The Apriori Algorithm: Basics

The Apriori Algorithm is an influential algorithm for mining frequent itemsets for boolean association rules.

Key Concepts:

- **Frequent Itemsets**: The sets of item which has minimum support (denoted by \( L_i \) for \( i^{th} \)-Itemset).
- **Apriori Property**: Any subset of frequent itemset must be frequent.
- **Join Operation**: To find \( L_k \), a set of candidate \( k \)-itemsets is generated by joining \( L_{k-1} \) with itself.
The Apriori Algorithm in a Nutshell

• Find the frequent itemsets: the sets of items that have minimum support
  – A subset of a frequent itemset must also be a frequent itemset
    • i.e., if \{AB\} is a frequent itemset, both \{A\} and \{B\} should be a frequent itemset
  – Iteratively find frequent itemsets with cardinality from 1 to \(k\) (\(k\)-itemset)
• Use the frequent itemsets to generate association rules.
The Apriori Algorithm: Pseudo code

- **Join Step**: $C_k$ is generated by joining $L_{k-1}$ with itself.
- **Prune Step**: Any $(k-1)$-itemset that is not frequent cannot be a subset of a frequent $k$-itemset.

**Pseudo-code**:

$C_k$: Candidate itemset of size $k$
$L_k$: frequent itemset of size $k$

$L_1 = \{\text{frequent items}\}$;

for $(k = 1; L_k \neq \emptyset; k++)$ do begin
  $C_{k+1} = \text{candidates generated from } L_k$;
  for each transaction $t$ in database do
    increment the count of all candidates in $C_{k+1}$ that are contained in $t$
  end
  $L_{k+1} = \text{candidates in } C_{k+1} \text{ with min} \_ \text{support}$
end
return $\bigcup_k L_k$;
The Apriori Algorithm: Example

Consider a database, D, consisting of 9 transactions.

Suppose min. support count required is 2 (i.e. min_sup = 2/9 = 22%)

Let minimum confidence required is 70%.

We have to first find out the frequent itemset using Apriori algorithm.

Then, Association rules will be generated using min. support & min. confidence.
Step 1: Generating 1-itemset Frequent Pattern

The set of frequent 1-itemsets, \( L_1 \), consists of the candidate 1-itemsets satisfying minimum support.

In the first iteration of the algorithm, each item is a member of the set of candidate.
Step 2: Generating 2-itemset Frequent Pattern

Generate $C_2$ candidates from $L_1$

$C_2$

<table>
<thead>
<tr>
<th>Itemset</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>${I_1, I_2}$</td>
<td>4</td>
</tr>
<tr>
<td>${I_1, I_3}$</td>
<td>4</td>
</tr>
<tr>
<td>${I_1, I_4}$</td>
<td>1</td>
</tr>
<tr>
<td>${I_1, I_5}$</td>
<td>2</td>
</tr>
<tr>
<td>${I_2, I_3}$</td>
<td>4</td>
</tr>
<tr>
<td>${I_2, I_4}$</td>
<td>2</td>
</tr>
<tr>
<td>${I_2, I_5}$</td>
<td>2</td>
</tr>
<tr>
<td>${I_3, I_4}$</td>
<td>0</td>
</tr>
<tr>
<td>${I_3, I_5}$</td>
<td>1</td>
</tr>
<tr>
<td>${I_4, I_5}$</td>
<td>0</td>
</tr>
</tbody>
</table>

Scan $D$ for count of each candidate

<table>
<thead>
<tr>
<th>Itemset</th>
<th>Sup. Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>${I_1, I_2}$</td>
<td>4</td>
</tr>
<tr>
<td>${I_1, I_3}$</td>
<td>4</td>
</tr>
<tr>
<td>${I_1, I_4}$</td>
<td>1</td>
</tr>
<tr>
<td>${I_1, I_5}$</td>
<td>2</td>
</tr>
<tr>
<td>${I_2, I_3}$</td>
<td>4</td>
</tr>
<tr>
<td>${I_2, I_4}$</td>
<td>2</td>
</tr>
<tr>
<td>${I_2, I_5}$</td>
<td>2</td>
</tr>
<tr>
<td>${I_3, I_4}$</td>
<td>0</td>
</tr>
<tr>
<td>${I_3, I_5}$</td>
<td>1</td>
</tr>
<tr>
<td>${I_4, I_5}$</td>
<td>0</td>
</tr>
</tbody>
</table>

Compare candidate support count with minimum support count

<table>
<thead>
<tr>
<th>Itemset</th>
<th>Sup Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>${I_1, I_2}$</td>
<td>4</td>
</tr>
<tr>
<td>${I_1, I_3}$</td>
<td>4</td>
</tr>
<tr>
<td>${I_1, I_5}$</td>
<td>2</td>
</tr>
<tr>
<td>${I_2, I_3}$</td>
<td>4</td>
</tr>
<tr>
<td>${I_2, I_4}$</td>
<td>2</td>
</tr>
<tr>
<td>${I_2, I_5}$</td>
<td>2</td>
</tr>
</tbody>
</table>

$L_2$
Step 2: Generating 2-itemset Frequent Pattern

• To discover the set of frequent 2-itemsets, $L_2$, the algorithm uses $L_1$ Join $L_1$ to generate a candidate set of 2-itemsets, $C_2$.

• Next, the transactions in $D$ are scanned and the support count for each candidate itemset in $C_2$ is accumulated (as shown in the middle table).

• The set of frequent 2-itemsets, $L_2$, is then determined, consisting of those candidate 2-itemsets in $C_2$ having minimum support.

• Note: We haven’t used Apriori Property yet.
Step 3: Generating 3-itemset Frequent Pattern

- The generation of the set of candidate 3-itemsets, $C_3$, involves use of the Apriori Property.
- In order to find $C_3$, we compute $L_2 \text{ Join } L_2$.
- $C_3 = L_2 \text{ Join } L_2 = \{\{I_1, I_2, I_3\}, \{I_1, I_2, I_5\}, \{I_1, I_3, I_5\}, \{I_2, I_3, I_4\}, \{I_2, I_3, I_5\}, \{I_2, I_4, I_5\}\}$.
- Now, Join step is complete and Prune step will be used to reduce the size of $C_3$. Prune step helps to avoid heavy computation due to large $C_k$. 
Step 3: Generating 3-itemset Frequent Pattern

- Based on the Apriori property that all subsets of a frequent itemset must also be frequent, we can determine that four latter candidates cannot possibly be frequent. How?
- For example, let's take \{I_1, I_2, I_3\}. The 2-item subsets of it are \{I_1, I_2\}, \{I_1, I_3\} & \{I_2, I_3\}. Since all 2-item subsets of \{I_1, I_2, I_3\} are members of L_2, we will keep \{I_1, I_2, I_3\} in C_3.
- Let's take another example of \{I_2, I_3, I_5\} which shows how the pruning is performed. The 2-item subsets are \{I_2, I_3\}, \{I_2, I_5\} & \{I_3, I_5\}.
- BUT, \{I_3, I_5\} is not a member of L_2 and hence it is not frequent violating Apriori Property. Thus we will have to remove \{I_2, I_3, I_5\} from C_3.
- Therefore, C_3 = \{\{I_1, I_2, I_3\}, \{I_1, I_2, I_5\}\} after checking for all members of result of Join operation for Pruning.
- Now, the transactions in D are scanned in order to determine L_3, consisting of those candidates 3-itemsets in C_3 having minimum support.
Step 4: Generating 4-itemset Frequent Pattern

- The algorithm uses $L_3 \ join L_3$ to generate a candidate set of 4-itemsets, $C_4$. Although the join results in $\{I1, I2, I3, I5\}$, this itemset is pruned since its subset $\{I2, I3, I5\}$ is not frequent.

- Thus, $C_4 = \emptyset$, and algorithm terminates, having found all of the frequent items. This completes our Apriori Algorithm.

- What’s Next?
  These frequent itemsets will be used to generate strong association rules (where strong association rules satisfy both minimum support & minimum confidence).
Step 5: Generating Association Rules from Frequent Itemsets

• Procedure:
  • For each frequent itemset “I”, generate all nonempty subsets of I.
  • For every nonempty subset s of l, output the rule “s → (l-s)” if \( \text{support_count}(l) / \text{support_count}(s) \geq \text{min_conf} \) where min_conf is minimum confidence threshold.

• Back To Example:
  We had \( L = \{\{I1\}, \{I2\}, \{I3\}, \{I4\}, \{I5\}, \{I1,I2\}, \{I1,I3\}, \{I1,I5\}, \{I2,I3\}, \{I2,I4\}, \{I2,I5\}, \{I1,I2,I3\}, \{I1,I2,I5\}\}.
  • Lets take \( I = \{I1,I2,I5\}\).
  • Its all nonempty subsets are \( \{I1,I2\}, \{I1,I5\}, \{I2,I5\}, \{I1\}, \{I2\}, \{I5\}\).
Step 5: Generating Association Rules from Frequent Itemsets

- Let minimum confidence threshold is , say 70%.
- The resulting association rules are shown below, each listed with its confidence.
  - $R_1: I_1 \land I_2 \Rightarrow I_5$
    - Confidence = $\frac{sc\{I_1,I_2,I_5\}}{sc\{I_1,I_2\}} = \frac{2}{4} = 50\%$
    - $R_1$ is Rejected.
  - $R_2: I_1 \land I_5 \Rightarrow I_2$
    - Confidence = $\frac{sc\{I_1,I_2,I_5\}}{sc\{I_1,I_5\}} = \frac{2}{2} = 100\%$
    - $R_2$ is Selected.
  - $R_3: I_2 \land I_5 \Rightarrow I_1$
    - Confidence = $\frac{sc\{I_1,I_2,I_5\}}{sc\{I_2,I_5\}} = \frac{2}{2} = 100\%$
    - $R_3$ is Selected.
Step 5: Generating Association Rules from Frequent Itemsets

- R4: \( l_1 \rightarrow l_2 \land l_5 \)
  - Confidence = \( \frac{\text{sc}\{l_1,l_2,l_5\}}{\text{sc}\{l_1\}} = \frac{2}{6} = 33\% \)
  - R4 is Rejected.

- R5: \( l_2 \rightarrow l_1 \land l_5 \)
  - Confidence = \( \frac{\text{sc}\{l_1,l_2,l_5\}}{\{l_2\}} = \frac{2}{7} = 29\% \)
  - R5 is Rejected.

- R6: \( l_5 \rightarrow l_1 \land l_2 \)
  - Confidence = \( \frac{\text{sc}\{l_1,l_2,l_5\}}{\{l_5\}} = \frac{2}{2} = 100\% \)
  - R6 is Selected.

In this way, We have found three strong association rules.
Methods to Improve Apriori’s Efficiency

- **Hash-based itemset counting**: A \( k \)-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent.

- **Transaction reduction**: A transaction that does not contain any frequent \( k \)-itemset is useless in subsequent scans.

- **Partitioning**: Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB.

- **Sampling**: mining on a subset of given data, lower support threshold + a method to determine the completeness.

- **Dynamic itemset counting**: add new candidate itemsets only when all of their subsets are estimated to be frequent.
Mining Frequent Patterns Without Candidate Generation

• Compress a large database into a compact, Frequent-Pattern tree (FP-tree) structure
  – highly condensed, but complete for frequent pattern mining
  – avoid costly database scans
• Develop an efficient, FP-tree-based frequent pattern mining method
  – A divide-and-conquer methodology: decompose mining tasks into smaller ones
  – Avoid candidate generation: sub-database test only!
Consider the same previous example of a database, D, consisting of 9 transactions. Suppose min. support count required is 2 (i.e. min_sup = 2/9 = 22 %).

The first scan of database is same as Apriori, which derives the set of 1-itemsets & their support counts. The set of frequent items is sorted in the order of descending support count. The resulting set is denoted as \( L = \{l2:7, l1:6, l3:6, l4:2, l5:2\} \).
FP-Growth Method: Construction of FP-Tree

- First, create the root of the tree, labeled with “null”.
- Scan the database D a second time. (First time we scanned it to create 1-itemset and then L).
- The items in each transaction are processed in L order (i.e. sorted order).
- A branch is created for each transaction with items having their support count separated by colon.
- Whenever the same node is encountered in another transaction, we just increment the support count of the common node or Prefix.
- To facilitate tree traversal, an item header table is built so that each item points to its occurrences in the tree via a chain of node-links.
- Now, The problem of mining frequent patterns in database is transformed to that of mining the FP-Tree.
FP-Growth Method: Construction of FP-Tree

An FP-Tree that registers compressed, frequent pattern information
Mining the FP-Tree by Creating Conditional (sub) pattern bases

Steps:
1. Start from each frequent length-1 pattern (as an initial suffix pattern).
2. Construct its conditional pattern base which consists of the set of prefix paths in the FP-Tree co-occurring with suffix pattern.
3. Then, Construct its conditional FP-Tree & perform mining on such a tree.
4. The pattern growth is achieved by concatenation of the suffix pattern with the frequent patterns generated from a conditional FP-Tree.
5. The union of all frequent patterns (generated by step 4) gives the required frequent itemset.
FP-Tree Example Continued

<table>
<thead>
<tr>
<th>Item</th>
<th>Conditional pattern base</th>
<th>Conditional FP-Tree</th>
<th>Frequent pattern generated</th>
</tr>
</thead>
<tbody>
<tr>
<td>I5</td>
<td>{(I2 I1: 1),(I2 I1 I3: 1)}</td>
<td>&lt;I2:2 , I1:2&gt;</td>
<td>I2 I5:2, I1 I5:2, I2 I1 I5: 2</td>
</tr>
<tr>
<td>I4</td>
<td>{(I2 I1: 1),(I2: 1)}</td>
<td>&lt;I2 : 2&gt;</td>
<td>I2 I4: 2</td>
</tr>
<tr>
<td>I3</td>
<td>{(I2 I1: 1),(I2: 2), (I1: 2)}</td>
<td>&lt;I2 : 4, I1: 2&gt;,<a href="">I1:2</a></td>
<td>I2 I3:4, I1, I3: 2 , I2 I1 I3: 2</td>
</tr>
<tr>
<td>I2</td>
<td>{(I2: 4)}</td>
<td>&lt;I2: 4&gt;</td>
<td>I2 I1: 4</td>
</tr>
</tbody>
</table>

**Mining the FP-Tree by creating conditional (sub) pattern bases**

Now, Following the above mentioned steps:

- Lets start from I5. The I5 is involved in 2 branches namely {I2 I1 I5: 1} and {I2 I1 I3 I5: 1}.
- Therefore considering I5 as suffix, its 2 corresponding prefix paths would be {I2 I1: 1} and {I2 I1 I3: 1}, which forms its conditional pattern base.
• Out of these, Only I1 & I2 is selected in the conditional FP-Tree because I3 is not satisfying the minimum support count.
  For I1, support count in conditional pattern base = 1 + 1 = 2
  For I2, support count in conditional pattern base = 1 + 1 = 2
  For I3, support count in conditional pattern base = 1
  Thus support count for I3 is less than required min_sup which is 2 here.
• Now, We have conditional FP-Tree with us.
• All frequent pattern corresponding to suffix I5 are generated by considering all possible combinations of I5 and conditional FP-Tree.
• The same procedure is applied to suffixes I4, I3 and I1.
• Note: I2 is not taken into consideration for suffix because it doesn’t have any prefix at all.
Why Frequent Pattern Growth Fast?

• Performance study shows
  – FP-growth is an order of magnitude faster than Apriori, and is also faster than tree-projection

• Reasoning
  – No candidate generation, no candidate test
  – Use compact data structure
  – Eliminate repeated database scan
  – Basic operation is counting and FP-tree building