Supine and Prone Colon Registration Using Quasi-Conformal Mapping

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Supine-Prone Colons

Data Acquisition
- CT Scans
- Slices → CT Volume
- Res.: 0.7x0.7x1.0mm
- Typical Size: 512x512x450

Representation
- Cleansing, Segmentation ...
- Colon Wall Surface Extraction
  - Topological cylinder
  - Triangular mesh

Supine (facing-up)  Prone (facing-down)
Registration Purpose

- To Clarify Situations

3D Endoluminal View

Supine

Prone
Registration Purpose cont.

- To Confirm Polyp Finding

Polyp Detection

Supine  Prone
3D Colon Registration Framework

Conformal Geometry
Scope

• **Situation** - Colorectal Cancer: 3rd most incident in US

• **Virtual Colonoscopy (VC)**
  • *Advantage*: Non-invasive precancerous polyps screening
  • *Key*: *Computer-aided detection* (CAD)

• **Visualization Tools by Supine-Prone Registration**
  • *Goal*: Correlation to enhance polyp detection, or confirm a finding
  • *More*: Distortion measurement of colon tissues and muscles
Overview

- **Problem** - Supine and Prone Colon Registration
  - **Challenge**: Non-rigid deformation and substantial distortion, due to position shifting

- **Solution** - Conformal Geometry Based Method
  - **Formulation**: Matching between 3D topological quadrilaterals
  - **Key**: $3D \Rightarrow 2D$ image matching problem
  - **Goal**: Diffeomorphism: Quasi-conformal elastic deformation

- **Contribution** - Diffeomorphism between Surfaces
  - **Advantage**: One-to-one and onto mapping of whole surface
  - **Efficiency**: Linear time complexity
Related Works

• **Conventional Methods** - Centerline Based Registration
  • Linear stretching and shrinking; Correlating individual points along centerline; Taeniae coli correlation; Free-form deformation grid modeling; Deforming fields …

• **Our Method** - *Conformal Geometry* Based Registration
  • Conformal mapping using holomorphic 1-forms [Gu&Yau 2003]
  • Colon conformal flattening and colonic polyp detection in 2006
Algorithm Overview

Two Colon Surfaces

1. Anatomical Landmark Extraction

   → 3D Supine & Prone Colon Segments (S₁, S₂)

2. Rectangle Conformal Mapping

   → 2D Rectangle Conformal Mappings (D₁, D₂)

3. Internal Feature Detection & Matching

   → Feature Correspondence Constraints (F₁, F₂)

4. Harmonic Mapping Registration

   → 2D Image Matching \( \tilde{f} \)

   3D Surface Registration \( f \)
Quasi-Conformal Registration
1. Anatomical Landmarks Extraction

- *Taenia coli* – 3 bands of muscles, as ridge breakers for haustral folds

- **Idea** - Detecting Haustral Folds
  - 1) Heat Diffusion
  - 2) Fuzzy C-Means Clustering

- *To slice the colon open*
Anatomical Landmarks Extraction Cont.

- **Flexures** – Bends of colon
- **Idea** - Utilizing 3D Centerline
  - **Projection** onto
    1) Z-X plane;
    2) Y-Z plane.
- To divide colon into 5 segments
2. Conformal Mapping Process

Colon Surface, Harmonic Function, Conjugate Harmonic, Holomorphic 1-Form, Rectangle Conf-Map, Circle Packing
Conformal Mappings for Segments

- Colon segments
  - topological cylinder, denoted as triangular mesh
3. Feature Points Detection-Matching

- Feature points – endpoints of hastral folds
- **Idea** Perform detection and matching on 2D rectangle images by **mean curvature** of 3D surface.

- **Methods**
  1) **Graph-Cut Segmentation**
     - Min-cut/Max-flow algorithm for energy minimization problem
  2) **Graph Matching**
     - Feature Correspondence
2D Conformal Mappings | Haustral Folds | Feature Points | Feature Correspondence

Graph-Cut Segmentation | Endpoints Detection | Graph Matching | Optimization
4. Quasi-Conformal Registration
Harmonic Mapping for Registration

- **Idea** Perform harmonic mapping between two 2D maps with feature correspondence constraints
  - Diffeomorphism: One-to-one and Onto Surface Mapping
  - Computation: Solving Linear System
2D Flattened Registration Result

Diffeomorphism (One-to-One, Onto)
More 2D Registration Results

Seg. 1
Deformed Supine
Prone
Seg. 2
Seg. 3
3D Endoluminal Consistent Views

- **Idea** 3D Correlation
  - Camera Frame \((c_1; e_1, e_2, e_3)\)
    - \((c_0, p_0, p_1) \rightarrow (c_1, q_0, q_1)\)
    - \(c_0 \rightarrow c_1; p_0 \rightarrow q_0, p_1 \rightarrow q_1\)
  
  \[
  e_3 = \frac{q_0 - c_1}{|q_0 - c_1|}, \quad e_1 = \frac{(q_1 - c_1) \times e_3}{|(q_1 - c_1) \times e_3|} \\
  e_2 = e_3 \times e_1
  \]

- **GPU Volumetric Rendering**
  - Ray casting through CT volume

View-1

View-2

Supine

Prone
Quasi-Conformal Deformation

- Characteristic of deformation
  - Elasticity: strain-stress relation
- Conformal
  - Orientation independent
  - *Conformal images directly induce the registration*
- Quasi-Conformal
  - Bounded angle distortion
  - *Quasi-Conformal adjustment of conformal image*
- Supine-Prone deformation: Quasi-Conformal
Quasi-Conformal Mapping

- Quasi-Conformal: Infinitesimal Circles $\rightarrow$ Infinitesimal Ellipses
- Beltrami Coefficient $\mu \rightarrow [\text{Eccentricity } K; \text{Angle Distortion } \theta ]$

$K = \frac{1 + |\mu|}{1 - |\mu|}$

$\theta = \frac{1}{2} \text{arg } \mu$
Quasi-Conformal Registration

- **Harmonic mapping with feature constraints is Quasi-Conformal.**

- **Anisotropy of deformation**

  → **Anatomic characteristic of the colon wall tissue**
Quasi-Conformal Morphing

Eccentricity

Checker-Board

Angle Distortion

Circle-Packing
Experimental Results

- **Data** - 6 pairs of public supine-prone CT scans
  - National Institute of Biomedical Imaging and Bioengineering (NIBIB) Image and Clinical Data Repository by National Institute of Health (NIH)

- **Accuracy** - Averaged distance error in $R^3$ (mm)
  - Additional feature points alignment
  - Colonic polyp verification
# Experiment – Registration Accuracy

**Accuracy** - *Averaged distance error in $R^3$ (mm)*

<table>
<thead>
<tr>
<th>Methods</th>
<th>Distance Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our Quasi-Conformal Mapping Method</td>
<td>7.85</td>
</tr>
<tr>
<td>Centerline registration + statistical analysis [Li et al. 04]</td>
<td>12.66</td>
</tr>
<tr>
<td>Linear stretching / shrinking of centerline [Acar et al. 01]</td>
<td>13.20</td>
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<tr>
<td>Centerline feature matching + lumen deformation [Suh &amp; Wyatt 09]</td>
<td>13.77</td>
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<tr>
<td>Centerline point correlation [de Vries et al. 06]</td>
<td>20.00</td>
</tr>
<tr>
<td>Taenia coli correlation [Huang et al. 05]</td>
<td>23.33</td>
</tr>
</tbody>
</table>
Conclusion

- **Supine-Prone Surface Registration**
  - *Diffeomorphism*: One-to-One and Onto
  - *Quasi-Conformality*: Elastic Deformation

  *To the best of our knowledge, first to apply quasi-conformal mapping in supine-prone colon registration!*

- **Computation Efficiency**
  - *Linear* Conformal map. & Quasi-Conformal harmonic map.

- **Registration Accuracy**
  - *Outperform* other existing methods

- **Visualization Tool**
  - 2D Flattened & 3D Endoluminal Consistent Views
Future Work

• **Explore** the elastic deformation analysis based on quasi-conformal theory for morphology study of general objects

• **Apply** to Computer-Aided Detection for better sensitivity and specificity

• **Extend** to 3D volumetric registration for outer colon wall segmentations
Thank You for Your Attention! 😊

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