is compositional with respect to hiding of submodes:

THEOREM 2. (Compositionality of hiding) Suppose that $M[P] \preceq N[Q]$. Then $\overline{M[P]} \preceq \overline{N[Q]}.$

PROOF. Let $t \in L_{\overline{M[P]}}$. Then by definition traces, there is a trace $u \in L_{M[P]}$, such that $t = u \Uparrow M$. By hypothesis $u \in N[Q]$ holds, and by compatibility of M and $N, t = u \Uparrow N[Q]$. Again by definition of traces of modes $t \in L_{\overline{N[Q]}}$.

The next theorem states that refinement is also compositional with respect to submodes.

THEOREM 3. (Mode compositionality) Given a generic mode M[Q] and a mode P such that $P \preceq Q$. Then $M[P] \preceq M[Q]$.

PROOF. Let $t \in M[P]$. Then by Lemma 1, $t \uparrow P \in L_P$. By compatibility and refinement hypothesis it follows that $t \uparrow Q \in L_Q$. Hence, by Lemma 1, $t \in P[Q]$.

The refinement rule is shown in a visual way in Figure 7, left. By applying Theorems 2 and 3 one immediately obtains that $\overline{M[P]} \preceq \overline{M[Q]}$ provided that $P \preceq Q$. Hence, refinement is compositional with mode encapsulation too.

Example 4. (Village telephone system) A refinement of the mode toggle is the mode ctoggle shown in Figure 8, left. While in toggle, the switch to on is enabled whenever h is off, in ctoggle the switch to on is enabled whenever c is disconnected. The transitions of ctoggle are given below.

def $h = on \rightarrow h := off$ on2off $coff2on \stackrel{def}{=}$ c != disconnected -> h := on

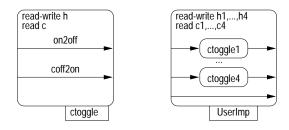


Fig. 8. An implementation of UserSpec

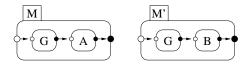


Fig. 9. Context refinement counter example

To prove refinement, the additional read variable c may be added to the mode toggle without any harm because it is not read by this mode. In a more general setting, the refinement rules would take additional variables into account.

Since $ctoggle \leq toggle$ we obtain by compositionality and the fact that renaming does not change refinement that UserImp \leq UserSpec.

The refinement operation is also compositional with respect to mode contexts, i.e., with respect to generic modes construction.

THEOREM 4. (Context Compositionality) Suppose that $M[Q] \preceq M'[Q]$ and $P \preceq Q$. Then $M[P] \preceq M'[P]$.

PROOF. Let $t \in L_{M[P]}$. Then by the second hypothesis and Theorem 3, $t \in L_{M[Q]}$. Using now the first hypothesis it follows that $t \in L_{M'[Q]}$. Since $t \Uparrow P \in L_P$ we conclude by Lemma 1 that $t \in L_{M'[P]}$. \square

To see how this theorem can be used, consider the middle picture of Figure 7. Here, the submode N is reused in two different contexts M and M'. The rule says that the refinement $M[N] \leq M'[N]$ can be established by replacing N by the most general mode G (or any mode that N refines), thereby simplifying the system.

Similarly to the mode compositionality rule where the interaction of the submodes with their environment is observable in the rule's hypothesis, the interaction between the mode context and its submodes has to be observable in the hypothesis of the context compositionality rule too. In general, from $\overline{M[G]} \preceq \overline{M'[G]}$ we cannot conclude that $\overline{M[P]} \preceq \overline{M'[P]}$. For example, consider the modes M[G]and M'[G] defined as in Figure 9, where the set of states $Q = \{a, b, c\}$ is the same for all modes. Define the macro transition relations as follows: $G.mT.\alpha =$ $Q \times Q$, $A.mT.\alpha = \{(a, b), (b, c), (c, a)\}$, $B.mT.\alpha = \{(a, c), (b, a), (c, b)\}$. It is easy to observe that $\overline{M[G]} = \overline{M'[G]} = G$. Now consider the identity relation I with $I.mT.\alpha = \{(a, a), (b, b), (c, c)\}$. Clearly, $\overline{M[I]} = A \not\subseteq \overline{M'[I]} = B$.

The compositionality rules allow us to decompose the proof obligation into refinement of submodes in the most general context, and refinement of contexts under

the most general submode. Can we allow circular assume-guarantee reasoning in the style of Proposition 2? The answer is yes, if we can observe the interaction between the context M and submode N.

THEOREM 5. (Assume Guarantee) Let M[N] and M'[N'] be compatible generic modes. If both $M[N'] \preceq M'[N']$ and $M'[N] \preceq M'[N']$ then $M[N] \preceq M'[N']$.

PROOF. The proof is by induction over the length of traces $t \in L_{M[N]}$.

Base case: Suppose t is the empty trace. This is a trace both of $L_{M[N]}$ and of $L_{M'[N']}$.

Induction step: Suppose t is of length n + 1 of the form $t = t'(c_{n+1}, s_{n+1})$. By induction hypothesis, t is a trace of both M[N] and M'[N']. Let α be an execution of M[N] corresponding to t, and let β be an execution of M'[N'] corresponding to t. Now we do a case analysis for the last transition in α .

Case: Environment transition. An environment transition only updates the global variables. Let s' be a state that agrees with the last state in β on all private variables of M'[N'], and agrees with s_{n+1} on the global variables of M'. Then $\beta(c_{n+1}, s')$ is an execution of M'[N'], and hence, t is a trace of M'[N'].

Case: Transition of M. Observe that α , with private variables of N projected out, is an execution of M[G], and hence, t is a trace of M[G]. We will establish that $t \uparrow N'$ is a trace of N', and then, by lemma 1, $t \in L_{M[N']}$. Since t' is trace of M'[N'], by lemma 1, $t' \uparrow N'$ is a trace of N'. If c_{i+1} is not a control point of N, then $t \uparrow N'$ is the same as $t' \uparrow N'$. Suppose c_{i+1} is an entry point of N. Consider the execution γ of N' that corresponds to the trace $t' \uparrow N'$. Let s' be a state that agrees with the last state in γ on all private variables of N', and agrees with s_{n+1} on the global variables of N'. Then $\gamma(c_{n+1}, s')$ is an execution of N', and hence, $t \uparrow N'$ is a trace of N'.

Thus, $t \in L_{M[N']}$. Since $M[N'] \preceq M'[N']$, t must be in $L_{M'[N']}$.

Case: Transition of N. Since t is a trace of M[N], by lemma 1, $t \uparrow N$ is a trace of N. Let β' be the execution β with each state projected by deleting the private variables of N', and s' be a state of M'[G] that agrees with s_{n+1} on global variables of N and with the last state in β' on the rest of the variables. Now, $\beta(c_{n+1}, s')$ is an execution of M'[G], and hence, t is a trace of M'[G]. By lemma 1, t is also a trace of M'[N]. Since $M'[N] \preceq M'[N']$, t must be in $L_{M'[N']}$. \Box

A visual representation of this rule is shown in Figure 7, right. Intuitively, to establish that $\overline{M[N]}$ refines $\overline{M'[N']}$, we first prove that the generic mode M[N'] refines the generic mode M'[N'], and then prove that M'[N] refines M'[N']. Thus, to establish that the implementation context M refines the specification context M', we assume the submode specification N', and to prove that the implementation submode N refines the specification submode N', we assume that the context meets the specification M'. In both cases, the interaction between the submode and its context must be observable, and thus, we need to consider traces of the generic mode.

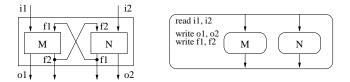


Fig. 10. Parallel composition by interleaving

4. CONJUNCTIVE MODES

In the model we have discussed so far the architectural hierarchy of the system is described by parallel composition of modules, and the behavioral hierarchy within atomic modules is described by sequential composition of modes. Languages such as Statecharts allow mixed hierarchies where *and* (conjunctive or concurrent) and *or* (disjunctive or sequential) modes can be arbitrarily nested. In this section, we show that the semantics of modes presented in the previous sections is a very expressive setting in which we can define various interesting parallel composition operators found in the literature. Moreover, the user may define himself the desired composition and can even mix different ways of compositions in the same setting. The main characteristics of the semantics that allow us to do this are hierarchy, control interfaces and the fact that closed modes never get "stuck" and take "no time" to execute. In this setting, a module is a top mode that communicates with its environment via shared variables. In the following subsections we discuss two common forms of parallel composition.

4.1 Concurrency by interleaving.

In the interleaving semantics, a round (macrostep) of a composed mode is a round of one of its submodes. The choice between the submodes is arbitrary. The set of write variables of the composed mode is the union of the sets of write variables of the submodes. The set of read variables of the composed mode is the union of the sets of read variables of the submodes minus the sets of write variables of the submodes. The set of initial states is the product of the sets of initial states of the components. This semantics is easily accommodated in our framework as shown in Figure 10 right.

Definition 16. (Interleaving composition) Given two top-level modes M and N, the interleaving composition M|N of the modes M and N is a hierarchic mode defined as follows.

Control points. The sets of entry/exit points $dE = \{de\}$ and $dX = \{dx\}$. Variables. The sets of write, read and local variables V_w , V_r and V_l :

 $V_w = M.V_w \cup N.V_w, \quad V_r = (M.V_r \cup N.V_r) \setminus V_w, \quad V_l = \emptyset.$

Submodes. The set $\{M, N\}$.

Transitions. The set $T = \{(de, i, M.de), (de, i, N.de), (M.dx, i, dx), (N.dx, i, dx)\}$ where i is the identity action.

Initial states and modes. The sets $I_s = (M.I).V_w \times (N.I).V_w$ and $I_m = \emptyset$. From the above definition it follows easily that $M \preceq M | N$ and that interleaving composition is compositional with respect to refinement.

Modular Refinement of Hierarchic Reactive Machines · 25

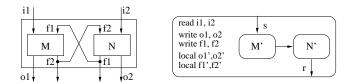


Fig. 11. Lock-step parallel composition

THEOREM 6. If $M_1 \preceq N_1$ and $M_2 \preceq N_2$ are top modes, then $M_1 | M_2 \preceq N_1 | N_2$.

4.2 Lock-step parallel composition

In the lock-step composition, a round (macro step) of a composed mode consists of a sequence of rounds of the submodes, one round for each submode. The order of execution of the submodes is arbitrary and has no consequence on the trace semantics of the composed mode. As a consequence, each linearization of the submodes is a valid translation of the corresponding composed module. A linearization that gives priority to the left mode is shown in Figure 11, right. The associated module diagram is given in Figure 11, left. Note that the module diagram explicitly shows the communication variables. To make sure that each mode in the sequence accesses the same values of the variables, namely the values at the beginning of the round, we have to latch the write variables. This is easily accomplished as follows.

First, we add for each write variable of a submode a local primed variable to the composed mode. The primed variable is used to keep the current value, i.e., the value computed by the submode in the current round. The entry transition s of the composed mode initializes the primed variables with the value of the corresponding unprimed variables. The exit transition r assigns the value of the primed variable back to the unprimed one.

Second, since the submodes were defined in terms of unprimed variables, they have to be renamed such that each occurrence of an unprimed write variable is replaced by a primed variable. Note that only one submode is renamed with respect to a write variable because composability assures that the two submodes write to disjoint sets of variables. In Figure 11 the renamed modes are marked as primed.

Definition 17. (Lock-step composition) Given two top-level modes M and N such that $M.V_w \cap N.V_w = \emptyset$. The lock-step composition $M \parallel N$ of the modes M and N is defined as follows.

Control points. The sets $dE = \{de\}, dX = \{dx\}$ of entry and exit points .

Variables. The sets V_w , V_r and V_l of write, read and local variables:

 $V_w = M.V_w \cup N.V_w, \quad V_r = (M.V_r \cup N.V_r) \setminus V_w, \quad V_l = M.V'_w \cup N.V'_w.$

Submodes. The set $\{M', N'\}$ where:

$$M' = M[M.V'_w/M.V_w], \quad N' = N[N.V'_w/N.V_w]$$

That is, M' is obtained from M by replacing, for each variable x in $M.V_w$, every reference to x in M by x'.

Transitions. The set $T = \{(de, s, M'.de), (M'.dx, i, N'.de), (N'.dx, r, dx)\}$ where i is the identity action, s, r are defined by

$$s \stackrel{\text{def}}{=} x_1' := x_1; \dots; x_n' := x_n$$
$$r \stackrel{\text{def}}{=} x_1 := x_1'; \dots; x_n := x_n'$$

and $x_1 \ldots x_n$ are the variables in V_w .

Initial states and modes. The sets $I_s = (M.I).V_w \times (N.I).V_w$ and $I_m = \emptyset$. \Box

From the above definition it follows that parallel composition is compositional with respect to refinement.

THEOREM 7. If $M_1 \leq N_1$ and $M_2 \leq N_2$ are top modes, then $M_1 || M_2 \leq N_1 || N_2$.

PROOF. Let $Q[M_1, N_1]$ be the mode corresponding to $M_1 || N_1$. Since $M_1 \leq M_2$ and $N_1 \leq N_2$, $Q[M_1, N_1] \leq Q[M_2, N_2]$. But $Q[M_2, N_2]$ is the mode corresponding to $M_2 || N_2$. \Box

Remark 7. (Await dependencies) In the above theorem we considered only Moorelike modules and their translation to Moore-like modes. In general, reactive modules in MOCHA allow to define a partial order (await dependencies) between atomic modules called atoms such that the values computed by an atom in a round may influence the values computed by any atom greater in the partial order in the same round. We can easily handle this in our setting by considering that each submode in the composition has both an unprimed and a primed copy of each variable. Similarly to the transitions of a parallel mode, the submodes may then use both primed and unprimed variables in their transitions. The partial order, is nothing but a constraint that has to be satisfied by the linearization of modes. Note that in this setting, one may define events analogously to reactive modules. For example, testing for the occurrence of an event e? is translated to $e' \neq e$. Similarly, issuing an event e! is translated to $e' := \neg e$.

The lock-step composition of two top-level modes is essentially the same as the parallel composition of the modules defined by these modes. The only difference is of technical nature: the modules assume that the environment is not writing on their interface variables while the modes do not make this assumption for their write variables. This mismatch can be easily handled by defining two operators: mod2mdl and mdl2mod that convert top-level modes to modules and reciprocally modules to modes.

The *mod2mdl* operation only adds the environment assumption. The *mdl2mod* operation works recursively. First it replaces each submodule by a submode that does not impose the environment assumption. Then it replaces the parallel composition of modules by the lock-step composition of modes. Since any atom in reactive modules is also a mode, this gives an algorithm to completely convert any module to a mode.

Example 5. (Village telephone system) The recursive conversion of the village telephone system module to a mode is shown in Figure 12. \Box

Hence, a module diagram may be regarded as a convenient shorthand for a particular mode diagram. However, since a module diagram shows the observable variables

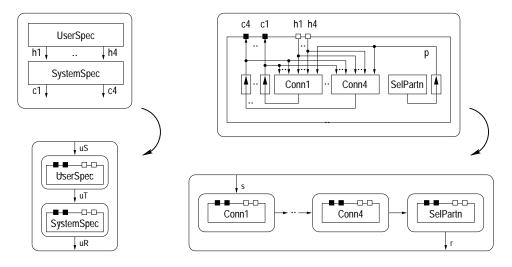


Fig. 12. Recursive translation of modules to modes

and their sharing among modes, a module diagram is more convenient for the representation of parallel modes. The fact that a module diagram represents a mode allows us to integrate module diagrams inside of mode diagrams and the other way around. As a consequence, we can construct arbitrary and/or hierarchies of mixed synchronous or asynchronous components.

The construction of such hierarchies is an important step towards modeling modern concepts like dynamical network reconfiguration or mobility. For example, creating a new process can be easily accomplished by a transition that takes a module diagram into another module diagram that has an additional module. To model mobility we also need to distinguish between location of computation modules and software modules. It is important to note that this expressive power comes together with a clean semantics that can be used successfully in analysis.

Example 6. *(Hot-lines)* A possible implementation of the module SystemSpec is by hot lines, as shown in Figure 13, right. The mode definition of the module Line1 is shown in Figure 13, left. The initial state and the associated transitions are given below. The definition of the module Line2 is similar.

```
write c1,c2: connType = disconnected;
hloff? \stackrel{\text{def}}{=} hl = off -> skip;
h2off? \stackrel{\text{def}}{=} h2 = off -> skip;
           \stackrel{\text{def}}{=}
                h1 = on \rightarrow skip;
h1on?
           \stackrel{\text{def}}{=}
                 h2 = on \rightarrow skip;
h2on?
           \stackrel{\mathsf{def}}{=}
                 h2 = off \rightarrow c1 := 2; c2 := 1;
conn2
           \stackrel{\mathsf{def}}{=}
                  h1 = off \rightarrow c1 := 2; c2 := 1;
conn1
           def
=
                  h1 = on -> c1 := disconnected; c2 := drooping;
diDr1
```

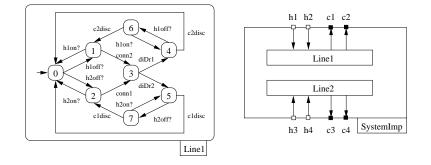


Fig. 13. Hot lines implementation

diDr2 $\stackrel{\text{def}}{=}$ h2 = on -> c2 := disconnected; c1 := drooping; c2disc $\stackrel{\text{def}}{=}$ h2 = on -> c2 := disconnected; c1disc $\stackrel{\text{def}}{=}$ h1 = on -> c1 := disconnected;

We would like now to prove by assume/guarantee that UserImp \parallel SystemImp \preceq UserSpec \parallel SystemSpec. This can be done in a mixed module/modes setting or solely in a modes setting by converting the above modules to modes. In this case, we can use the assume/guarantee rule for modes.

Let SystemImpPar be the top level mode for the parallel composition together with the mode for SystemImp and SystemSpecPar be the top level mode for the parallel composition together with the mode for SystemSpec. Then we have to prove that

> SystemImpPar[UserSpec] ≤ SystemSpecPar[UserSpec] SystemSpecPar[UserImp] ≤ SystemSpecPar[UserSpec]

It is easy to see that in this case the assume/guarantee rule for modes is the same as the one for modules. $\hfill \Box$

5. CONCLUSIONS

The notion of hierarchy is useful for structuring architecture of component connections as well as for describing behavior of individual components. While architectural hierarchy has been well understood in context of modular reasoning, there has been no basis for modular reasoning about behavior hierarchy. Existing languages for hierarchic state-machines have complex operational semantics and no notion of observational refinement. We show that hierarchy can be preserved in observational trace semantics even in presence of powerful features such as mode hierarchy, exceptions, history retention, conjunctive modes, and mode reuse. Our language has powerful rules for refinement of modes, and should provide a basis for systematic development and formal analysis of hierarchic descriptions.

The current proposal builds on our previous work on the language of reactive modules, the toolkit MOCHA that supports assume-guarantee refinement checks and the relational semantics for hierarchic machines in [Grosu et al. 1998]. The operations of building a mode by connecting submodes, scoping of local variables, and

mode instantiation, are direct analogs of parallel composition of modules, variable hiding, and module instantiation, respectively. Indeed, the same graphical-user-interface can be used for both the module diagrams and mode diagrams. In [Alur et al. 2000] we report on the GUI and a model checker for hierarchic modes.

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