Association Analysis (chapter 6)

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Lecture Notes

Association Rules Mining
An Introduction

- This is an intuitive (more or less) introduction
- It contains explanation of the main ideas:
- Frequent item sets, association rules, how we construct the association rules,
- how we judge the goodness of the rules
- Example of an intuitive "run" of the Appriori Algorithm and association rules generation
- Discussion of the relationship between the Association and Correlation analysis

What Is Association Mining?

Association rule mining:

» Finding frequent patterns called associations, among sets of items or objects in transaction databases, relational databases, and other information repositories.

- **Applications:**
  - Basket data analysis, cross-marketing, catalog design, loss-leader analysis, clustering, classification, etc.

Association Rules

- Rule general form:

  "Body $\rightarrow$ Head [support, confidence]"

  Rule Predicate form:

  buys(x, "diapers") $\rightarrow$ buys(x, "beer") [0.5%, 60%]

  major(x, "CS") ^ takes(x, "DB") $\rightarrow$ grade(x, "A") [1%, 75%]

  Rule Attribute form:

  Diapers $\rightarrow$ beer [1%, 75%]

Association Analysis: Basic Concepts

- **Given: a** database of transactions, where each transaction is a list of items
- **Find:** all rules that associate the presence of one set of items with that of another set of items
- **Example**

  98% of people who purchase tires and auto accessories also get automotive services done

Association Model

- **I** ={i1, i2, ..., in} a set of **items**
- **J** = P(**I**) set of all subsets of the set of items, elements of **J** are called **itemsets**
- **Transaction T:** **T** is subset of set **I** of items
- **Data Base:** set of transactions
- **An association rule** is an implication of the form : **X-> Y,** where **X, Y** are disjoint subsets of items **I** (elements of **J**)
- **Problem:** Find rules that have support and confidence greater that user-specified minimum support and minimun confidence

How we find the rules? (1)
Apriori Algorithm

- **Apriori Algorithm:**
- **First Step:** we find all frequent item-sets
- An item-set is frequent if it has a support greater or equal a fixed minimum support.
- We fix minimum support usually low.
- **Rules generation** from the frequent item-sets is a separate problem and our book doesn't really talk much about it.

How we find the rules? (2)
Apriori Algorithm

- In order calculate efficiently frequent item-sets:
- 1-item-sets (one element item-sets)
- 2-item-sets (two elements item-sets)
- 3-item-sets (three elements item-sets)
- etc….
- we use a principle, called an **Apriori Principle** (hence the name: Apriori Algorithm):
- ANY SUBSET OF A FREQUENT ITEMSET IS A FREQUENT ITEMSET

How we find the rules? (3)
Apriori Process

- Appriori Algorithm **stops** after the **First Step**
- **Second Step** in the Appriori Proces (item-sets generarion AND rules generation) is the rule generation:
- We calculate, from the frequent item-sets a set of the strong rules .
- Strong rules: rules with at least minimum support (low) and minimum confidence (high)
- Apriori Process is then **finished** .

How we find the rules? (4)

Apriori Process

- **The Aprriori Process** problem is:
- how do we form the association rules (A =>B) from the frequent item sets?
- **Remark:** A, B are disjoint subsets of the set **I** of items in general, and of the set 2-frequent, 3-frequent item sets ….. etc, … as generated by the Apriori Algorithm

How we find the rules? (5)

- 1-frequent item set: {i1}- no rule
- 2-frequent item set {i1, i2}: there are two rules:
- {i1} => {i2} and {i2} => {i2}
- We write them also as
- i1 => i2 and i2 => i2
- We decide which rule we accept by calculating its support (greater= minimum support) and confidence (greater= minimum confidence)

How we find the rules? (6)

- 3-frequent item set: {i1, i2, i3}
- The rules, by definition are of the form (A =>B) where A and B are disjoint subsets of {i1, i2, i3}, i.e.
- we have to find all subsets A,B of {i1, i2, i3} such that $A \cup B$ = {i1, i2, i3} and $A \cap B = \Phi$
- **For example**,
- let A= {i1, i2} and B= {i3}. The rule is
- {i1, i2} => {i3}),
- and we write it in a form:

(i1 $\cap$ i2 => i3) or milk $\cap$ bread => vodka

if item i1 is milk, item i2 is bread and item i3 is vodka

How we find the rules? (7)

- Another choice for A and B is, for example:
- A= {i1} and B= {12, i3}.
- The rule is
- {i1} => {i2, i3}, and we write it in a form:

i1 => (i2$\cap$ i3) or milk => (bread $\cap$ vodka)

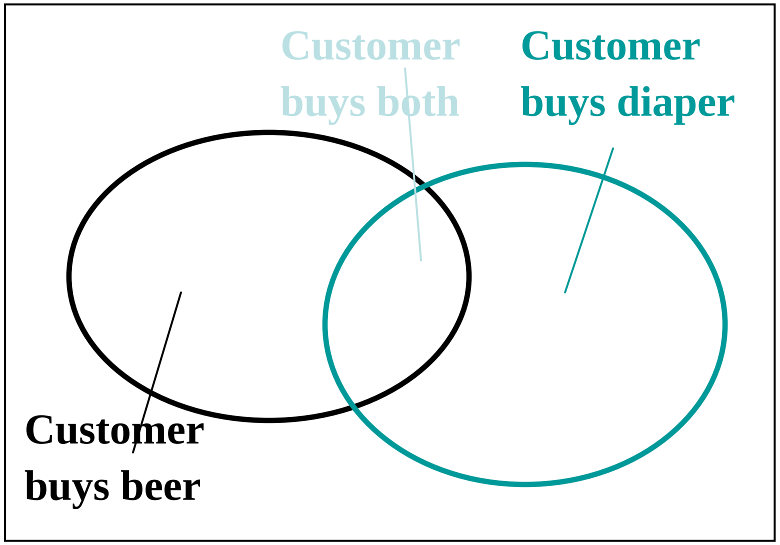
if item i1 is milk, item i2 is bread and item i3 is vodka

- REMEMBER:
- We have to cover all the choices for A and B!
- **Which rule we accept is being decided by calculating its support (greater = minimum support) and confidence (greater = minimum confidence)**

Confidence and Support (1)

- **Confidence:**
- the rule **X->Y** holds in the database **D** with confidence c if the c% of the transactions in **D** that contain **X** also contain **Y**
- **Support:** The rule **X->Y** has support s in **D** if s% of the transaction contain **XUY**

Support and Confidence (2)



- Find all the rules *X & Y* $\Rightarrow$ *Z* with minimum confidence and support
  - Support *s:* **probability** that a transaction contains {X, Y, Z}
  - confidence *c:* **conditional probability** that a transaction containing {X, Y} also contains *Z*

Support (definition)

- Support of a rule (A=>B) in the database D of transactions is given by formula (where sc=support count)
- Support(A => B) = P(A U B) =

$$\frac{sc(A \cup B)}{\#D}$$

Frequent Item sets: sets of items with a support support > MINIMAL support
We (user) fix MIN support usually low and Min Confidence high

Confidence (definition)

- **Confidence** of a rule (A=>B) in the database D of transactions is given by formula (where sc=support count)

- **Conf(A => B)** = P(B|A) = $\boxed{\dfrac{P(A \cup B)}{P(A)}}$

- = $\boxed{\dfrac{sc(AUB)}{\#D} \text{ divided by } \dfrac{scA}{\#D}}$ = $\boxed{\dfrac{sc(AUB)}{scA}}$

Example

- Let consider a data base **D** ={ T1, T2, …. T9}, where
- T1={ 1,2,5} (we write k for item ik),
- T2= {2, 4}, T3={2, 3}, T4={1, 2, 4}, T5={1, 3}
- T6={2, 3}, T7={1, 3}, T8={1, 2, 3,5}, T9={1,2,3}
- To find association rules we follow the following steps
- **STEP 1**: Count occurrences of items in **D**
- **STEP2:**   Fix Minimum support (usually low)
- **STEP 3:** Calculate frequent 1-item sets
- **STEP 4:** Calculate frequent 2-item sets
- **STEP 5:** Calculate frequent 3-item sets
- **STOP** when there is no more frequent item sets.
- This is the end of Apriori Algorithm phase.

Example (c.d)

- **STEP 6:** Fix the minimum confidence (usually high)
- **STEP 7:** Generate strong rules (support >min support and confidence> min confidence)
- **END** of rules generation phase.
- Lets now calculate all steps of our process for the data base D. We represent our transactional data base as relational data base (a table) and put the occurrences of items as an extra row, on the bottom (STEP 1)

Example (c.d)

STEP 1: items occurrences=sc (in a table)

its	1	2	3	4	5
T1	+	+	0	0	+
T2	0	+	0	+	0
T3	0	+	+	0	0
T4	+	+	0	+	0
T5	+	0	+	0	0
T6	0	+	+	0	0
T7	+	0	+	0	0
T8	+	+	+	0	+
T9	+	+	+	0	0
sc	**6**	**7**	**6**	**2**	**2**

Example (c.d.)

- **STEP 2:** Fix minimal support count, for example
- msc = 2
- Minimal support = msc/#D= 2/9=22%
- ms=22%
- Observe: minimal support of an item set is determined uniquely by the minimal support count (msc) and we are going to use only msc to choose our frequent k-itemsets

Example (c.d.)

- **STEP 3:** calculate frequent 1-item sets: look at the count – we get that all 1-item sets are frequent.
- **STEP 4:** calculate frequent 2-item sets.
- First we calculate **2-item sets candidates from frequent 1-item sets.**
- As our all 1-item sets are frequent so all subsets of any 2-item set are frequent and we have to find counts of all 2-item sets.
- If for example we set our msc=6, i.e we would have only {1}, {2} and {3} as frequent item sets then by Apriori Principle: “if A is a frequent item set, then each of its subsets is a frequent item set” we would be examining only those 2-item sets that have {1}, {2}, {3} as subsets
- Apriori Principle reduces the complexity of the algorithm

Example (c.d)

- **STEP 4 :** All 2-item sets = all 2-element subsets of {1,2,3,4,5,} are **candidates** and we evaluate their sc=support counts (in red). They are called **2-item set candidates**
- {1,2,} **(4)**, {1,3} **(4)**, {1,4} **(1)**, {1,5} **(2)**,
- {2,3} **(4)**, {2,4} **(2)**, {2,5} **,**
- {3,4} **(0)**, {3,5} **(1)**,
- {4,5} **(0)**
- msc=2 and we get the folowing
- **Frequent 2- item sets:** {1,2}, {1,3}, {1,5}, {2,3}, {2,4}, {2,5}

Example (c.d.)

- **STEP 5:** generate all frequent 3-item sets.
- FIRST: use the frequent 2-item sets to generate all **3-item set candidates.**
- SECOND: use Apriori Principle to prune the candidates set
- THIRD: Evaluate the count of the pruned set
- STOP: list the frequent 3-item sets
- **STEP 6:** repeat the procedure for 4-itemsets etc (if any)

Example (c.d.)

- **STEP 5 (c.d):** generate all frequent 3-item sets.
- FIRST: **3-item set candidates are**: {1,2,3}, {1,2,4}, {1,2,5}, {1,3,4}, {1,3,5}, {2,3,4}, {2,3,5}, {2,4,5}
- SECOND: pruned the candidates set is:
- {1,2,3}, {1,2,5}
- THIRD: the sc=support count of the pruned set is:
- {1,2,3} **(2)**, {1,2,5} **(2)**
- STOP: list the frequent 3-item sets:
- {1,2,3}, {1,2,5}
- **STEP 6:** there is no 4-item sets.
- **STEP 7:** Use the confidence to generate Apriori Rules.
- We fix minimum confidence = 70%

Example: Association Rules Generation

- We will generate, as an example rules only from one frequent 2-item set: {1,2}. Rule generation for other 2-item sets is similar.
- **Reminder:** conf(A=>B) = $\frac{sc(A \cup B)}{scA}$
- We split {1,2} into disjoint subsets A and B as follows: A={1} and B={2} or A={2} and B={1} and get two possible rules:
- {1}=>{2} or {2}=>{1}

Example: Association Rules Generation (c.d)

- Conf(1=>2)= $\boxed{\dfrac{sc\{1,2\}}{sc\{1\}}}$ = 4/6 = 66%

The rule is not accepted (min conf= 70%)

- Conf(2=>1) = $\boxed{\dfrac{sc\{1,2\}}{sc\{2\}}}$ = 4/7= 57%

The rule is not accepted

Example: Association Rules Generation (c.d.)

- Now we use one frequent 3-item set
- {1,2,5} to show how to generate strong rules.
- First we evaluate all possibilities how to split the set {1,2,5} into to disjoint subsets A,B to obtain all possible rules (A=>B).
- For each rule we evaluate its confidence and choose only those with conf $\geq$ 70% (our minimal confidence).
- The minimal support condition is fulfilled as we deal only with frequent items.
- The rules such obtained are strong rules.

Example: Association Rules Generation (c.d.)

- The rules (for {1,2,5}) are the following:
- **R1:** {1,2}=>{5}
- conf(R1)=sc{1,2,3}/sc{1,2}= 2/4 = ½ = 50%
- **R1 is rejected**
- **R2:** {1,5} => {2}
- conf(R2)=sc{1,2,3}/sc{1,5}= 2/2 = 100%
- **R2 is a strong rule (keep)**
- **R3:** {2,5} => {1}
- conf(R3)=sc{1,2,3}/sc{2,5}= 2/2 = 100%
- **R3 is a strong rule (keep)**
- **R4:** {1} => {2,5}
- conf(R4)=sc{1,2,3}/sc{1}= 2/6 = 33%
- **R4 is rejected**

Example: Association Rules Generation (c.d.)

- The next rules (for {1,2,5}) are the following:
- **R5:** {2}=>{1,5}
- conf(R5)=sc{1,2,3}/sc{2}= 2/7 = 27%
- **R5 is rejected**
- **R6:** {5} => {1,2}
- conf(R6)=sc{1,2,3}/sc{5}= 2/2 = 100%
- **R6 is a strong rule (keep)**
- As the last step we evaluate the exact support for the strong rules (we know that already that it is greater or equal to minimum support, as rules were obtained from the frequent item sets)

Example: Association Rules Generation (c.d.)

- Exact support for the strong rules is:
- Sup({1,5}=>{2})=sc{1,2,5}/#D=2/9= 22%
- We write:
- $1 \cap 5 \Rightarrow 2$ [22%, 100%]
- Sup({2,5}=>{1}) =sc{1,2,5}/#D=2/9= 22%
- We write:
- $2 \cap 5 \Rightarrow 1$ [22%, 100%]
- Sup({5}=>{1,2}) =sc{1,2,5}/#D=2/9= 22%
- We write:
- $5 \Rightarrow 1 \cap 2$ [22%, 100%]
- **THE END**

Criticism to Support and Confidence

- **Example 1:** (Aggarwal & Yu, PODS98)
  - Among 5000 students
    - 3000 play basketball
    - 3750 eat cereal
    - 2000 both play basket ball and eat cereal

RULE: play basketball $\Rightarrow$ *eat cereal* [40%, 66.7%] is misleading because the overall percentage of students eating cereal is 75% which is higher than 66.7%.

RULE: play basketball $\Rightarrow$ *not eat cereal* [20%, 33.3%] is far more accurate, although with lower support and confidence

	basketball	not basketball	sum(row)
cereal	2000	1750	3750
not cereal	1000	250	1250
sum(col.)	3000	2000	5000

Association and Correlation

- As we can see support-confidence framework can be misleading;
- it can identify a rule (A=>B) as interesting (strong) when, in fact the occurrence of A might not imply the occurrence of B.
- Correlation Analysis provides an alternative framework for finding interesting relationships,
- or to improve understanding of meaning of some association rules (a lift of an association rule)

Correlation and Association

- Definition: Two item sets A and B are independent (the occurrence of A is independent of the occurrence of item set B) iff probability P
- $P(A \cup B) = P(A) \cdot P(B)$
- Otherwise A and B are dependent or correlated
- The measure of correlation, or correlation between A and B is given by the formula:
- $\text{Corr}(A,B) = \boxed{\dfrac{P(A \cup B)}{P(A)P(B)}}$

Correlation and Association (c.d.)

- **corr(A,B) >1** means that A and B are positively correlated i.e.the occurrence of one implies the occurrence of the other
- **corr(A,B) < 1** means that the occurrence of A is negatively correlated with (or discourages) the occurrence of B
- **corr(A,B) =1** means that A and B are independent.

Correlation and Association (c.d.)

- The correlation formula can be re-written as
- Corr(A,B) = $\boxed{\dfrac{P(B|A)}{P(B)}}$
- Supp(A=>B)= P(AUB)
- Conf(A=>B)= P(B|A), i.e.
- Conf(A=>B)= corr(A,B) P(B)
- So correlation, support and confidence are all different, but the correlation provides an extra information about the association rule (A=>B)
- We say that the correlation corr(A,B) provides the LIFT of the association rule (A=>B), i.e.
- A is said to increase (or LIFT) the likelihood of B by the factor of the value returned by the formula for corr(A,B)

Correlation Rules

- A correlation rule is a set of items
- {i1, i2, ….in}, where the items occurrences are correlated.
- The correlation value is given by the correlation formula and we use X square test to determine if correlation is statistically significant.
- The X square test can also determine the negative correlation.We can also form minimal correlated item sets, etc…
- Limitations: X square test is less accurate on the data tables that are sparse and can be misleading for the contingency tables larger then 2x2

Mining Association Rules in Large Databases (Chapter 6)

- Association rule mining
- Mining single-dimensional Boolean association rules from transactional databases
- Mining multilevel association rules from transactional databases
- Mining multidimensional association rules from transactional databases and data warehouse
- From association mining to correlation analysis
- Constraint-based association mining
- Summary

Support and Confidence (book slide)

Transaction ID	Items Bought
2000	A,B,C
1000	A,C
4000	A,D
5000	B,E,F

Let minimum support 50%, and minimum confidence 50%, we have rules

$A \Rightarrow C$ (50%, 66.6%)

$C \Rightarrow A$ (50%, 100%)

Association Rule Mining: A Road Map

- **Boolean (Qualitative) vs. quantitative associations** (Based on the types of values handled)

  buys(x, "SQLServer") ^ income(x, "DMBook") => buys(x, "DBMiner") [0.2%, 60%] (**Boolean/Qualitative**)

  age(x, "30..39") ^ income(x, "42..48K") => buys(x, "PC") [1%, 75%] **(quantitative)**

- Single dimension (one predicate) vs. multiple dimensional associations (multiple predicates)

Association Rule Road Map (c.d)

- **Single level vs. multiple-level analysis**
  - What brands of beers are associated with what brands of diapers – single level
  - **Various extensions**

1. Correlation analysis (just discussed)
2. Association does not necessarily imply correlation or causality
3. Constraints enforced

   Example:

   smallsales (sum < 100) implies bigbuys (sum >1,000)?

Chapter 6: Mining Association Rules

- Association rule mining
- Mining single-dimensional Boolean association rules from transactional databases
- Mining multilevel association rules from transactional databases
- Mining multidimensional association rules from transactional databases and data warehouse
- From association mining to correlation analysis
- Constraint-based association mining
- Summary

An Example

Transaction ID	Items Bought
2000	A,B,C
1000	A,C
4000	A,D
5000	B,E,F

Min. support 50%
Min. confidence 50%

Frequent Itemset	Support
{A}	75%
{B}	50%
{C}	50%
{A,C}	50%

For rule ***A*** $\Rightarrow$ ***C***:

support = support({*A*, *C*}) = 50%

confidence = sc({*A*, *C*})/sc({*A*}) = 66.6%

The Apriori principle:

Any subset of a frequent itemset must be frequent

Mining Frequent Itemsets: the Key Step

- **Find the frequent item sets:** the sets of items that have minimum support
  - A subset of a frequent item set must also be a frequent item set
    - **i.e., if {*A*, *B*} is a frequent item set, both {*A*} and {*B*} should be a frequent item set**
  - Iteratively find frequent item sets with cardinality from 1 to k (k-item set)
- **Use the frequent item sets to generate association rules.**

Apriori Algorithm — Book Example of frequents items sets generation

Database D

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

Scan D

C_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{4}	1
{5}	3

L_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{5}	3

C_2

itemset
{1 2}
{1 3}
{1 5}
{2 3}
{2 5}
{3 5}

Scan D

C_2

itemset	sup
{1 2}	1
{1 3}	2
{1 5}	1
{2 3}	2
{2 5}	3
{3 5}	2

L_2

itemset	sup
{1 3}	2
{2 3}	2
{2 5}	3
{3 5}	2

C_3

itemset
{2 3 5}

Scan D

L_3

itemset	sup
{2 3 5}	2

The Apriori Algorithm (Book)

- **Pseudo-code**:

C_k: Candidate itemset of size k
L_k : frequent itemset of size k

L_1 = {frequent items};
for (k = 1; L_k !=$\varnothing$; k++) **do begin**
C_{k+1} = candidates generated from L_k;
for each transaction t in database do
increment the count of all candidates in C_{k+1} that are contained in t
L_{k+1} = candidates in C_{k+1} with min_support
end
return $\cup_k L_k$;

Generating Candidates: C_k

- Join Step: C_k is generated by joining $L_{k-}$with itself
- Prune Step: Any (k-1)-item set that is not frequent cannot be a subset of a frequent k-item set

Example of Generating Candidates

- L_3={*abc, abd, acd, ace, bcd*}
- We write abc for {a,b,c}, etc…
- **Self-joining**: $L_3 * L_3$
  - *abcd* from *abc* and *abd*
  - *acde* from *acd* and *ace*
- **Pruning**:
  - *acde* is removed because *ade* is not frequent: is not in L_3
- C_4={*abcd*}

Appriori Performance Bottlenecks

- The core of the Apriori algorithm:
  - Use frequent (*k* – 1)-item sets to generate candidate frequent *k*-item sets
  - Use database scan and pattern matching to collect counts for the candidate item sets
- The bottleneck of *Apriori*: **candidate generation**
  - Huge candidate sets:
    - 10^4 frequent 1-itemset will generate 10^7 candidate 2-itemsets
    - To discover a frequent pattern of size 100, e.g., $\{a_1, a_2, \ldots, a_{100}\}$, one needs to generate $2^{100} \approx 10^{30}$ candidates.
  - Multiple scans of database:
    - Needs (n +1) scans, n is the length of the longest pattern

How to Count Supports of Candidates?

- **Why counting supports of candidates is a problem?**
  - The total number of candidates can be very huge
  - One transaction may contain many candidates
- **Method:**
  - Candidate itemsets are stored in a *hash-tree*
  - *Leaf* node of hash-tree contains a list of itemsets and counts
  - *Interior* node contains a hash table
  - *Subset function*: finds all the candidates contained in a transaction

Methods to Improve Apriori's Efficiency

- Hash-based itemset counting: A *k*-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent
- Transaction reduction: A transaction that does not contain any frequent k-itemset is useless in subsequent scans
- Partitioning: Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB
- Sampling: mining on a subset of given data, lower

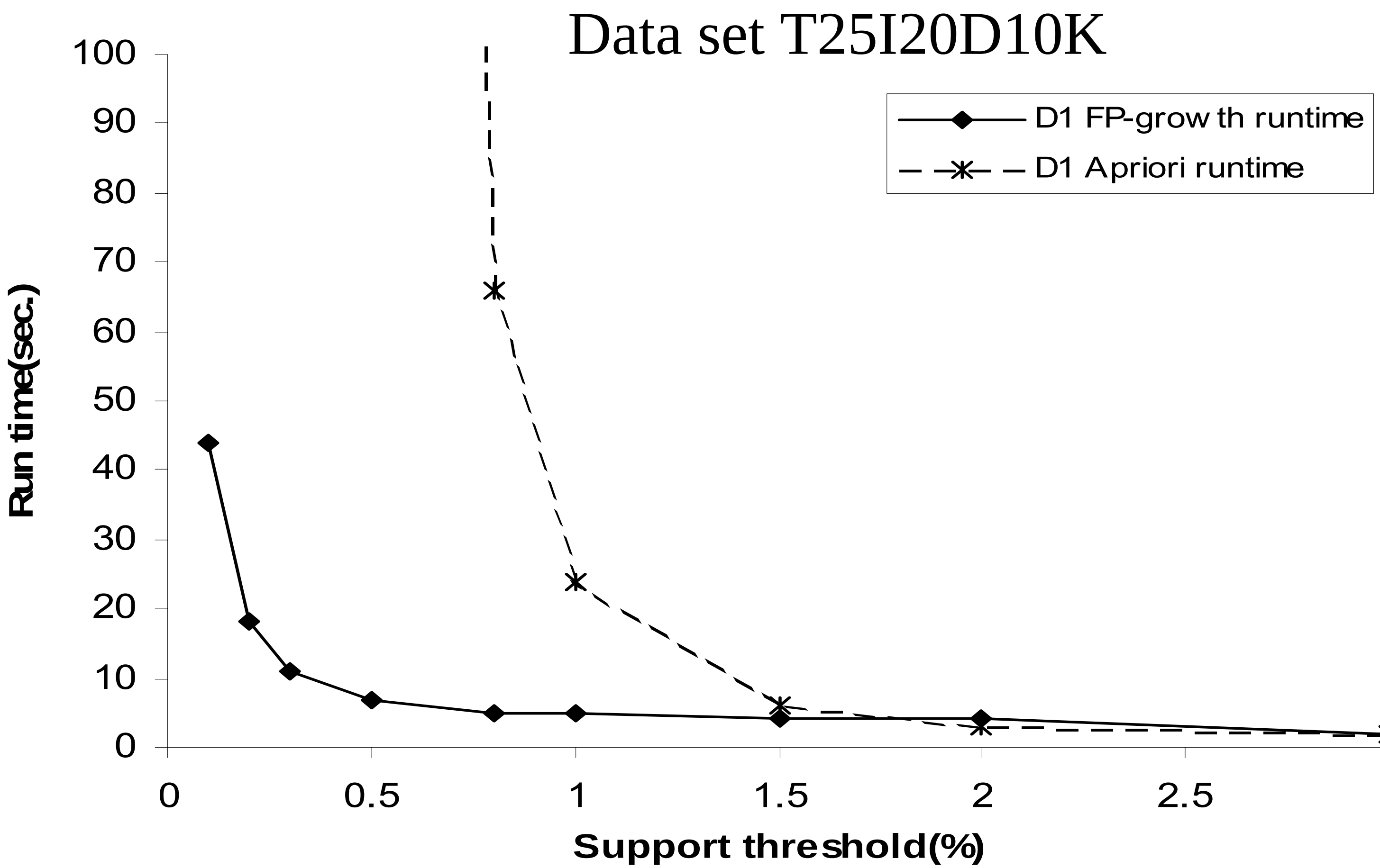
An Alternative: Mining Frequent Patterns Without Candidate Generation

- Compress a large database into a compact, **Frequent-Pattern tree** (**FP-tree**) structure
  - highly condensed, but complete for frequent pattern mining
  - avoid costly database scans
- Develop an efficient, FP-tree-based frequent pattern mining method
  - A divide-and-conquer methodology: decompose mining tasks into smaller ones
  - Avoid candidate generation: sub-database test only!

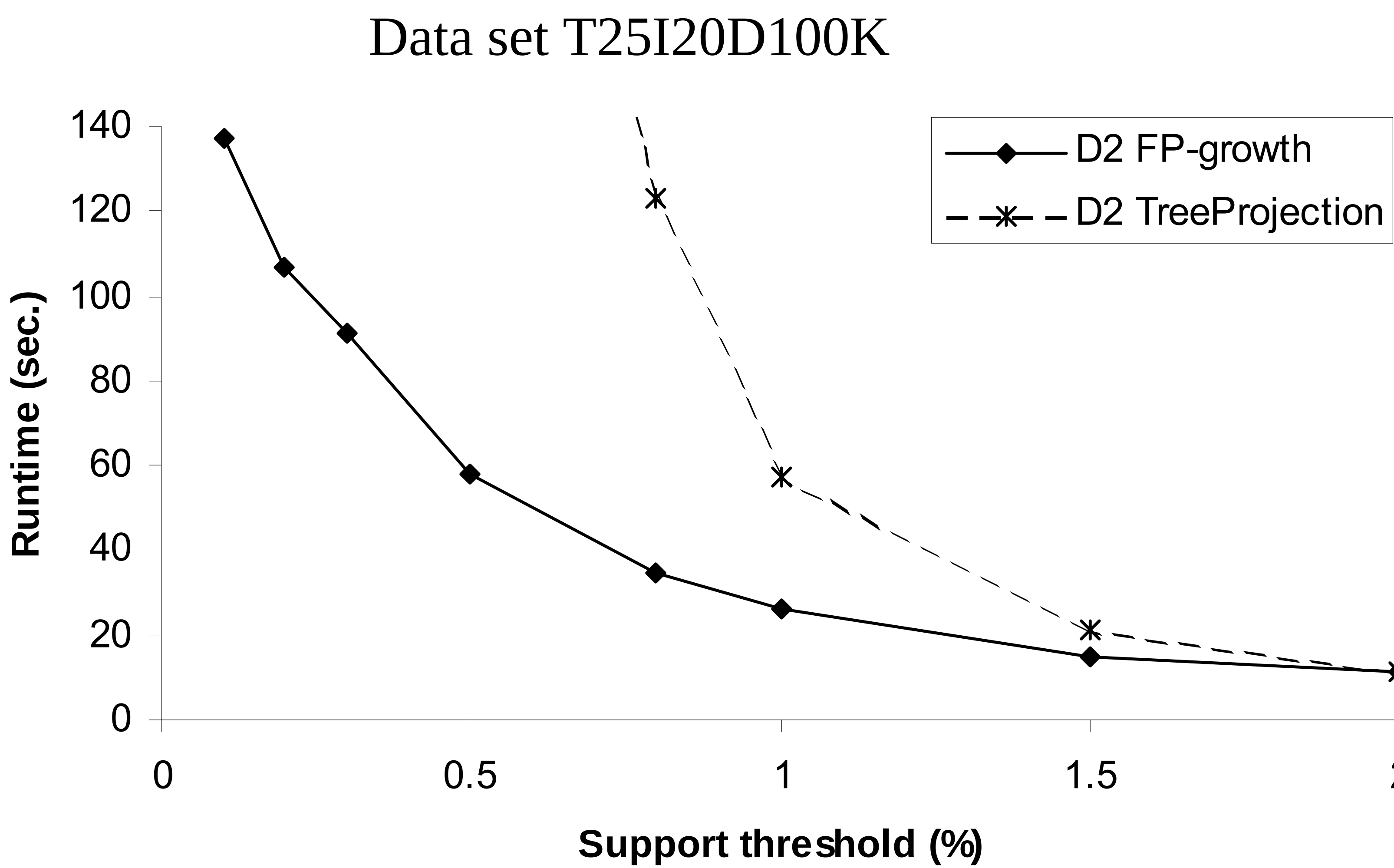
Why Is Frequent Pattern Growth Fast?

- **Performance study shows**
  - FP-growth is an order of magnitude faster than Apriori, and is also faster than tree-projection
- **Reasoning**
  - No candidate generation, no candidate test
  - Use compact data structure
  - Eliminate repeated database scan
  - Basic operation is counting and FP-tree building

FP-growth vs. Apriori: Scalability With the Support Threshold



FP-growth vs. Tree-Projection: Scalability with Support Threshold



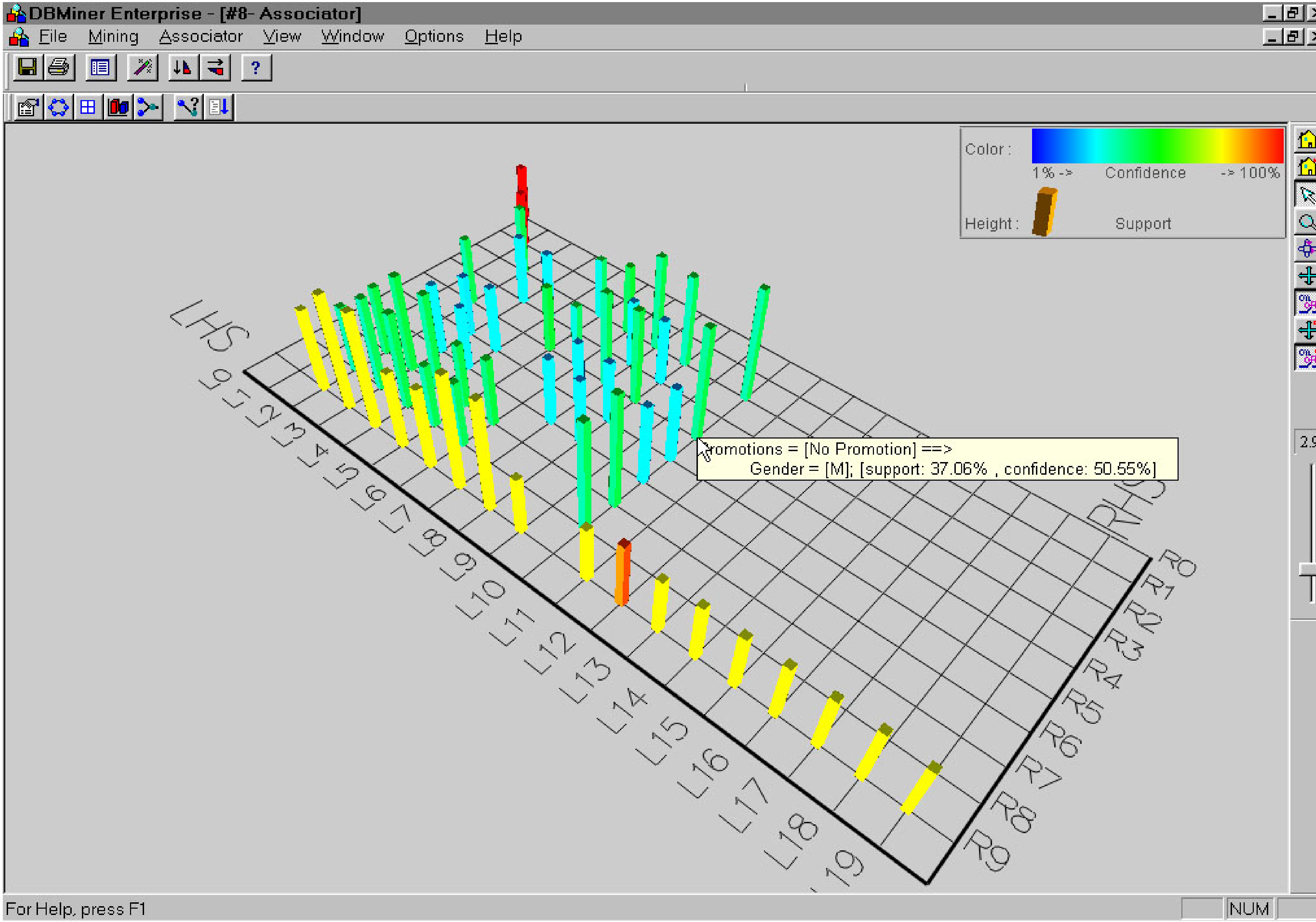
Presentation of Association Rules

(Table Form)

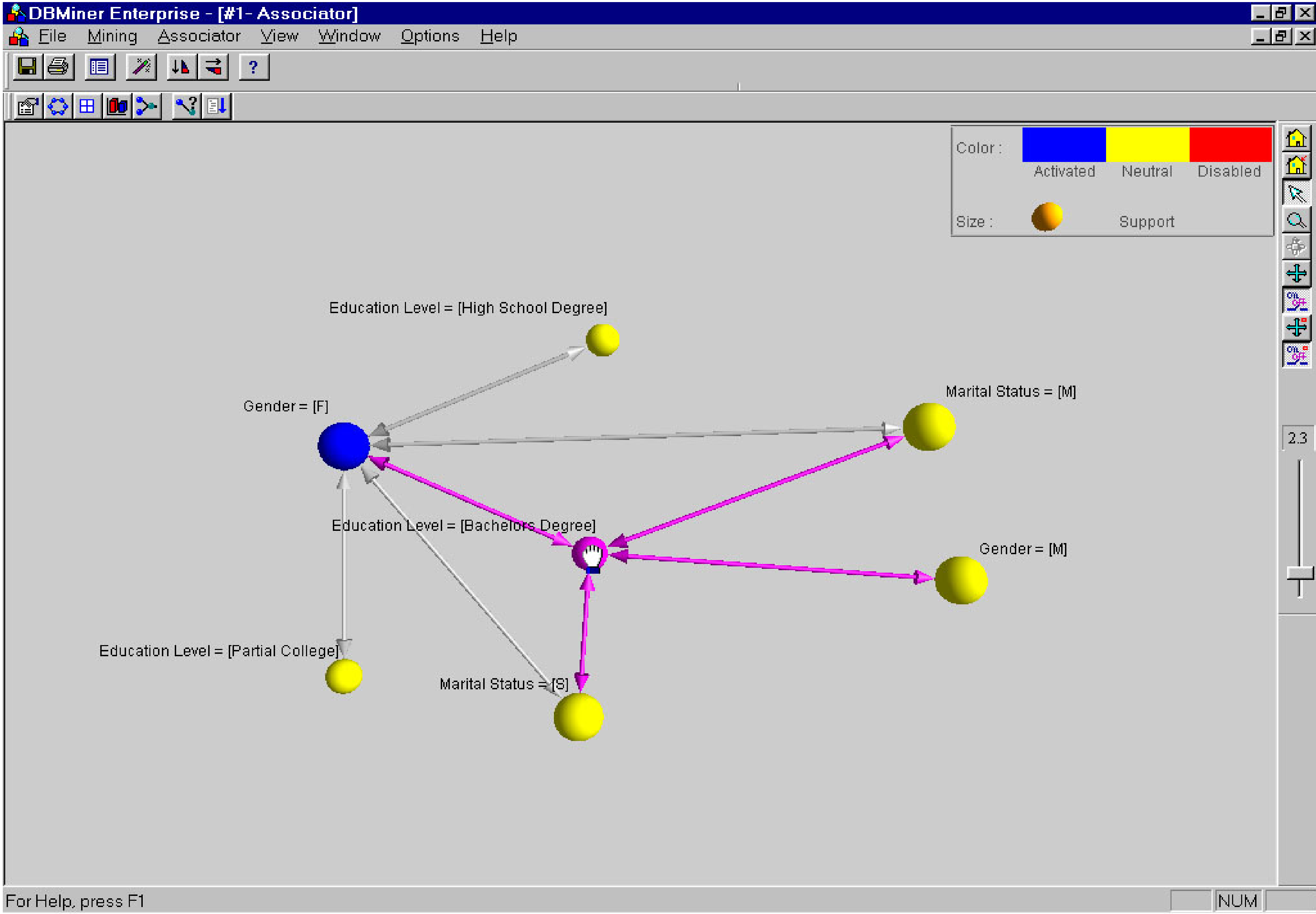
	Body	Implies	Head	Supp (%)	Conf (%)	F	G	H	I
1	cost(x) = '0.00~1000.00'	==>	revenue(x) = '0.00~500.00'	28.45	40.4				
2	cost(x) = '0.00~1000.00'	==>	revenue(x) = '500.00~1000.00'	20.46	29.05				
3	cost(x) = '0.00~1000.00'	==>	order_qty(x) = '0.00~100.00'	59.17	84.04				
4	cost(x) = '0.00~1000.00'	==>	revenue(x) = '1000.00~1500.00'	10.45	14.84				
5	cost(x) = '0.00~1000.00'	==>	region(x) = 'United States'	22.56	32.04				
6	cost(x) = '1000.00~2000.00'	==>	order_qty(x) = '0.00~100.00'	12.91	69.34				
7	order_qty(x) = '0.00~100.00'	==>	revenue(x) = '0.00~500.00'	28.45	34.54				
8	order_qty(x) = '0.00~100.00'	==>	cost(x) = '1000.00~2000.00'	12.91	15.67				
9	order_qty(x) = '0.00~100.00'	==>	region(x) = 'United States'	25.9	31.45				
10	order_qty(x) = '0.00~100.00'	==>	cost(x) = '0.00~1000.00'	59.17	71.86				
11	order_qty(x) = '0.00~100.00'	==>	product_line(x) = 'Tents'	13.52	16.42				
12	order_qty(x) = '0.00~100.00'	==>	revenue(x) = '500.00~1000.00'	19.67	23.88				
13	product_line(x) = 'Tents'	==>	order_qty(x) = '0.00~100.00'	13.52	98.72				
14	region(x) = 'United States'	==>	order_qty(x) = '0.00~100.00'	25.9	81.94				
15	region(x) = 'United States'	==>	cost(x) = '0.00~1000.00'	22.56	71.39				
16	revenue(x) = '0.00~500.00'	==>	cost(x) = '0.00~1000.00'	28.45	100				
17	revenue(x) = '0.00~500.00'	==>	order_qty(x) = '0.00~100.00'	28.45	100				
18	revenue(x) = '1000.00~1500.00'	==>	cost(x) = '0.00~1000.00'	10.45	96.75				
19	revenue(x) = '500.00~1000.00'	==>	cost(x) = '0.00~1000.00'	20.46	100				
20	revenue(x) = '500.00~1000.00'	==>	order_qty(x) = '0.00~100.00'	19.67	96.14				
21									
22									
23	cost(x) = '0.00~1000.00'	==>	revenue(x) = '0.00~500.00' AND order_qty(x) = '0.00~100.00'	28.45	40.4				
24	cost(x) = '0.00~1000.00'	==>	revenue(x) = '0.00~500.00' AND order_qty(x) = '0.00~100.00'	28.45	40.4				
25	cost(x) = '0.00~1000.00'	==>	revenue(x) = '500.00~1000.00' AND order_qty(x) = '0.00~100.00'	19.67	27.93				
26	cost(x) = '0.00~1000.00'	==>	revenue(x) = '500.00~1000.00' AND order_qty(x) = '0.00~100.00'	19.67	27.93				
27	cost(x) = '0.00~1000.00' AND order_qty(x) = '0.00~100.00'	==>	revenue(x) = '500.00~1000.00'	19.67	33.23				

Sheet1

Visualization of Association Rule Using Plane Graph



Visualization of Association Rule Using Rule Graph



Iceberg Queries

- Iceberg query: Compute aggregates over one or a set of attributes only for those whose aggregate values is above certain threshold
- Example:

```
select P.custID, P.itemID, sum(P.qty)
from purchase P
group by P.custID, P.itemID
having sum(P.qty) >= 10
```

- Compute iceberg queries efficiently by Apriori:
  - First compute lower dimensions
  - Then compute higher dimensions only when all the lower ones are above the threshold

Chapter 6: Mining Association Rules in Large Databases

- Association rule mining
- Mining single-dimensional Boolean association rules from transactional databases
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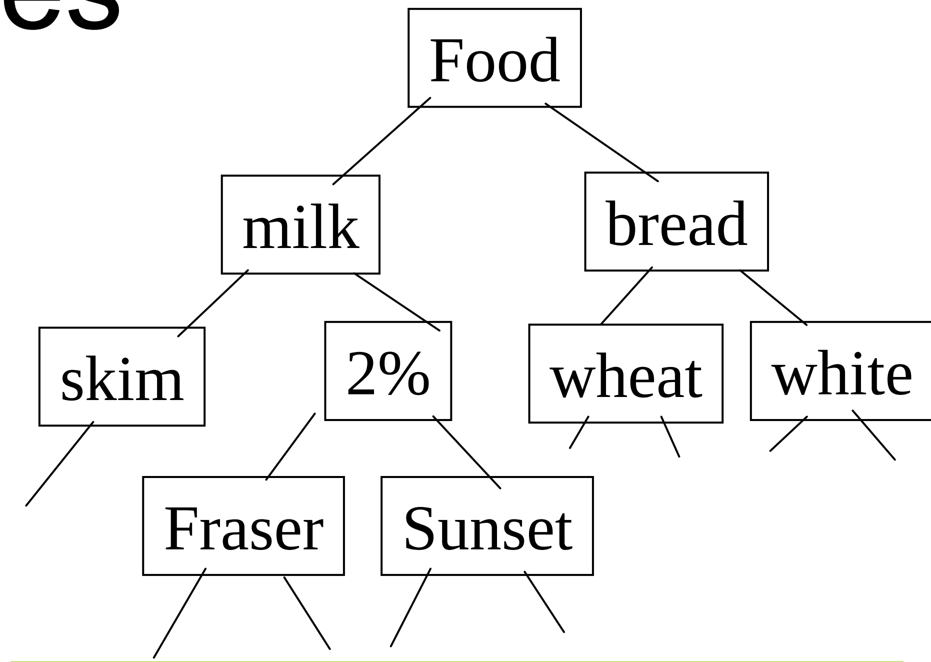
Multiple-Level Association Rules

- **Items often form hierarchy**
- Items at the lower level are expected to have lower support.
- Rules regarding itemsets at

  appropriate levels could be quite useful.
- Transaction database can be encoded based



TID	Items
T1	{111, 121, 211, 221}
T2	{111, 211, 222, 323}
T3	{112, 122, 221, 411}
T4	{111, 121}
T5	{111, 122, 211, 221, 413}

Mining Multi-Level Associations

- A top_down, progressive deepening approach:
  - **First find high-level strong rules**:
    milk $\rightarrow$ bread [20%, 60%].
  - **Then find their lower-level "weaker" rules**:
    2% milk $\rightarrow$ wheat bread [6%, 50%].
- **Variations at mining multiple-level association rules**.
  - **Level-crossed association rules:**

    2% *milk* $\rightarrow$ *Wonder wheat bread*
  - **Association rules with multiple, alternative hierarchies:**

    2% *milk* $\rightarrow$ *Wonder bread*

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Multi-Dimensional Association (1)

- **Single-dimensional rules**:
  - buys(X, “milk”) $\Rightarrow$ buys(X, “bread”)
- **Multi-dimensional rules**: Involve 2 or more dimensions or predicates
  - **Inter-dimension association rules (*no repeated predicates*)**
    - age(X,”19-25”) $\wedge$ occupation(X,“student”) $\Rightarrow$ buys(X,“coke”)

Multi-Dimensional Association

- **Hybrid-dimension association rules (*repeated predicates*)**
  - age(X,"19-25") $\wedge$ buys(X, "popcorn") $\Rightarrow$ buys(X, "coke")

- **Categorical (qualitative) Attributes**
  - finite number of possible values, no ordering among values
- **Quantitative Attributes**
  - numeric, implicit ordering among values

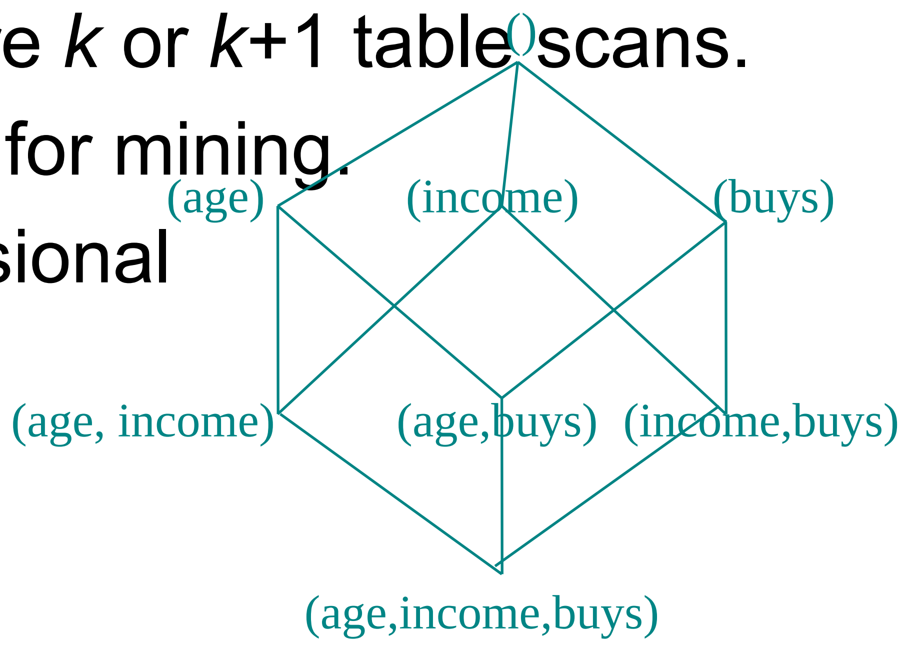
Techniques for Mining MD Associations

- Search for frequent *k*-predicate set:
  - Example: {age, occupation, buys} is a 3-predicate set.
  - Techniques can be categorized by how age are treated.

1. Using static discretization of quantitative attributes
   - Quantitative attributes are statically discretized by using predefined concept hierarchies.
2. Quantitative association rules
   - Quantitative attributes are dynamically discretized into “bins” based on the distribution of the data.
3. Distance-based association rules
   - This is a dynamic discretization process that considers the distance between data points.

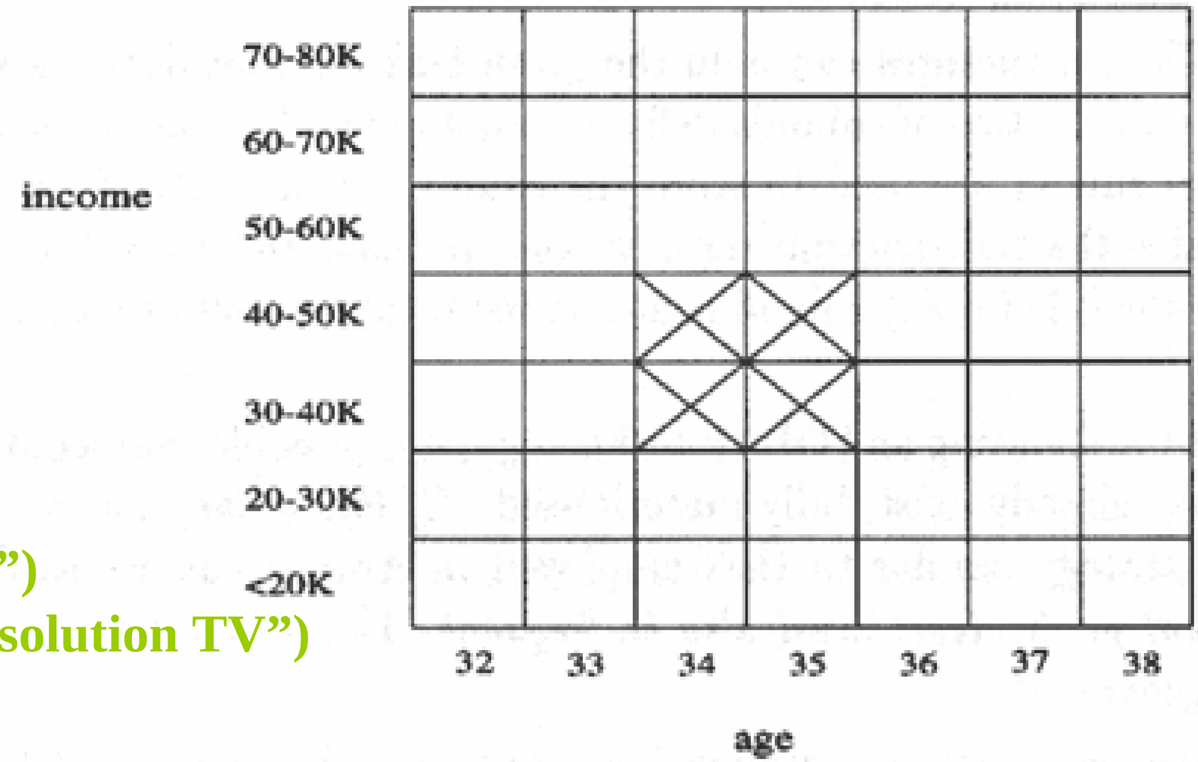
Static Discretization of Quantitative Attributes

- Discretized prior to mining using concept hierarchy.
- Numeric values are replaced by ranges.
- In relational database, finding all frequent k-predicate sets will require *k* or *k*+1 table scans.
- Data cube is well suited for mining.
- The cells of an n-dimensional cuboid correspond to the predicate sets.
- Mining from data cubes can be much faster.



Quantitative Association Rules

- Numeric attributes are *dynamically* discretized
  - Such that the confidence or compactness of the rules mined is maximized.
- 2-D quantitative association rules: $A_{quan1} \wedge A_{quan2} \Rightarrow A_{cat}$
- Cluster "adjacent" association rules to form general rules using a 2-D grid.
- Example:



age(X,"30-34") $\wedge$
income(X,"24K - 48K")
$\Rightarrow$ buys(X,"high resolution TV")

ARCS (Association Rule Clustering System)

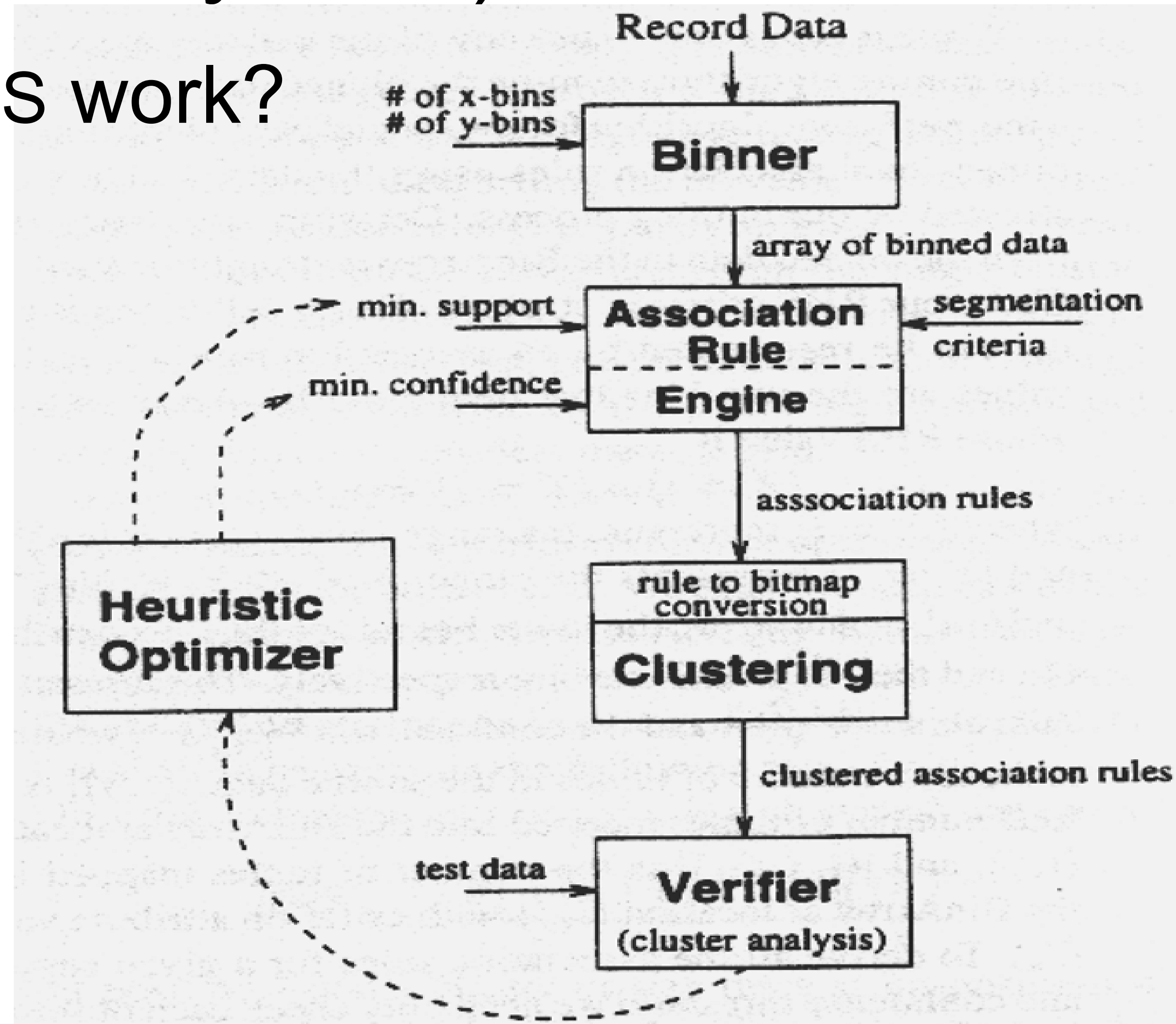
How does ARCS work?

1. Binning

2. Find frequent predicateset

3. Clustering

4. Optimize



Limitations of ARCS

- Only quantitative attributes on LHS of rules.
- Only 2 attributes on LHS. (2D limitation)
- An alternative to ARCS
  - Non-grid-based
  - equi-depth binning
  - clustering based on a measure of *partial completeness*.
  - "***Mining Quantitative Association Rules in Large Relational Tables***" by R. Srikant and R. Agrawal.

Mining Distance-based Association Rules

- Binning methods do not capture the semantics of interval data

Price($)	Equi-width (width $10)	Equi-depth (depth 2)	Distance-based
7	[0,10]	[7,20]	[7,7]
20	[11,20]	[22,50]	[20,22]
22	[21,30]	[51,53]	[50,53]
50	[31,40]		
51	[41,50]		
53	[51,60]		

- Distance-based partitioning, more meaningful discretization considering:
  - density/number of points in an interval
  - “closeness” of points in an interval

Clusters and Distance Measurements

- S[X] is a set of N tuples t_1, t_2, …, t_N , projected on the attribute set X
- The diameter of S[X]:

$$d(S[X]) = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} dist_X(t_i[X], t_j[X])}{N(N-1)}$$

  – $dist_x$:distance metric, e.g. Euclidean distance or

  Manhattan

Clusters and Distance Measurements(c.d.)

- The diameter, *d,* assesses the density of a cluster C_X , where

$$d(C_X) \leq d_0^X$$

$$|C_X| \geq s_0$$

- Finding clusters and distance-based rules
  – the density threshold, d_0 , replaces the notion of support
  – modified version of the BIRCH clustering algorithm

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Interestingness Measurements

- Objective measures

  Two popular measurements:

  ☆ *support;* and

  🕐 *confidence*

- Subjective measures (Silberschatz & Tuzhilin, KDD95)

  A rule (pattern) is interesting if

  ☆ it is *unexpected* (surprising to the user); and/or

  🕐 *actionable* (the user can do something with it)

Criticism to Support and Confidence (Cont.)

- Example 2:
  - X and Y: positively correlated,
  - X and Z, negatively related
  - support and confidence of X=>Z dominates

X	1	1	1	1	0	0	0	0
Y	1	1	0	0	0	0	0	0
Z	0	1	1	1	1	1	1	1

Rule	Support	Confidence
X=>Y	25%	50%
X=>Z	37.50%	75%

Other Interestingness Measures: Interest

- Interest $$\frac{P(A \wedge B)}{P(A)P(B)}$$
  - taking both $P(A)$ and $P(B)$ in consideration
  - $P(A \wedge B) = P(B) * P(A)$, if A and B are independent events
  - A and B negatively correlated, if the value is less than 1; otherwise A and B positively correlated.

X	1	1	1	1	0	0	0	0
Y	1	1	0	0	0	0	0	0
Z	0	1	1	1	1	1	1	1

Itemset	Support	Interest
X,Y	25%	2
X,Z	37.50%	0.9
Y,Z	12.50%	0.57

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- Summary

Constraint-Based Mining

- Interactive, exploratory mining giga-bytes of data?
  - Could it be real? — Making good use of constraints!
- What kinds of constraints can be used in mining?
  - Knowledge type constraint: classification, association, etc.
  - Data constraint: SQL-like queries
    - Find product pairs sold together in **Vancouver in Dec.'98**
  - Dimension/level constraints:
    - in relevance to **region, price, brand, customer category**
    - small sales (price $< \$10$) triggers big sales (sum $> \$200$).
  - Interestingness constraints:
    - strong rules (min_support $\geq 3\%$, min_confidence $\geq 60\%$).

Rule Constraints in Association Mining

- Two kind of rule constraints:
  - Rule form constraints: meta-rule guided mining.
    - $P(x, y) \wedge Q(x, w) \rightarrow$ takes(x, "database systems").
  - Rule (content) constraint: constraint-based query optimization (Ng, et al., SIGMOD'98).
    - sum(LHS) < 100 ^ min(LHS) > 20 ^ count(LHS) > 3 ^ sum(RHS) > 1000
- 1-variable vs. 2-variable constraints (Lakshmanan, et al. SIGMOD'99):
  - 1-var: A constraint confining only one side (L/R) of the rule, e.g., as shown above.
  - 2-var: A constraint confining both sides (L and R).
    - sum(LHS) < min(RHS) ^ max(RHS) < 5* sum(LHS)

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Why Is the Big Pie Still There?

- More on constraint-based mining of associations
  - **Boolean vs. quantitative associations**
    - Association on discrete vs. continuous data
  - **From association to correlation and causal structure analysis.**
    - Association does not necessarily imply correlation or causal relationships
  - **From intra-trasanction association to inter-transaction associations**
    - E.g., break the barriers of transactions (Lu, et al. TOIS'99).
  - **From association analysis to classification and clustering analysis**
    - E.g, clustering association rules

Summary

- **Association rule mining**
  - probably the most significant contribution from the database community in KDD
  - A large number of papers have been published
- Many interesting issues have been explored
- An interesting research direction
  - **Association analysis in other types of data:** spatial data, multimedia data, time series data, etc.