cse634
DATA MINING

Lecture Notes
Chapter 6: CLASSIFICATION by DECISION TREES
Introduction

BASIC ALGORITHM

Examples

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Classification Learning ALGORITHMS

Different Classifiers

- DESCRIPTIVE:
- Decision Trees (ID3, C4.5)
- Rough Sets
- Genetic Algorithms
- Classification by Association

- STATISTICAL:
- Neural Networks
- Bayesian Networks

Classification Data

- **Data format:** a data table with key attribute removed. Special attribute- class attribute must be distinguished

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

Classification (Training) Data with objects

rec	Age	Income	Student	Credit_rating	Buys_computer (CLASS)
r1	<=30	High	No	Fair	No
r2	<=30	High	No	Excellent	No
r3	31…40	High	No	Fair	Yes
r4	>40	Medium	No	Fair	Yes
r5	>40	Low	Yes	Fair	Yes
r6	>40	Low	Yes	Excellent	No
r7	31…40	Low	Yes	Excellent	Yes
r8	<=30	Medium	No	Fair	No
r9	<=30	Low	Yes	Fair	Yes
r10	>40	Medium	Yes	Fair	Yes
r11	<=30	Medium	Yes	Excellent	Yes
r12	31…40	Medium	No	Excellent	Yes
r13	31…40	High	Yes	Fair	Yes
r14	>40	Medium	No	Excellent	No

Classification by Decision Tree Induction

- Decision tree is

  A flow-chart-like tree structure;

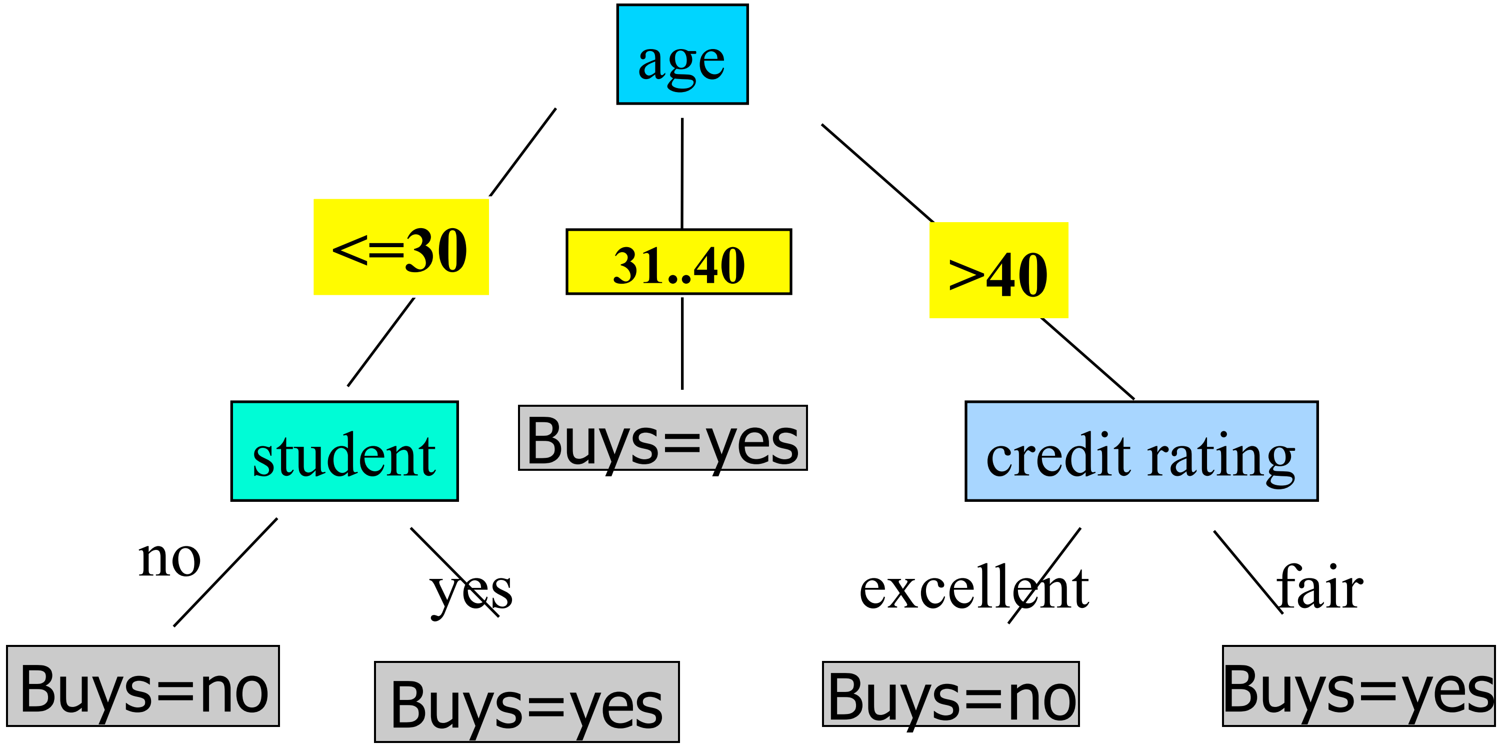
  Internal node denotes an attribute;

  Branch represents the values of the node attribute;

  Leaf nodes represent class labels or class distribution

DECISION TREE

An Example



Classification by Decision Tree Induction

Basic Algorithm

- The basic algorithm for **decision tree** construction is a greedy algorithm that constructs **decision trees** in a top-down **recursive** divide-and-conquer manner

- Given a **training set D** of classification data, i.e.
- a data table with a **distinguished class attribute**

- This **training set** is **recursively partitioned** into smaller subsets (data tables) as the **tree is being built**

Classification by Decision Tree Induction

Basic Algorithm

- **Tree** STARTS as a single node (**root**) representing all **training dataset** **D** (samples)

- We **choose** a **root attribute** from **D**
- It is called a **SPLIT attribute**

- **A branch** is created for **each value as defined in D** of the **node attribute** and is labeled by this values and the samples (it means the data table) are **partitioned** accordingly

- The **algorithm** uses the same process **recursively** to form a **decision tree** at **each partition**

- Once an attribute has occurred at a node, it **need not be** considered in any other of the node's descendants

Classification by Decision Tree Induction

Basic Algorithm

- The **recursive partitioning** **STOPS** only when any **one** of the following conditions is **true**

1. **All the samples** (records) in the partition are of the **same class**, then the node becomes **the leaf labeled with that class**

2. There is no remaining attributes on which the data may be further partitioned, i.e. **we have only** class attribute left
In this case we apply **MAJORITY VOTING** to **classify** the node

MAJORITY VOTING involves converting the node into a **leaf** and labeling it with the **most common class** in the **training data set**

3. There is no records (samples) left – a **LEAF** is created with **majority vote** for **training data set**

Classification by Decision Tree Induction

Crucial point

Good choice of the root attribute and internal nodes attributes is a **crucial point**

Bad choice may result, in the worst case in a just another knowledge representation:

a relational table re-written as a tree with class attributes (decision attributes) as the leaves.

- **Decision Tree Algorithms differ** on methods of **evaluating** and **choosing** the root and internal nodes attributes

Decision Tree Construction
Example 1

Consider our TRAING Dataset (next slide)

We START building the Decision Tree by **choosing** the attribute **age** as the **root** of the tree

Training Data with objects

rec	Age	Income	Student	Credit_rating	Buys_computer(CLASS)
r1	<=30	High	No	Fair	No
r2	<=30	High	No	Excellent	No
r3	31…40	High	No	Fair	Yes
r4	>40	Medium	No	Fair	Yes
r5	>40	Low	Yes	Fair	Yes
r6	>40	Low	Yes	Excellent	No
r7	31…40	Low	Yes	Excellent	Yes
r8	<=30	Medium	No	Fair	No
r9	<=30	Low	Yes	Fair	Yes
r10	>40	Medium	Yes	Fair	Yes
r11	<=30	Medium	Yes	Excellent	Yes
r12	31…40	Medium	No	Excellent	Yes
r13	31…40	High	Yes	Fair	Yes
r14	>40	Medium	No	Excellent	No

Building The Tree: we choose "age" as a root

age

<=30

>40

income	student	credit	class
high	no	fair	no
high	no	excellent	no
medium	no	fair	no
low	yes	fair	yes
medium	yes	excellent	yes

income	student	credit	class
medium	no	fair	yes
low	yes	fair	yes
low	yes	excellent	no
medium	yes	fair	yes
medium	no	excellent	no

31…40

income	student	credit	class
high	no	fair	yes
low	yes	excellent	yes
medium	no	excellent	yes
high	yes	fair	yes

Building The Tree: “age” as the root

age

<=30

income	student	credit	class
high	no	fair	no
high	no	excellent	no
medium	no	fair	no
low	yes	fair	yes
medium	yes	excellent	yes

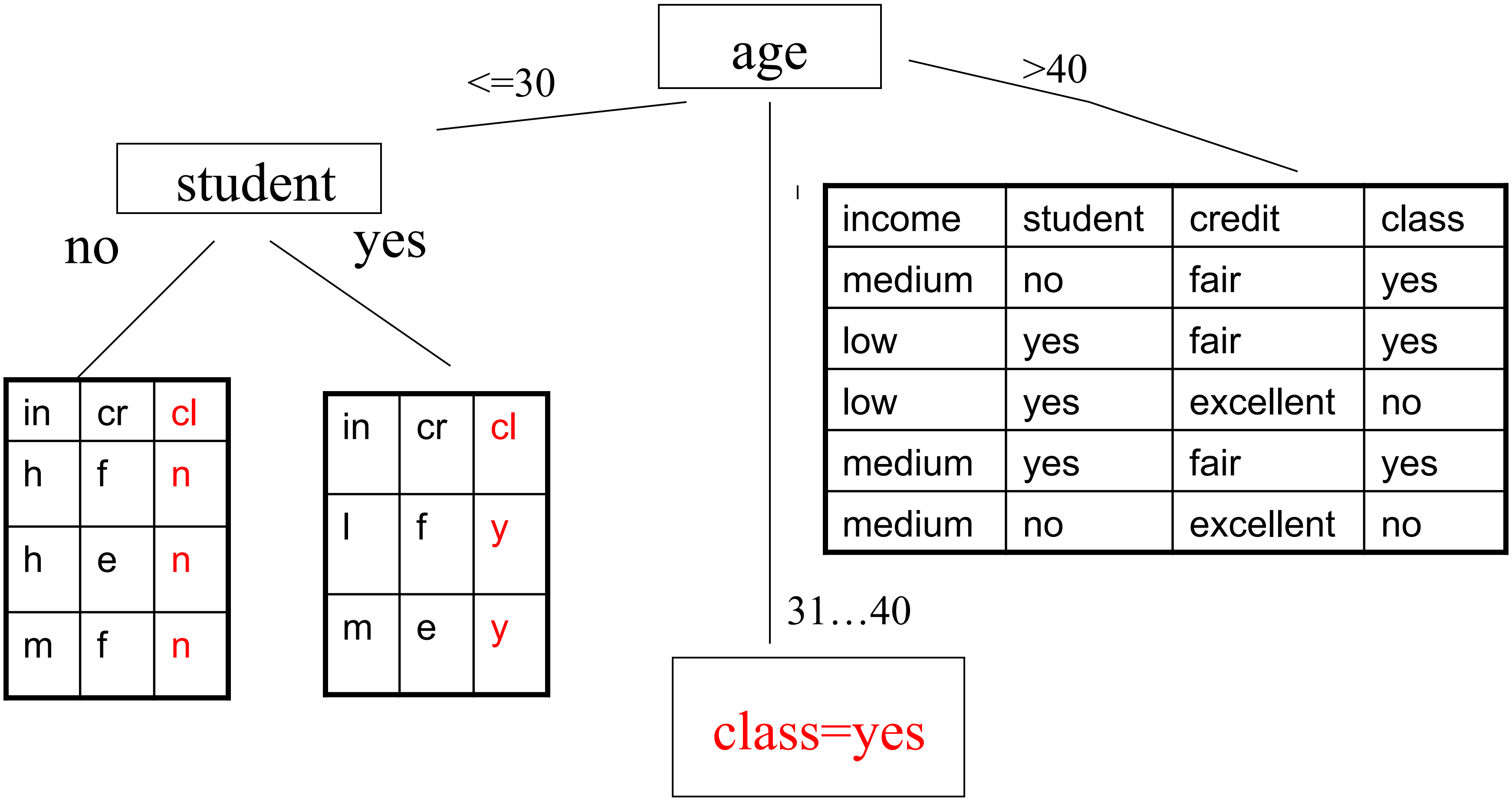
>40

income	student	credit	class
medium	no	fair	yes
low	yes	fair	yes
low	yes	excellent	no
medium	yes	fair	yes
medium	no	excellent	no

31…40

class=yes

Building The Tree: we chose "student" on <=30 branch

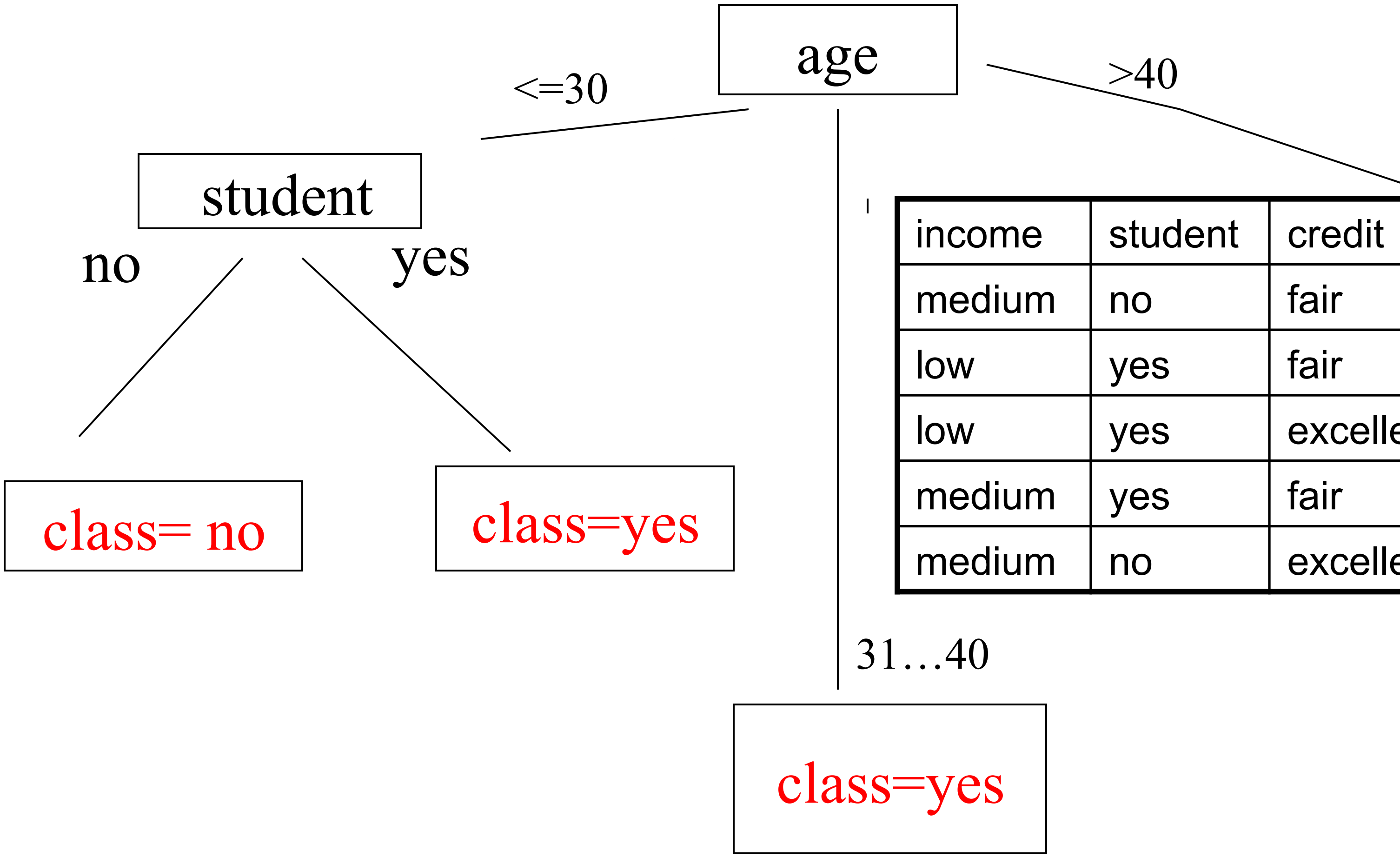


in	cr	cl
h	f	n
h	e	n
m	f	n

in	cr	cl
l	f	y
m	e	y

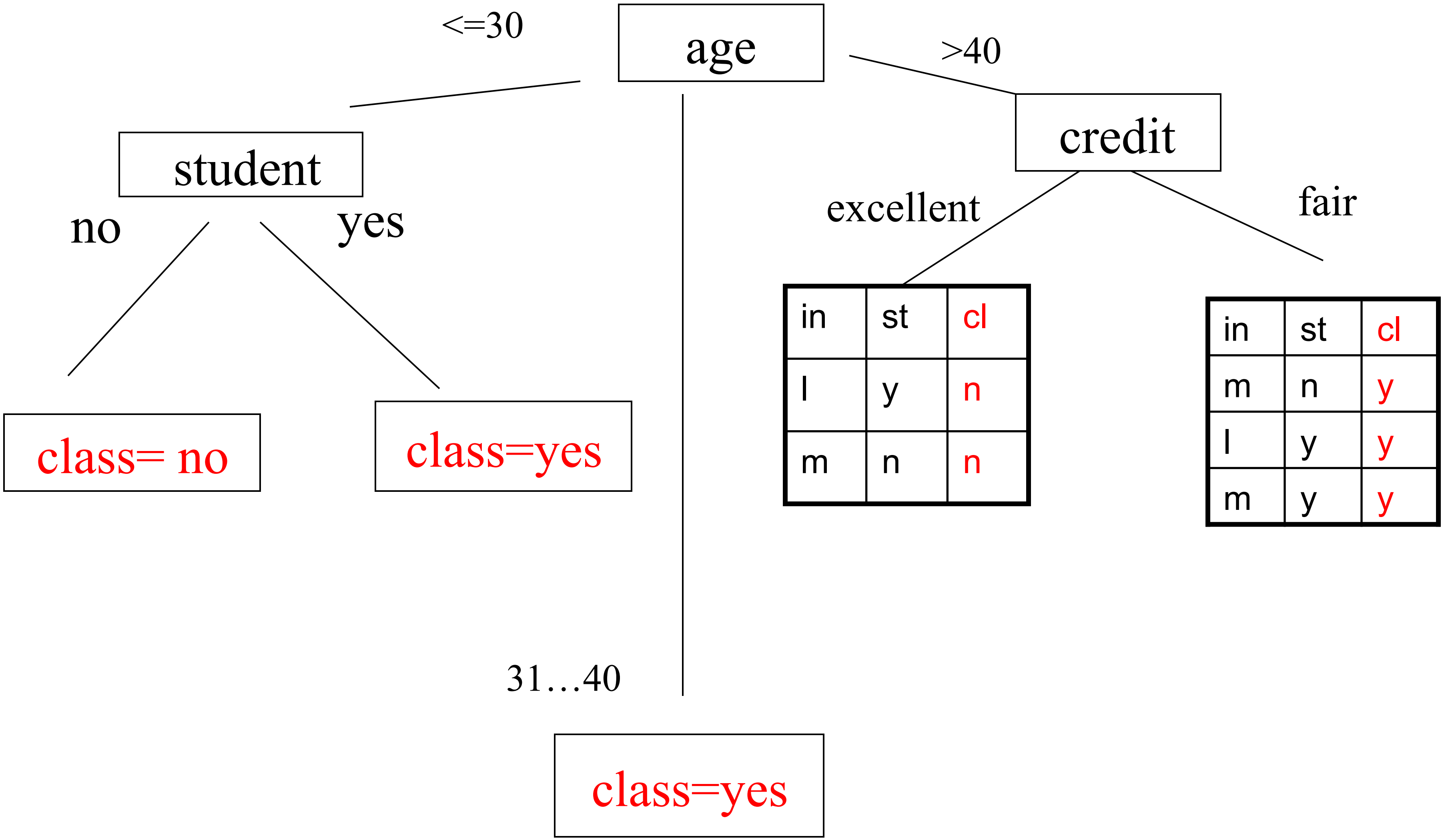
income	student	credit	class
medium	no	fair	yes
low	yes	fair	yes
low	yes	excellent	no
medium	yes	fair	yes
medium	no	excellent	no

Building The Tree: we chose "student" on <=30 branch



income	student	credit	class
medium	no	fair	yes
low	yes	fair	yes
low	yes	excellent	no
medium	yes	fair	yes
medium	no	excellent	no

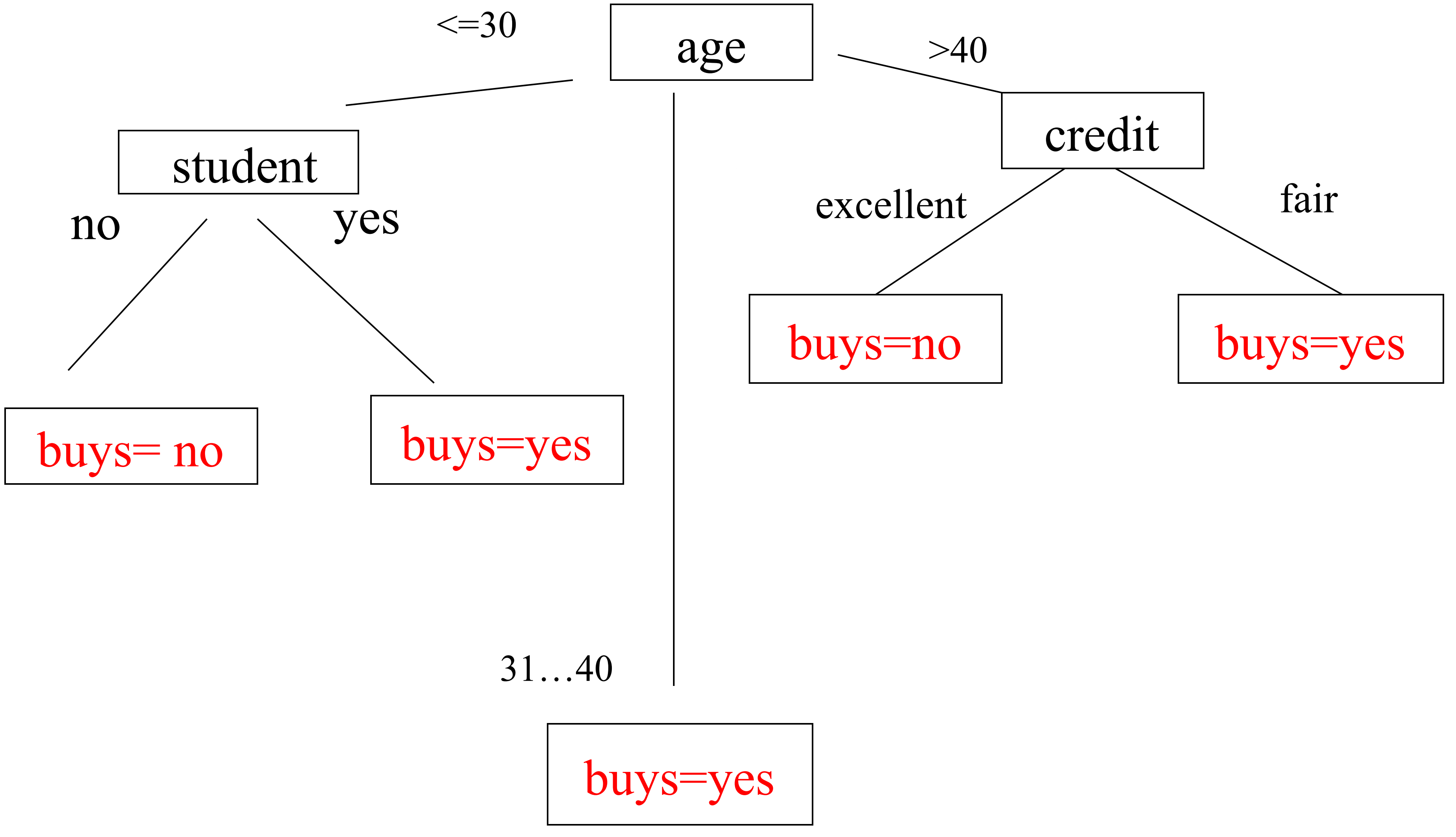
Building The Tree: we chose “credit” on >40 branch



in	st	cl
l	y	n
m	n	n

in	st	cl
m	n	y
l	y	y
m	y	y

Finished Tree for class="buys"



Extracting **Classification Rules** from Trees

- **Goal:** Represent the knowledge in the form of
- IF-THEN **discriminant** rules
- **One rule** is created for **each path** from the root to a leaf;
- **Each attribute-value** pair along a **path** forms a conjunction;
- The **leaf node** holds the class prediction

- Rules are easier to understand

Discriminant **RULES** extracted from our TREE

- The rules are:

IF *age* = “<=30” AND *student* = “*no*” THEN
buys_computer = “*no*”

IF *age* = “<=30” AND *student* = “*yes*” THEN
buys_computer = “*yes*”

IF *age* = “31…40” THEN
buys_computer = “*yes*”

IF *age* = “>40” AND *credit_rating* = “*excellent*” THEN
buys_computer = “no”

IF *age* = “>40” AND *credit_rating* = “*fair*” THEN
buys_computer = “yes”

Rules format for testing and applications

- In order to use rules for testing, and later when testing is done and predictive accuracy is acceptable we write rules in a **predicate form**:

IF *age*(x, <=30) AND *student(x, no)* THEN

buys_computer (x, *no*)

IF *age(x,* <=30) AND *student* (x, *yes*) THEN

buys_computer (x, *yes)*

- Attributes and their values of the new record x are **matched** with the **IF** part of the rule and the record x **is classified** accordingly to the **THEN** part of the rule

Exercise

Calculate the **predictive accuracy** of our set of rules with respect of the **TEST data** given by the next slide

R1: IF *age* = "<=30" AND *student* = "*no*" THEN *buys_computer* = "*no*"

R2: IF *age* = "<=30" AND *student* = "*yes*" THEN *buys_computer* = "*yes*"

R3: IF *age* = "31…40" THEN *buys_computer* = "*yes*"

R4: IF *age* = ">40" AND *credit_rating* = "*excellent*" THEN *buys_computer* = "no"

R5: IF *age* = ">40" AND *credit_rating* = "*fair*" THEN *buys_computer* = "yes"

TEST Data
for predictive accuracy evaluation

rec	Age	Income	Student	Credit_rating	Buys_computer(CLASS)
r1	<=30	Low	No	Fair	yes
r2	<=30	High	yes	Excellent	No
r3	<=30	High	No	Fair	Yes
r4	31…40	Medium	yes	Fair	Yes
r5	>40	Low	Yes	Fair	Yes
r6	>40	Low	Yes	Excellent	yes
r7	31…40	High	Yes	Excellent	Yes
r8	<=30	Medium	No	Fair	No
r9	31…40	Low	no	Excellent	Yes
r10	>40	Medium	Yes	Fair	Yes

Basic Idea of ID3/C4.5 Algorithm

- **The basic algorithm** for **decision tree induction** is a **greedy** algorithm that constructs decision trees in a **top-down recursive divide-and – conquer manner.**

- The **basic strategy** is as follows.

- Tree **STARTS** as a single **node** representing all training dataset (data table with records called samples)

- **IF** the samples (records in the data table) are **all in the same class, THEN** the **node** becomes **a leaf** and is **labeled with that class**

Basic Idea of ID3/C4.5 Algorithm

- OTHERWISE
- the algorithm uses an **entropy-based measure** known as **information gain** as a **heuristic** for selecting the **attribute** that will **best separate** the samples:
  split the data table **into** **individual classes**

- **This attribute** becomes the node-name:
  test, or **tree split decision** attribute

- **A branch** is created for **each value** of the **node-attribute** (as defined by the training data)
  and is labeled by this value
  and the **samples** (data table at the node) are **partitioned accordingly**

Basic Idea of ID3/C4.5 Algorithm Revisited

- The algorithm uses the same process **recursively**
- to form a decision tree at each **partition**

- **Once an attribute has occurred at a node, it need not be considered in any other of the node's descendants**

- The **recursive partitioning** **STOPS** only when **any one** of the following conditions is **TRUE**

Basic Idea of ID3/C4.5 Algorithm

Termination conditions:

1. All records (samples) for the given node belong to the **same class**

OR

2. There are **no remaining attributes left** on which the samples (records in the data table) may be further partitioned

In this case we **convert** the given node into a **LEAF** and **label it** with the **class** in **majority** among original **training samples**

- This is called a **majority voting**
- **OR**

3. There is **no records (samples) left** – a **LEAF** is created with **majority vote** for **training sample**

Heuristics: Attribute Selection Measures

- **Construction** of the tree **depends** on the **order** in which root attributes are **selected**

- **Different choices** produce different trees; some better, some worse
- **Shallower** trees are better; they are the ones in which classification is **reached** in fewer levels

- These trees are said to be more efficient and hence **termination** is reached quickly

Attribute Selection Measures

- Given a training data set (set of training samples) there are many ways **to choose** the root and nodes attributes while constructing the decision tree
- **Some possible choices:**
- Random
- Attribute with smallest/largest number of values
- **Following** certain order of attributes
- We present here a **special order**: **information gain**

  as a **measure** of goodness **of the split**
- The attribute with the **highest** **information gain** is **always chosen** as the split decision attribute for the current node while building the tree.

Information Gain Computation (ID3/C4.5): Case of Two Classes

- Assume there are two classes, ***P* (positive)** and ***N* (negative)**

  Let S be a training data set consisting of s examples (records):

  |S|=s

  And S contains p elements of class P and n elements of class N

  The amount of information, needed to **decide** if an arbitrary example in S belongs to P or N is defined as

$$I(p,n) = -\frac{p}{p+n}\log_2\frac{p}{p+n} - \frac{n}{p+n}\log_2\frac{n}{p+n}$$

- We use log_2 because the information is encoded in bits

Information Gain Measure

- Assume that using **attribute A** a set S will be partitioned into sets $S_1, S_2, \ldots, S_v$ (v is number of values of the attribute A)

  If S_i contains p_i examples of P and n_i examples of N

  the **entropy E(A)**, or the **expected information** needed to **classify objects** in all sub-trees S_i is

$$E(A) = \sum_{i=1}^{v} \frac{p_i + n_i}{p + n} I(p_i, n_i)$$

- The encoding information that would be gained by branching on *A*

$$Gain(A) = I(p, n) - E(A)$$

Attribute Selection: Information Gain

Data Mining Book slide

- Class P: buys_computer = "yes"
- Class N: buys_computer = "no"

$$Info(D) = I(9,5) = -\frac{9}{14}\log_2(\frac{9}{14}) - \frac{5}{14}\log_2(\frac{5}{14}) = 0.940$$

$$Info_{age}(D) = \frac{5}{14}I(2,3) + \frac{4}{14}I(4,0) + \frac{5}{14}I(3,2) = 0.694$$

age	p_i	n_i	$I(p_i, n_i)$
<=30	2	3	0.971
31…40	4	0	0
>40	3	2	0.971

$\frac{5}{14}I(2,3)$ means "age <=30" has 5 out of 14 samples, with 2 yes'es and 3 no's. Hence

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

$$Gain(age) = Info(D) - Info_{age}(D) = 0.246$$

Similarly,

$$Gain(income) = 0.029$$

$$Gain(student) = 0.151$$

$$Gain(credit\_rating) = 0.048$$

Attribute Selection by Information Gain Computation

- **Class P: buys_computer = "yes"**
- **Class N: buys_computer = "no"**
- **I(p, n) = I(9, 5) =0.940**
- **Compute the entropy for**

age	p_i	n_i	$I(p_i, n_i)$
<=30	2	3	0.971
31…40	4	0	0
>40	3	2	0.971

$$E(age) = \frac{5}{14} I(2,3) + \frac{4}{14} I(4,0) + \frac{5}{14} I(3,2) = 0.694$$

Hence

$$Gain(age) = I(p,n) - E(age)$$

Gain(age)=0.246

Similarly

$$Gain(income) = 0.029$$

$$Gain(student) = 0.151$$

$$Gain(credit\_rating) = 0.048$$

The attribute "age" becomes the root.

Decision Tree Induction, Predictive Accuracy and Information Gain

EXAMPLES

Decision Tree Construction
Example 2

TASK: Use Decision Tree Induction algorithm and
use different choices of the **root** and **nodes attributes** to
FIND discriminant rules that determine whether a person
buys a computer or not

Compute **Information gain** for all nodes of the tree.

1. We **choose** attribute *buys_computer* as the class attribute
2. We perform DT algorithm "by hand" using different choices of the root attribute, and different "by hand" choices of the following nodes
3. We build two trees with attributes: *Income* and *Credit Rating* respectively, as the **root** attribute to derive rules

Training Data

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

Training Data with objects

rec	Age	Income	Student	Credit_rating	Buys_computer
r1	<=30	High	No	Fair	No
r2	<=30	High	No	Excellent	No
r3	31…40	High	No	Fair	Yes
r4	>40	Medium	No	Fair	Yes
r5	>40	Low	Yes	Fair	Yes
r6	>40	Low	Yes	Excellent	No
r7	31…40	Low	Yes	Excellent	Yes
r8	<=30	Medium	No	Fair	No
r9	<=30	Low	Yes	Fair	Yes
r10	>40	Medium	Yes	Fair	Yes
r11	<=30	Medium	Yes	Excellent	Yes
r12	31…40	Medium	No	Excellent	Yes
r13	31…40	High	Yes	Fair	Yes
r14	>40	Medium	No	Excellent	No

EXAMPLE 2 **Incorrect Solutions**

- BOTH TREES of the following **Example 2 Solutions ARE NOT CORRECT !!!**

- **FIND STEPS** where the construction didn’t follow the ALGORITHM and **CORRECT THEM**

- Write the **CORRECT Solutions** for the EXAMPLE 2
- **Perform Exercises 1 and 2 for the corrected trees**

Index: 1

Income

Gain=0.027

Low

Age	Student	Credit	Class
>40	Yes	Fair	Yes
>40	Yes	Exc	No
31-40	Yes	Exc	Yes
<=30	Yes	Fair	Yes

High

Age	Student	Credit	Class
<=30	No	Fair	No
<=30	No	Exc	No
31-40	No	Fair	Yes
31-40	Yes	Fair	Yes

Med

Age	Student	Credit	Class
>40	No	Fair	Yes
<=30	No	Fair	No
>40	Yes	Fair	Yes
<=30	Yes	Exc	Yes
31-40	No	Exc	Yes
>40	No	Exc	No

CORRECT? – INCORRECT?

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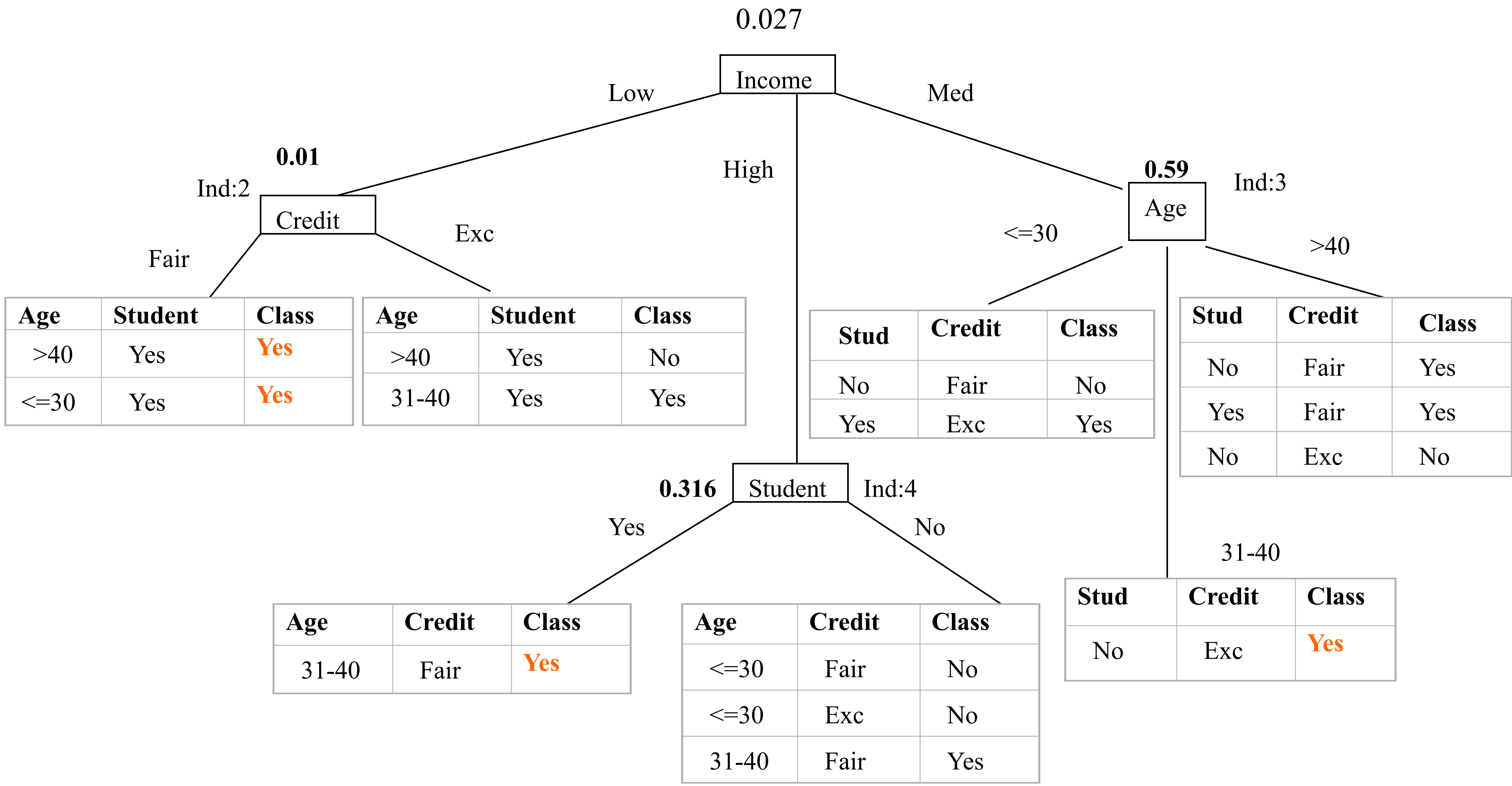


Age	Student	Class
>40	Yes	Yes
<=30	Yes	Yes

Age	Student	Class
>40	Yes	No
31-40	Yes	Yes

Stud	Credit	Class
No	Fair	No
Yes	Exc	Yes

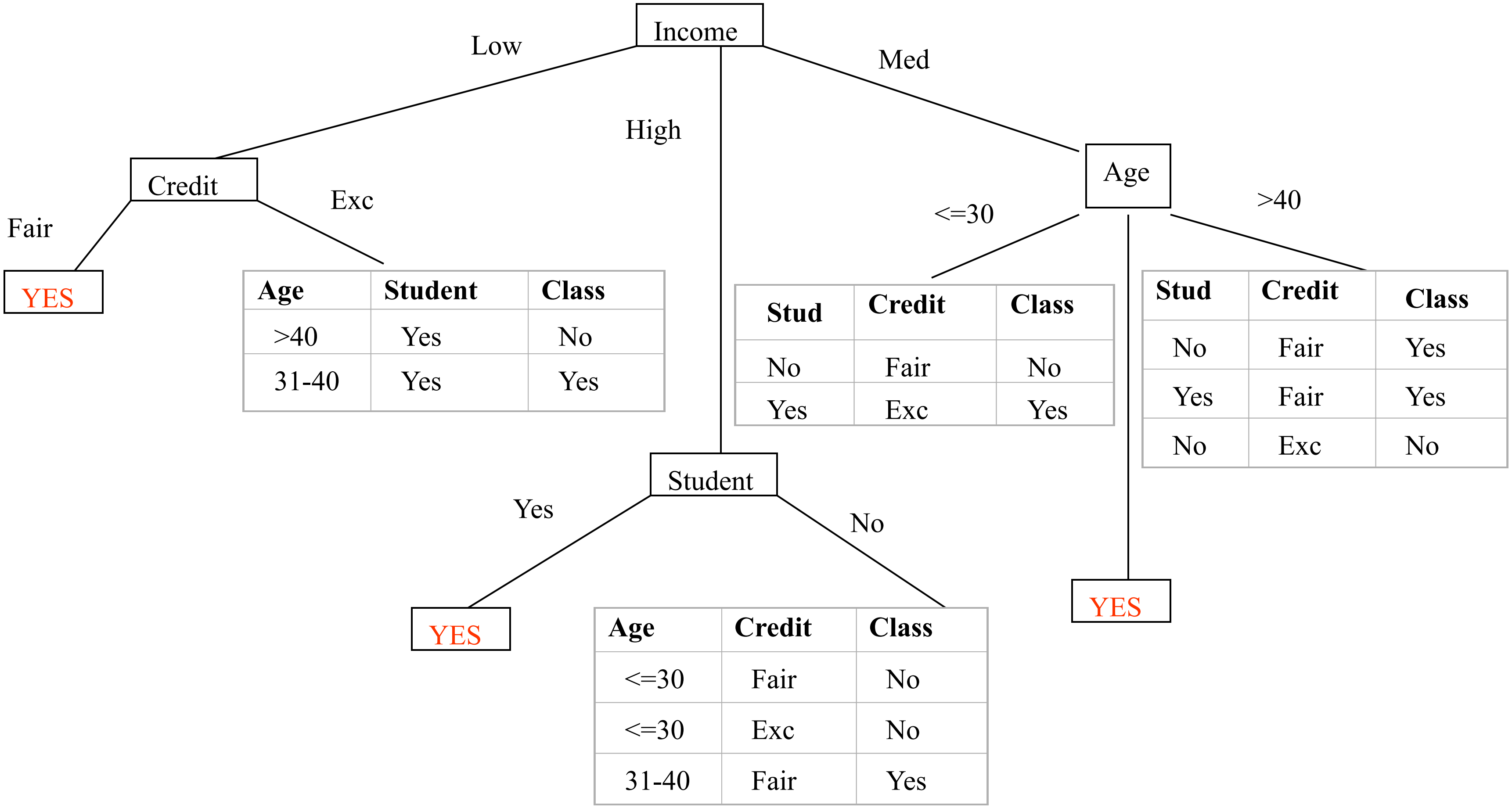
Stud	Credit	Class
No	Fair	Yes
Yes	Fair	Yes
No	Exc	No

Age	Credit	Class
31-40	Fair	Yes

Age	Credit	Class
<=30	Fair	No
<=30	Exc	No
31-40	Fair	Yes

Stud	Credit	Class
No	Exc	Yes

CORRECT? – INCORRECT?



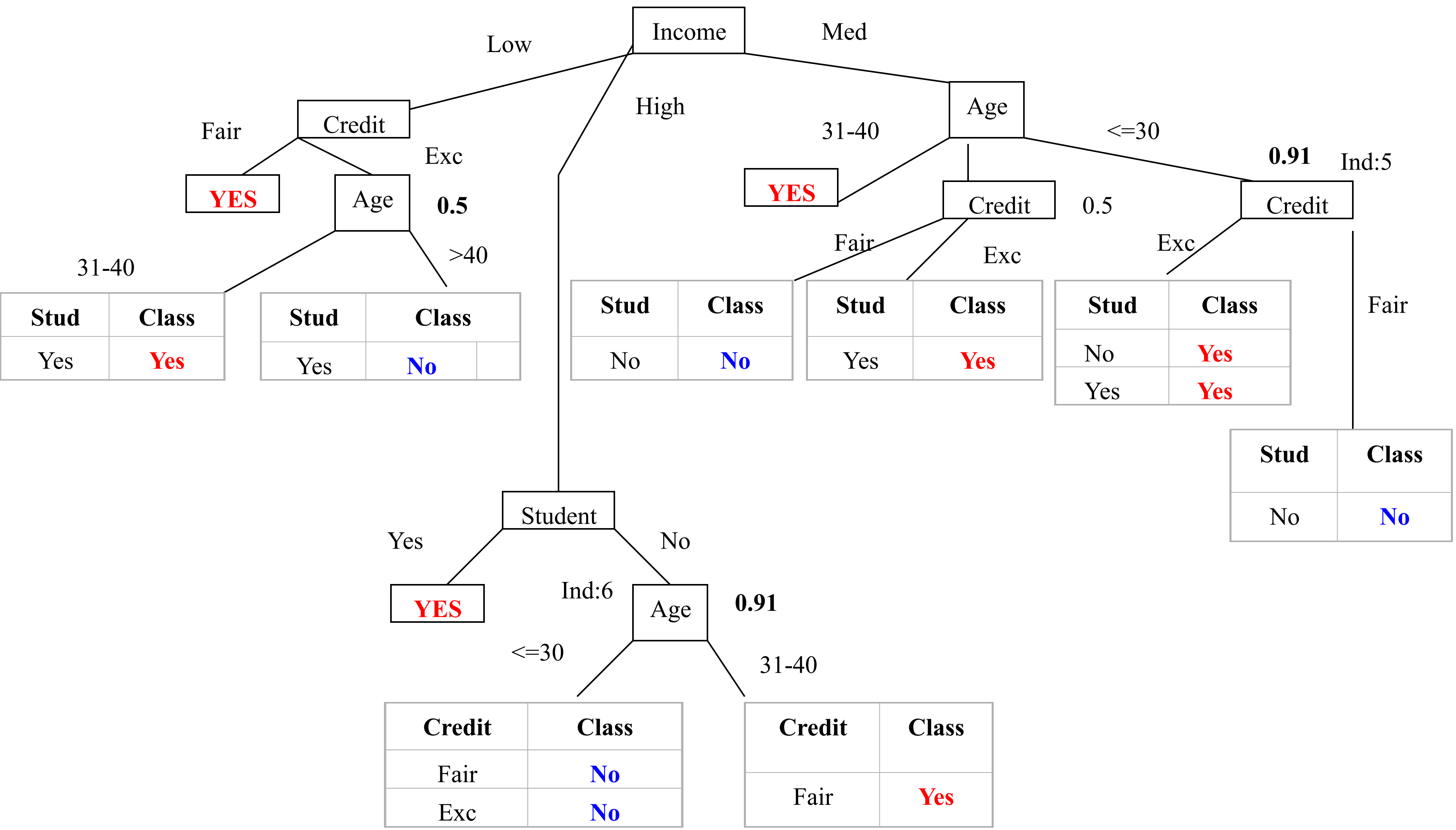
Age	Student	Class
>40	Yes	No
31-40	Yes	Yes

Stud	Credit	Class
No	Fair	No
Yes	Exc	Yes

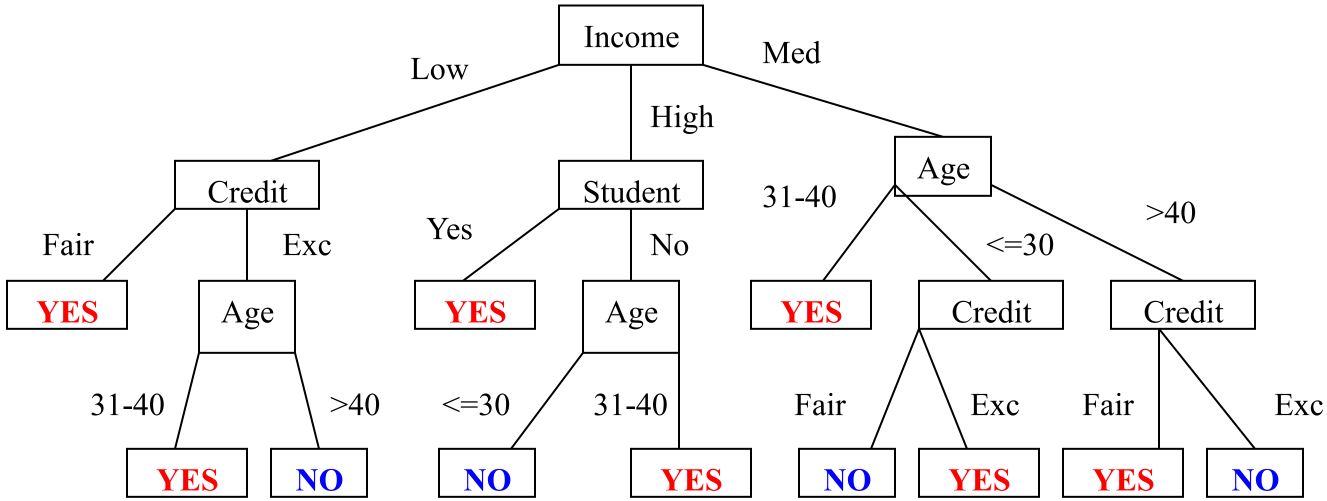
Stud	Credit	Class
No	Fair	Yes
Yes	Fair	Yes
No	Exc	No

Age	Credit	Class
<=30	Fair	No
<=30	Exc	No
31-40	Fair	Yes

CORRECT? – INCORRECT?



Tree 1 with root attribute Income



Rules derived from tree 1 (predicate form for testing)

1. Income(x, Low) ^ Credit(x, Fair) -> buysComputer(x, Yes).
2. Income(x, Low) ^ Credit(x, Exc) ^ Age(x, 31-40) -> buysComputer(x, Yes).
3. Income(x, Low) ^ Credit(x, Exc) ^ Age(>40) -> buysComputer(x, No).
4. Incomex, (High) ^ Student(x, Yes) -> buysComputer(x, Yes).
5. Income(x, High) ^ Student(x, No) ^ Agex(x, <=30) -> buysComputer(x, No).
6. Income(x, High) ^ Student(x, No) ^ Age(x, 31-40) -> buysComputer(x, Yes).
7. Income(x, Medium) ^ Age(x, 31-40) -> buysComputer(x, Yes).
8. Income(x, Medium) ^ Age(x, <=30) ^ Credit(x, Fair) -> buysComputer(x, No).
9. Income(x, Medium) ^ Age(x, <=30) ^ Credit(x, Exc) -> buysComputer(x, Yes).
10. Income(x, Medium) ^ Age(x, >40) ^ Credit(x, Fair) -> buysComputer(x, Yes).
11. Income(x, Medium) ^ Age(x, >40) ^ Credit(x, Exc) -> buysComputer(x, No).

Tree 2 with root attribute Credit Rating

Credit_Rating

Fair

Age	Income	Student	Class
<=30	High	No	No
31-40	High	No	Yes
>40	Med	No	Yes
>40	Low	Yes	Yes
<=30	Med	No	No
<=30	Low	Yes	Yes
>40	Med	Yes	Yes
31-40	High	Yes	Yes

Exc

Age	Income	Student	Class
<=30	High	No	No
>40	Low	Yes	Yes
31-40	Low	Yes	Yes
<=30	Med	Yes	Yes
31-40	Med	No	No
>40	Med	No	No

CORRECT? – INCORRECT?

CORRECT? – INCORRECT?

Credit_Rating

Fair

Income

Low

Age	Stud	Class
>40	Yes	YES
<=30	Yes	YES

High

Age	Stud	Class
<=30	No	No
31-40	No	Yes
31-40	Yes	Yes

Med

Age	Stud	Class
>40	No	Yes
<=30	No	No
>40	Yes	Yes

Exc

Student

Yes

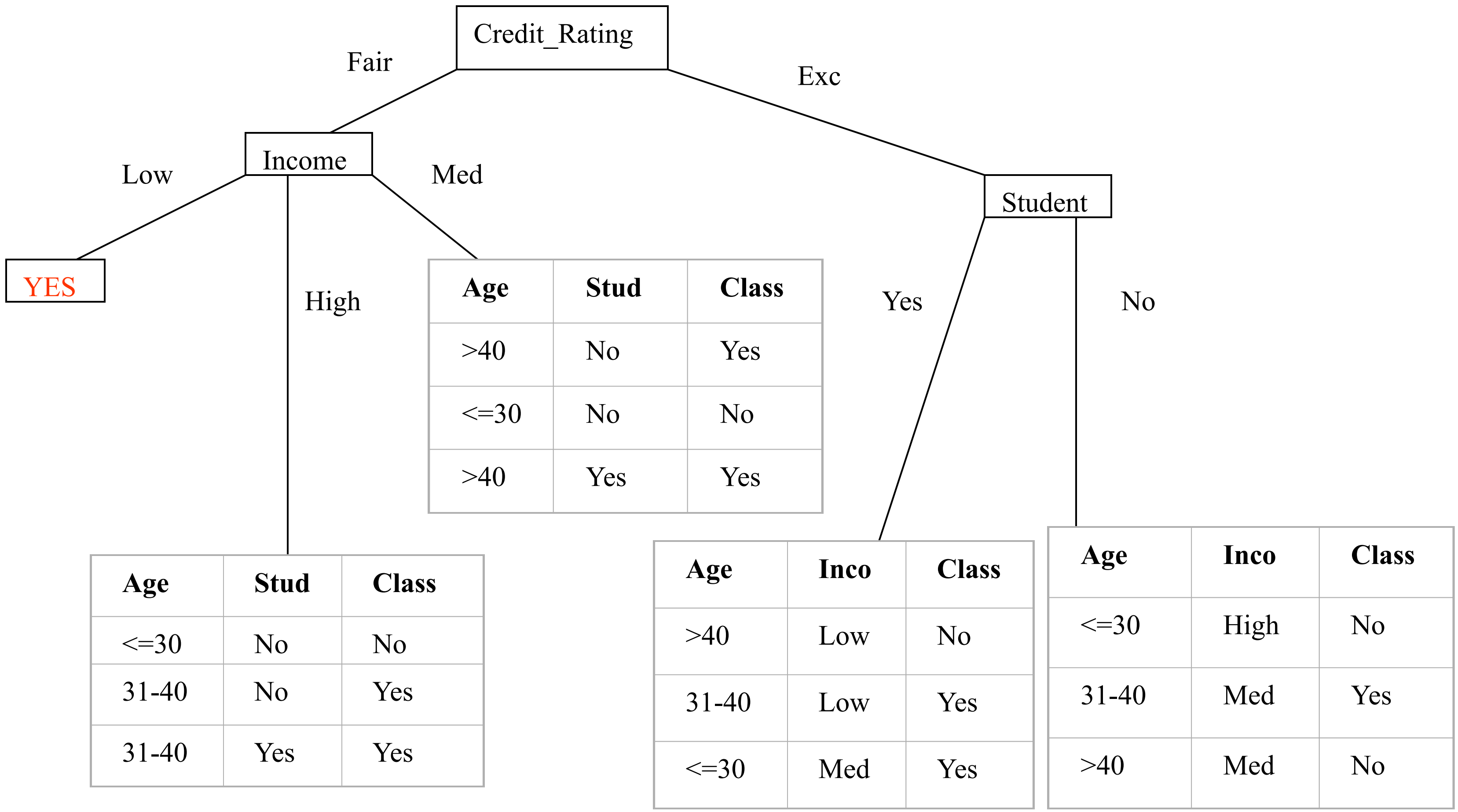
Age	Inco	Class
>40	Low	No
31-40	Low	Yes
<=30	Med	Yes

No

Age	Inco	Class
<=30	High	No
31-40	Med	Yes
>40	Med	No

Tree 2 with next level attributes Income and Student

CORRECT? – INCORRECT?



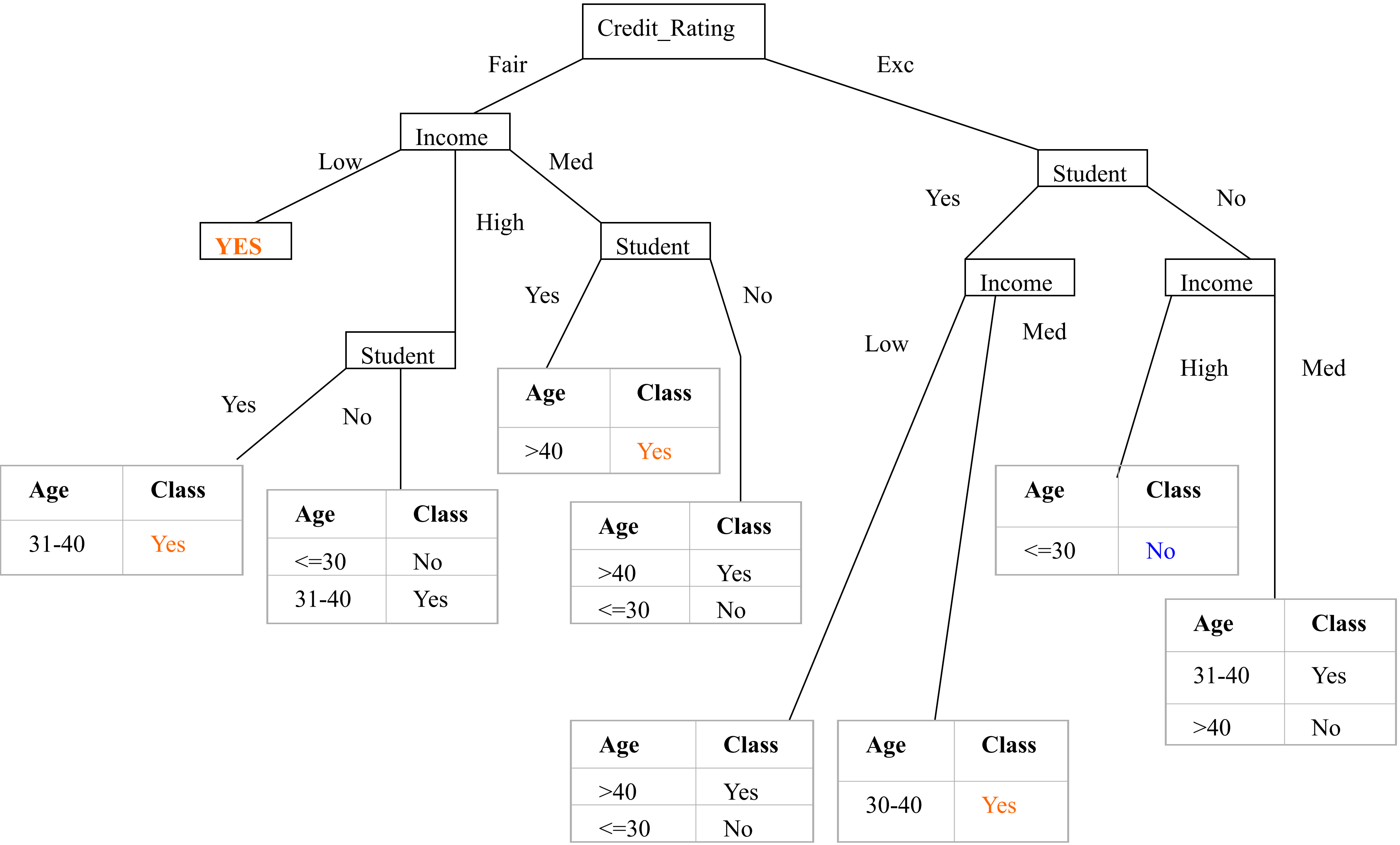
Age	Stud	Class
>40	No	Yes
<=30	No	No
>40	Yes	Yes

Age	Stud	Class
<=30	No	No
31-40	No	Yes
31-40	Yes	Yes

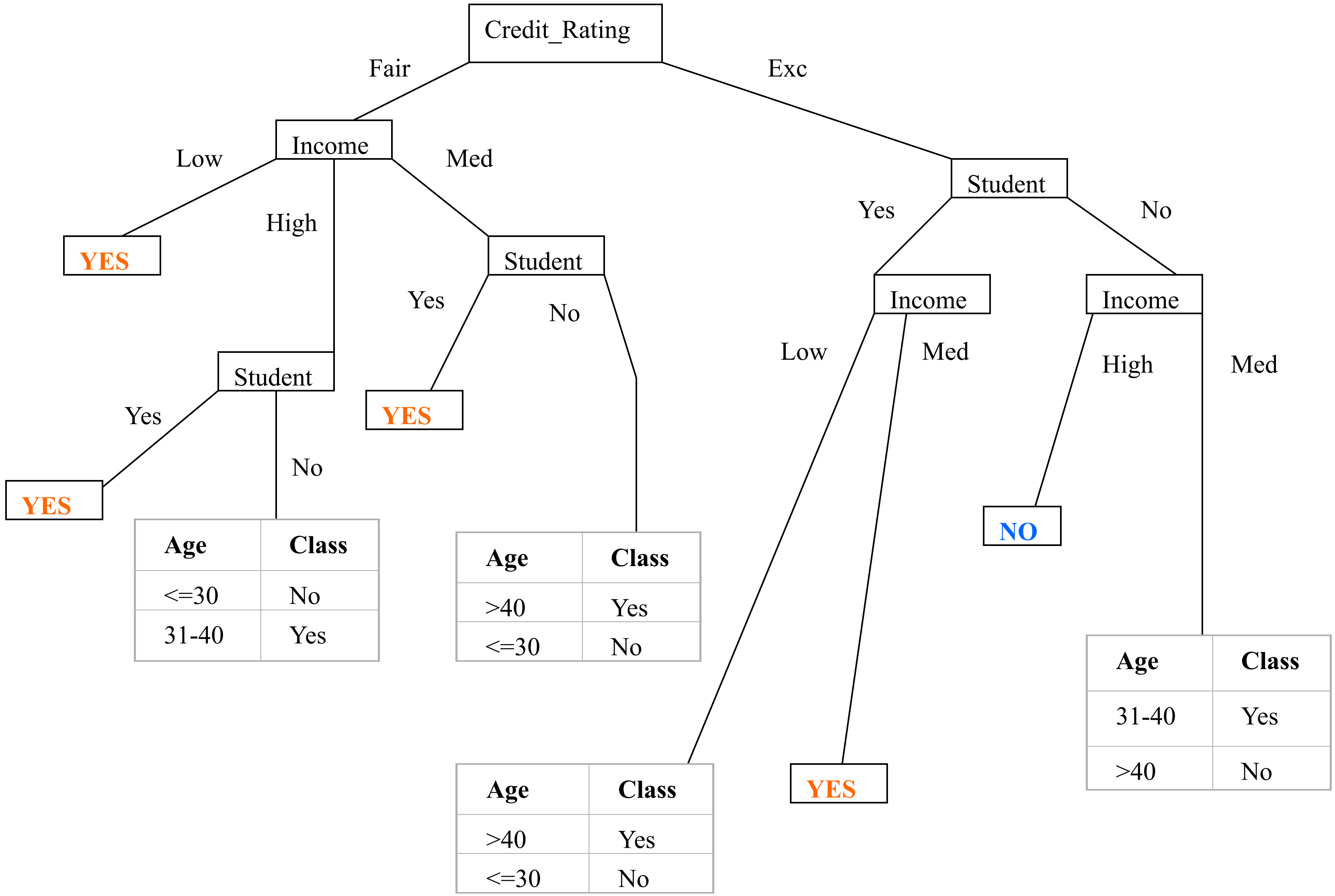
Age	Inco	Class
>40	Low	No
31-40	Low	Yes
<=30	Med	Yes

Age	Inco	Class
<=30	High	No
31-40	Med	Yes
>40	Med	No

Tree 2 with root attribute Credit Rating



CORRECT? – INCORRECT?



Age	Class
<=30	No
31-40	Yes

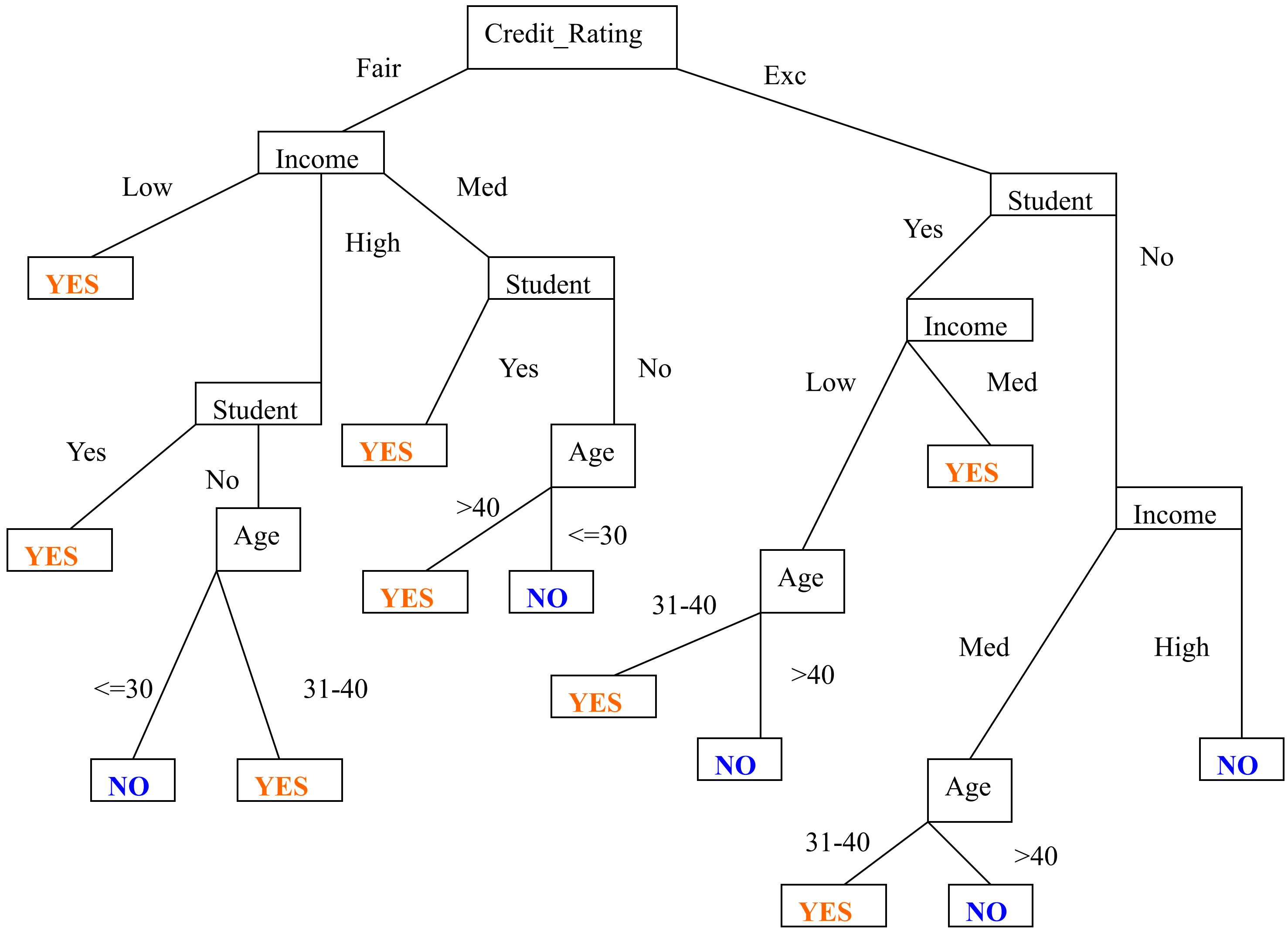
Age	Class
>40	Yes
<=30	No

Age	Class
>40	Yes
<=30	No

Age	Class
31-40	Yes
>40	No

CORRECT? – INCORRECT?

CORRECT? – INCORRECT?



Final Tree 2 with root attribute Credit Rating

The Decision tree with root attribute *Credit_Rating* has produced 13 rules, two more than with root attribute *Income*

1. Credit(x, Fair) ^ Income(x,Low) -> buysComp(x,Yes).
2. Credit(x,Fair) ^ Income(x, High) ^ Student(x,Yes) -> buysComp(x, Yes).
3. Credit(x,Fair) ^ Income(x, High) ^ Student(x, No) ^ Age(<=30) -> buysComp(x, No).
4. Credit(x,Fair) ^ Income(x, High) ^ Student(x, No) ^ Age(31-40) -> buysComp(x, Yes).
5. Credit(x, Fair) ^ Income(x, Med) ^ Student(x, Yes) -> buysComp(x, Yes).
6. Credit(x, Fair) ^ Income(x, Med) ^ Student(x, No) ^ Age(>40) -> buysComp(x, Yes).
7. Credit(x, Fair) ^ Income(x, Med) ^ Student(x, No) ^ Age(<=30) -> buysComp(x, No).
8. Credit(x, Exc) ^ Student(x, Yes) ^ Income(x, Low) ^ Age(31-40) -> buysComp(x, Yes).
9. Credit(x, Exc) ^ Student(x, Yes) ^ Income(x, Low) ^ Age(>40) -> buysComp(x, No).
10. Credit(x, Exc) ^ Student(x, Yes) ^ Income(x, Med) -> buysComp(x, Yes).
11. Credit(x, Exc) ^ Student(x, No) ^ Income(x, Med) ^ Age(x, 31-40) -> buysComp(x, Yes).
12. Credit(x, Exc) ^ Student(x, No) ^ Income(x, Med) ^ Age(x, >40) -> buysComp(x, No).
13. Credit(x, Exc) ^ Student(x, No) ^ Income(x, High) -> buysComp(x, No).

EXERCISE 1

- We use some random records (tuples) to calculate the Predictive Accuracy of the set

  of rules from the **Example 2**

Predictive Accuracy is the % of well classified records not from training set for which the class attribute is known

Random Tuples to Check Predictive Accuracy based on three sets of rules

Obj	Age	Income	Student	Credit_R	Class
1	<=30	High	Yes	Fair	Yes
2	31-40	Low	No	Fair	Yes
3	31-40	High	Yes	Exc	No
4	>40	Low	Yes	Fair	Yes
5	>40	Low	Yes	Exc	No
6	<=30	Low	No	Fair	No

Predictive accuracy:

1. Against Lecture Notes: 4/6 = 66.66%
2. Against Tree 1 rules with root att. *Income*: 3/6 = 50%
3. Against Tree 2 rules with root att. *Credit*: 5/6 = 83.33%

EXERCISE 2

- **Predictive accuracy depends heavily on a choice of the test and training data.**

- Find a small set of **TEST records** such that

they would give a **predictive accuracy 100%** for rules
From the Lecture Tree and Trees 1 and 2
from **Example 1**

1. <u>TEST DATA applied against rules in Lecture Notes</u> that gives predictive accuracy 100%

No	Age	Income	Student	Credit_R	Class
1	<=30	Med	No	Exc	No
2	<=30	High	Yes	Fair	Yes
3	31-40	Low	No	Exc	Yes
4	>40	High	Yes	Exc	No
5	<=30	Low	No	Fair	Yes
6	31-40	High	Yes	Fair	Yes

2. TEST DATA that applied against the rules with root attribute *Income* give **predictive accuracy 100%**

No	Age	Income	Student	Credit_R	Class
1	31-40	Low	Yes	Fair	Yes
2	>40	Low	No	Exc	No
3	<=30	High	Yes	Fair	Yes
4	31-40	High	No	Exc	Yes
5	31-40	Med	No	Fair	Yes
6	>40	Med	Yes	Exc	No

3.TEST DATA that applied against the rules with root attribute *Credit Rating* **gives predictive accuracy 100%**

No	Age	Income	Student	Credit_R	Class
1	31-40	Low	No	Fair	Yes
2	<=30	High	Yes	Fair	Yes
3	<=30	Med	No	Fair	No
4	31-40	High	Yes	Exc	Yes
5	>40	Med	Yes	Exc	No
6	>40	Med	No	Exc	No

Exercise 2 **Corrections**

We **FIXED** the following two points of the **Tree construction:**

1. We **choose recursively** internal nodes (attributes) with **all of their proper values** as branches

Mistake: NOT ALL attributes values were always used

2. there is **no more samples** (records) left
   In this case we apply **Majority Voting** to classify the node, where the

Majority Voting involves converting the node into a **leaf** and labeling it with the **most common class in the training set**

Mistake: NO MAJORITY Voting was used

CORRECT

Index: 1

Income

Gain=0.027

Low

Age	Student	Credit	Class
>40	Yes	Fair	Yes
>40	Yes	Exc	No
31-40	Yes	Exc	Yes
<=30	Yes	Fair	Yes

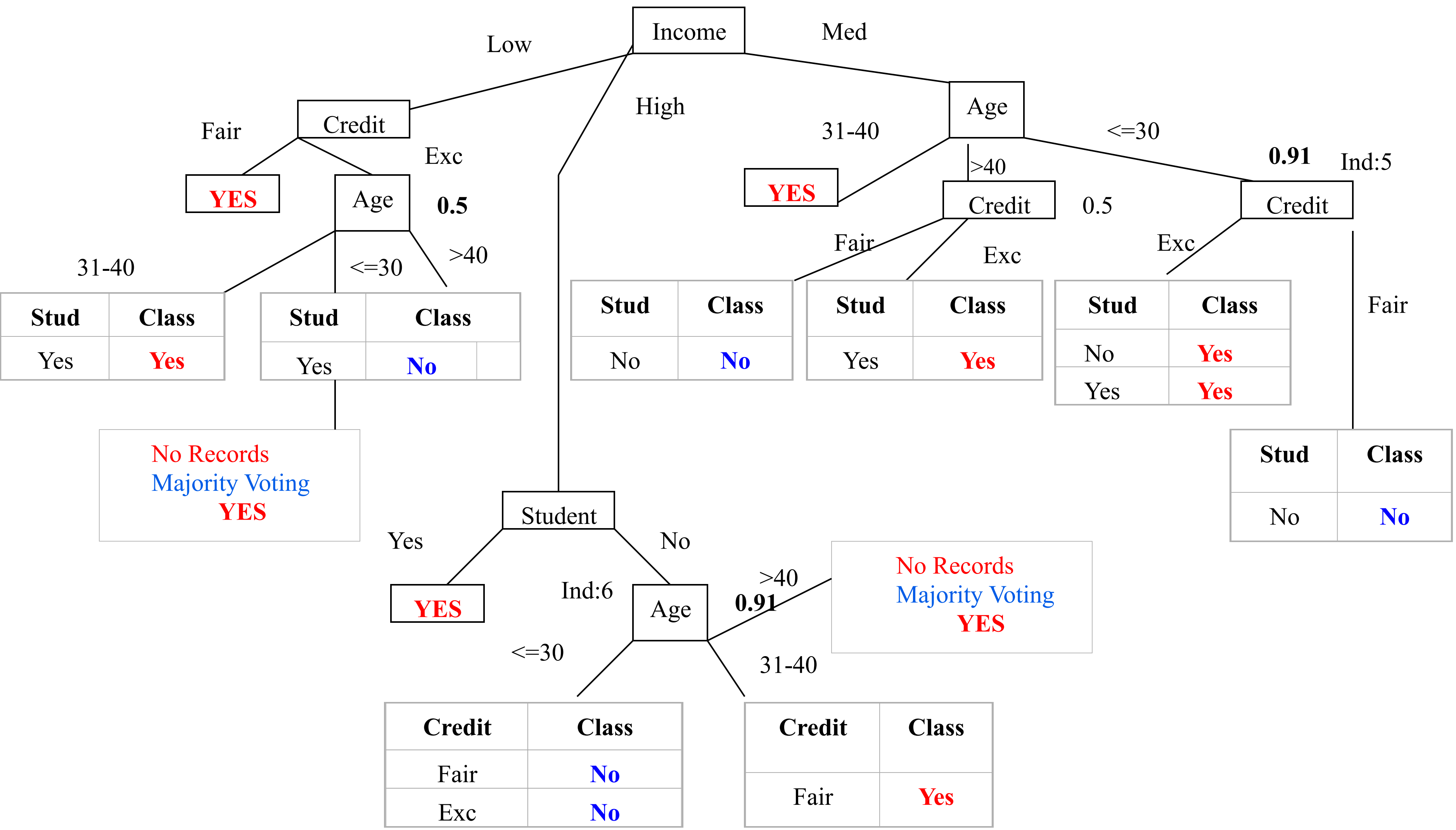
Med

Age	Student	Credit	Class
>40	No	Fair	Yes
<=30	No	Fair	No
>40	Yes	Fair	Yes
<=30	Yes	Exc	Yes
31-40	No	Exc	Yes
>40	No	Exc	No

High

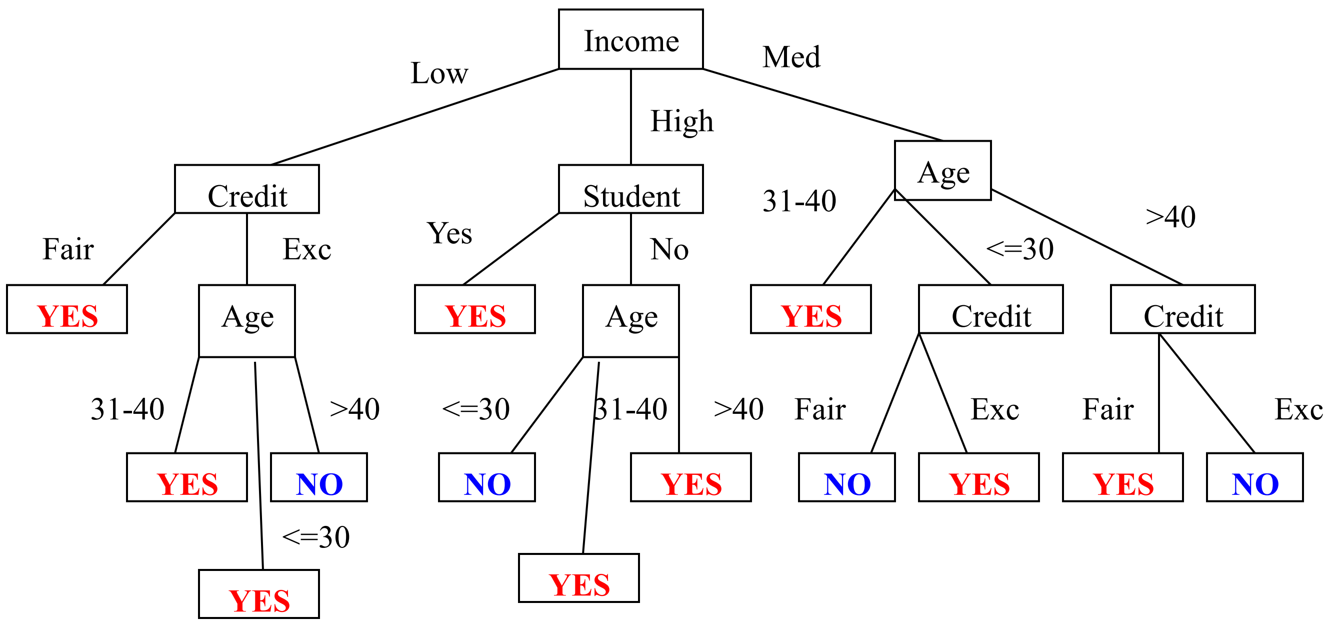
Age	Student	Credit	Class
<=30	No	Fair	No
<=30	No	Exc	No
31-40	No	Fair	Yes
31-40	Yes	Fair	Yes

CORRECTED



Income
Low
Med
High
Credit
Fair
Exc
YES
Age
0.5
31-40
<=30
>40
Stud Class
Yes Yes
Stud Class
Yes No
No Records
Majority Voting
YES
Age
31-40
<=30
YES
>40
Credit
0.5
0.91
Ind:5
Credit
Fair
Exc
Stud Class
No No
Stud Class
Yes Yes
Exc
Stud Class
No Yes
Yes Yes
Fair
Stud Class
No No
Student
Yes
No
YES
Ind:6
Age
>40
0.91
No Records
Majority Voting
YES
<=30
31-40
Credit Class
Fair No
Exc No
Credit Class
Fair Yes

CORRECT Tree 1 with root attribute Income



Rules derived from Tree 1 (predicate form for testing)

1. Income(x, Low) ^ Credit(x, Fair) -> buysComputer(x, Yes).
2. Income(x, Low) ^ Credit(x, Exc) ^ Age(x, 31-40) -> buysComputer(x, Yes).
3. Income(x, Low) ^ Credit(x, Exc) ^ Age(>40) -> buysComputer(x, No).
4. Incomex, (High) ^ Student(x, Yes) -> buysComputer(x, Yes).
5. Income(x, High) ^ Student(x, No) ^ Age(x, <=30) -> buysComputer(x, No).
6. Income(x, High) ^ Student(x, No) ^ Age(x, 31-40) -> buysComputer(x, Yes).
7. Income(x, Medium) ^ Age(x, 31-40) -> buysComputer(x, Yes).
8. Income(x, Medium) ^ Age(x, <=30) ^ Credit(x, Fair) -> buysComputer(x, No).
9. Income(x, Medium) ^ Age(x, <=30) ^ Credit(x, Exc) -> buysComputer(x, Yes).
10. Income(x, Medium) ^ Age(x, >40) ^ Credit(x, Fair) -> buysComputer(x, Yes).
11. Income(x, Medium) ^ Age(x, >40) ^ Credit(x, Exc) -> buysComputer(x, No).
12. Income(x, Low) ^ Age(x, <=30) ^ Credit(x, Exc) -> buysComputer(x, Yes). Majority Voting
13. Income(x, High) ^ Student(x, No) ^ Age(x>40) -> buysComputer(x, Yes). Majority Voting

Tree 2 with root attribute Credit Rating

Credit_Rating

Fair

Age	Income	Student	Class
<=30	High	No	No
31-40	High	No	Yes
>40	Med	No	Yes
>40	Low	Yes	Yes
<=30	Med	No	No
<=30	Low	Yes	Yes
>40	Med	Yes	Yes
31-40	High	Yes	Yes

Exc

Age	Income	Student	Class
<=30	High	No	No
>40	Low	Yes	Yes
31-40	Low	Yes	Yes
<=30	Med	Yes	Yes
31-40	Med	No	No
>40	Med	No	No

CORRECT

CORRECT

Credit_Rating

Fair

Income

Low

Age	Stud	Class
>40	Yes	YES
<=30	Yes	YES

High

Age	Stud	Class
<=30	No	No
31-40	No	Yes
31-40	Yes	Yes

Med

Age	Stud	Class
>40	No	Yes
<=30	No	No
>40	Yes	Yes

Exc

Student

Yes

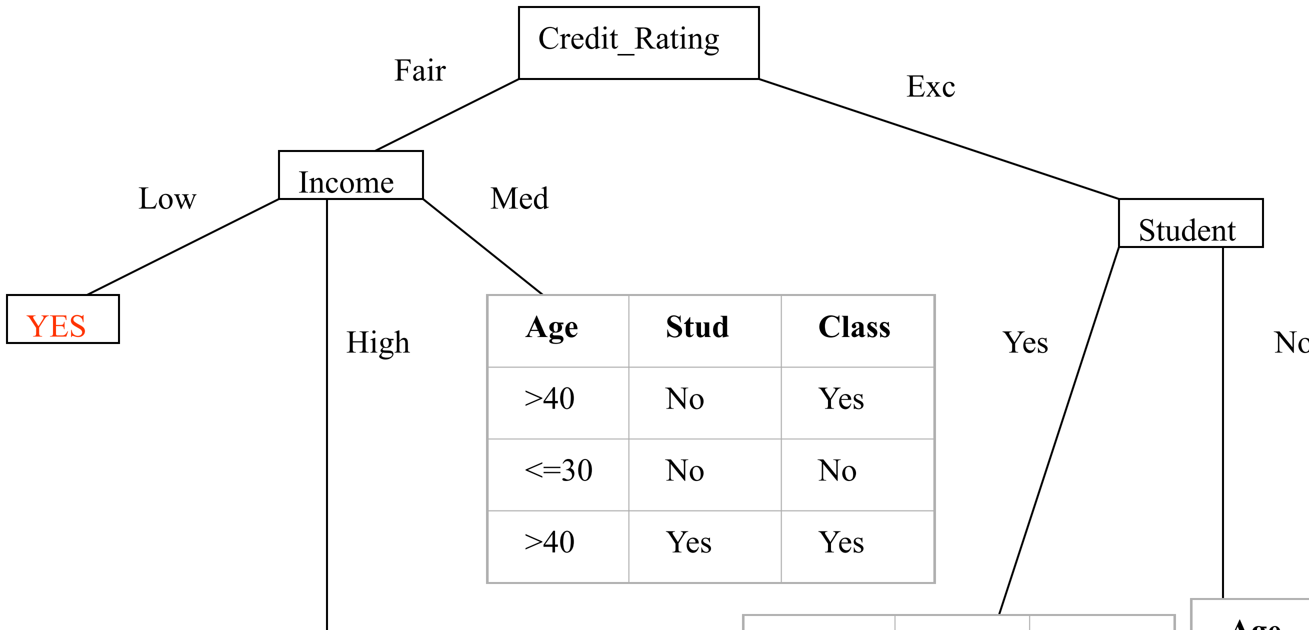
Age	Inco	Class
>40	Low	No
31-40	Low	Yes
<=30	Med	Yes

No

Age	Inco	Class
<=30	High	No
31-40	Med	Yes
>40	Med	No

Tree 2 with next level attributes Income and Student

CORRECT



Income = Med:

Age	Stud	Class
>40	No	Yes
<=30	No	No
>40	Yes	Yes

Income = High:

Age	Stud	Class
<=30	No	No
31-40	No	Yes
31-40	Yes	Yes

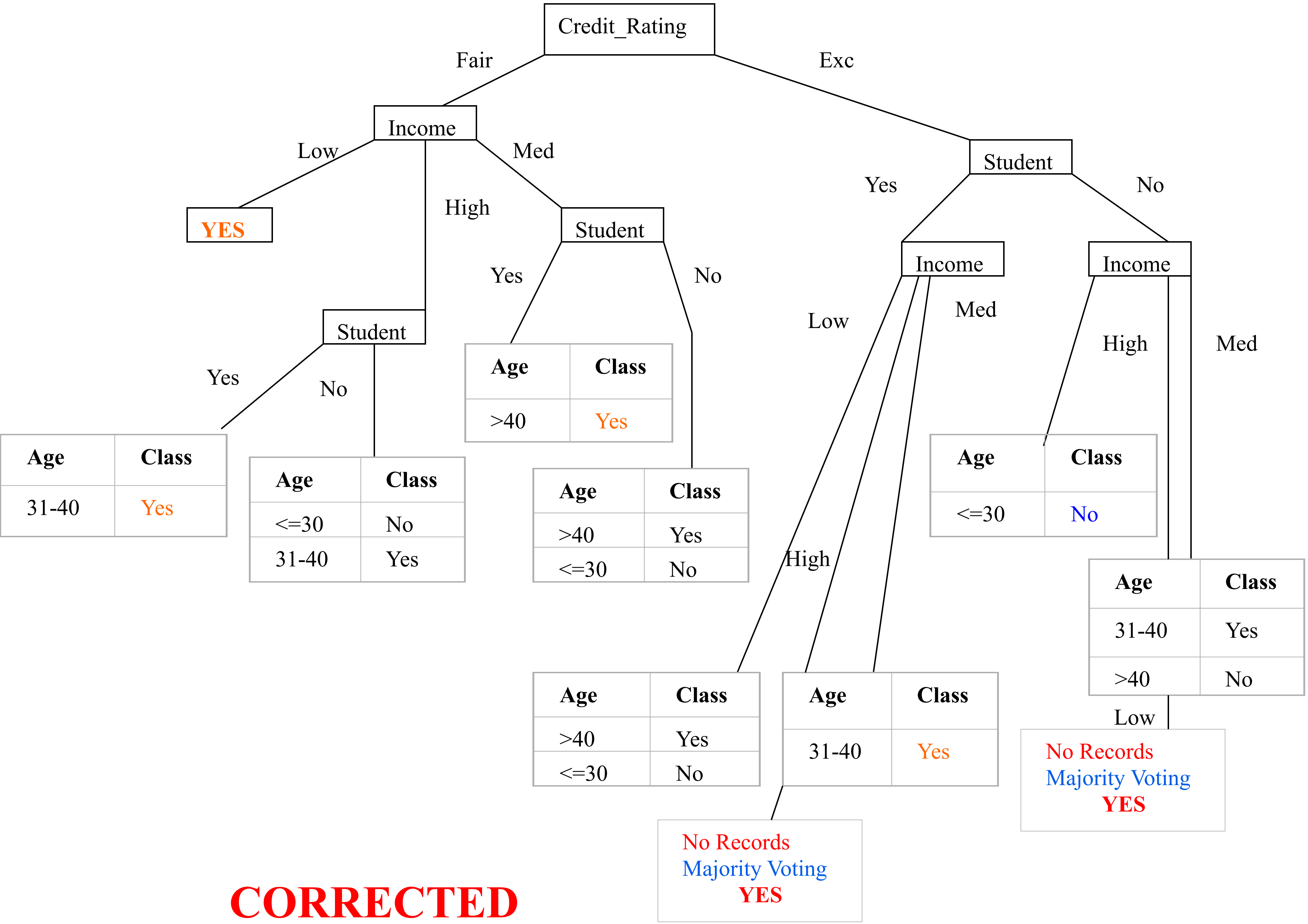
Student = Yes:

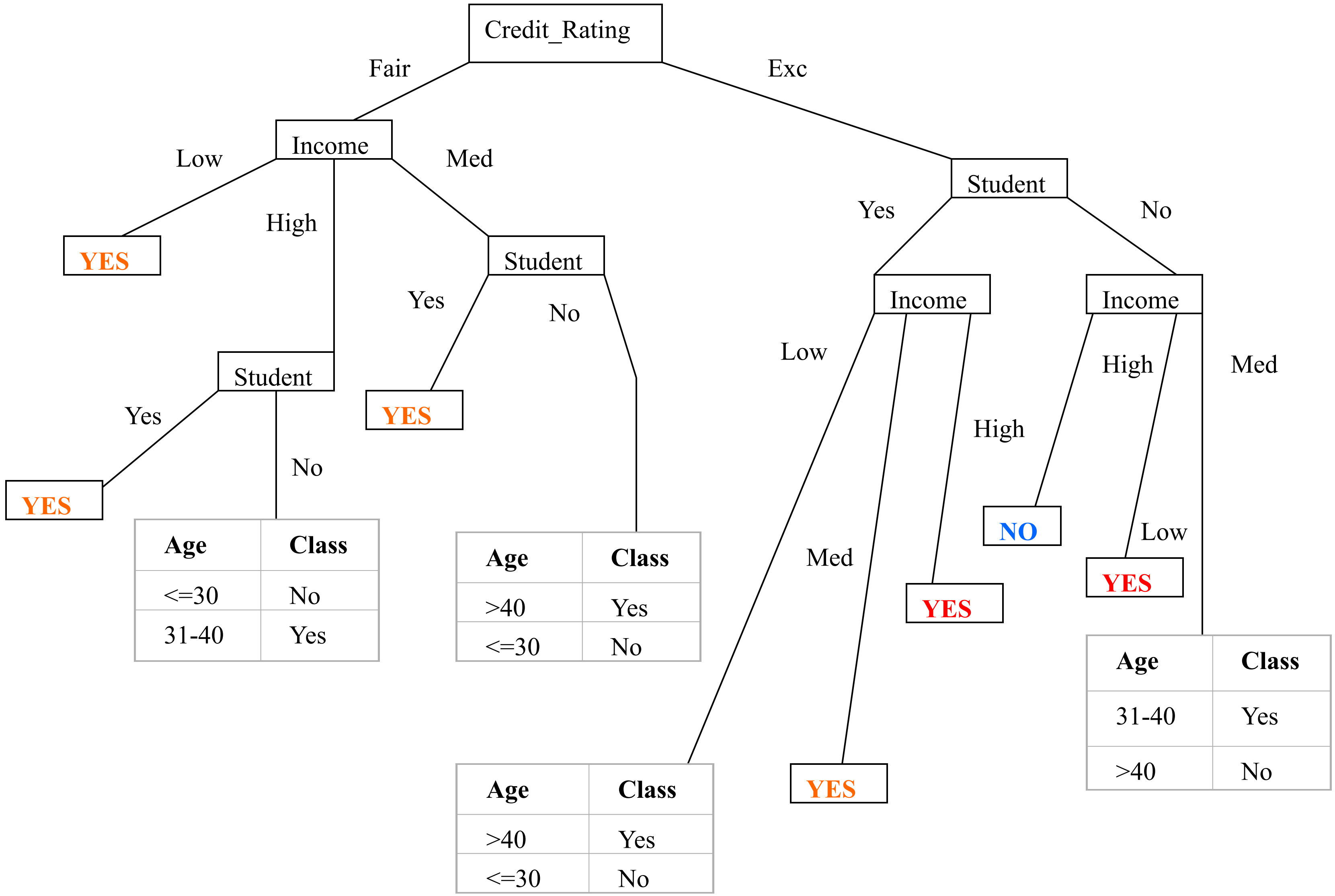
Age	Inco	Class
>40	Low	No
31-40	Low	Yes
<=30	Med	Yes

Student = No:

Age	Inco	Class
<=30	High	No
31-40	Med	Yes
>40	Med	No

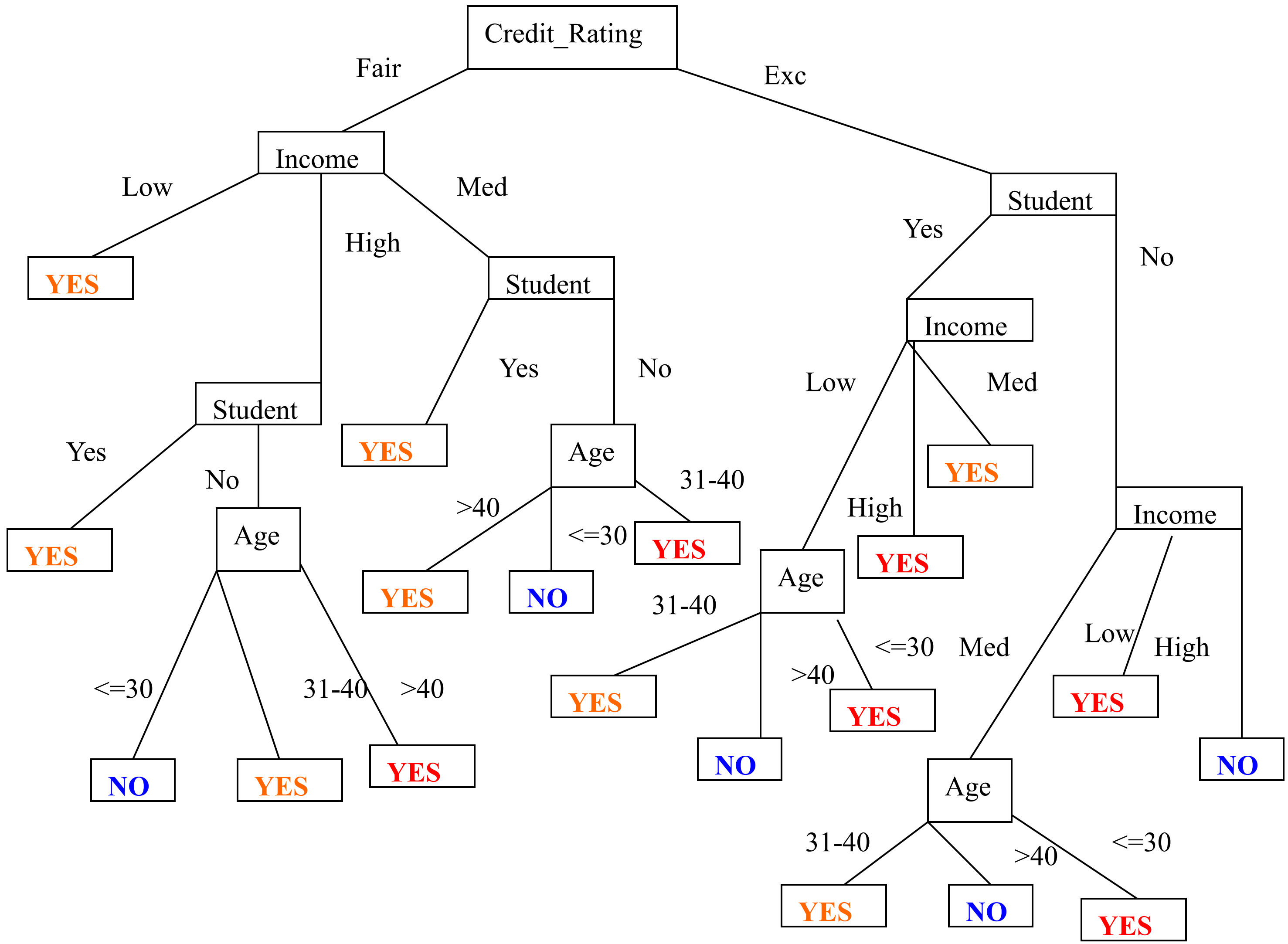
Tree 2 with root attribute Credit Rating


Credit_Rating
Fair
Exc
Income
Low
Med
High
YES
Student
Yes
No
Student
Yes
No
Age
Class
31-40
Yes
Age
Class
<=30
No
31-40
Yes
Age
Class
>40
Yes
Age
Class
>40
Yes
<=30
No
Student
Yes
No
Income
Low
Med
High
Income
High
Med
Low
Age
Class
<=30
No
Age
Class
31-40
Yes
>40
No
Age
Class
>40
Yes
<=30
No
Age
Class
31-40
Yes
No Records
Majority Voting
YES
No Records
Majority Voting
YES
CORRECTED



Credit_Rating
Fair
Exc
Income
Low
Med
High
YES
Student
Yes
No
YES
Student
Yes
No
YES
Age Class
<=30 No
31-40 Yes
Age Class
>40 Yes
<=30 No
Student
Yes
No
Income
Low
High
Med
YES
YES
Age Class
>40 Yes
<=30 No
Income
High
Low
Med
NO
YES
Age Class
31-40 Yes
>40 No

CORRECT



CORRECTED Tree 2 with root attribute Credit Rating

The Decision tree with root attribute *Credit_Rating* has produced 13 rules, two more than with root attribute *Income*

1. Credit(x, Fair) ^ Income(x,Low) -> buysComp(x,Yes).
2. Credit(x,Fair) ^ Income(x, High) ^ Student(x,Yes) -> buysComp(x, Yes).
3. Credit(x,Fair) ^ Income(x, High) ^ Student(x, No) ^ Age(<=30) -> buysComp(x, No).
4. Credit(x,Fair) ^ Income(x, High) ^ Student(x, No) ^ Age(31-40) -> buysComp(x, Yes).
5. Credit(x, Fair) ^ Income(x, Med) ^ Student(x, Yes) -> buysComp(x, Yes).
6. Credit(x, Fair) ^ Income(x, Med) ^ Student(x, No) ^ Age(>40) -> buysComp(x, Yes).
7. Credit(x, Fair) ^ Income(x, Med) ^ Student(x, No) ^ Age(<=30) -> buysComp(x, No).
8. Credit(x, Exc) ^ Student(x, Yes) ^ Income(x, Low) ^ Age(31-40) -> buysComp(x, Yes).
9. Credit(x, Exc) ^ Student(x, Yes) ^ Income(x, Low) ^ Age(>40) -> buysComp(x, No).
10. Credit(x, Exc) ^ Student(x, Yes) ^ Income(x, Med) -> buysComp(x, Yes).
11. Credit(x, Exc) ^ Student(x, No) ^ Income(x, Med) ^ Age(x, 31-40) -> buysComp(x, Yes).
12. Credit(x, Exc) ^ Student(x, No) ^ Income(x, Med) ^ Age(x, >40) -> buysComp(x, No).
13. Credit(x, Exc) ^ Student(x, No) ^ Income(x, High) -> buysComp(x, No).
14. Credit(x,Fair) ^ Income(x, High) ^ Student(x, No) ^ Age(>40) -> buysComp(x, Yes). Majority Voting
15. Credit(x, Fair) ^ Income(x, Med) ^ Student(x, No) ^ Age(31-40) -> buysComp(x, Yes). Majority Voting
16. Credit(x, Exc) ^ Student(x, Yes) ^ Income(x, Low) ^ Age(<=30) -> buysComp(x, Yes). Majority Voting
17. Credit(x, Exc) ^ Student(x,No) ^ Income(x, Med) ^ Age(x<=30) -> buysComp(x, Yes). Majority Voting
18. Credit(x, Exc) ^ Student(x, Yes) ^ Income(x, High) -> buysComp(x, Yes). Majority Voting
19. Credit(x, Exc) ^ Student(x, No) ^ Income(x, Low) -> buysComp(x, Yes). Majority Voting

Random Tuples to Check Predictive Accuracy based on three sets of rules

Obj	Age	Income	Student	Credit_R	Class
1	<=30	High	Yes	Fair	Yes
2	31-40	Low	No	Fair	Yes
3	31-40	High	Yes	Exc	No
4	>40	Low	Yes	Fair	Yes
5	>40	Low	Yes	Exc	No
6	<=30	Low	No	Fair	No

Predictive accuracy:

1. Against Lecture Notes: 4/6 = 66.66%
2. Against **Tree 1** rules with root att. *Income*: 3/6 = 50%
3. Against **Tree 2** rules with root att. *Credit*: 4/6 = 66.66%
4. Against OLD Tree 2 rules with root att. *Credit*: 5/6 = 83.33%

Calculation of Information gain at each level of tree with root attribute *Income*

1. Original Table:

Class P: *buys_computer* = yes; Class N: *buys_computer* = No

I(P,N) = -P/P+N log $_2$ (P/P+N) – N/P+N log$_2$N/P+N--------(equation 1)

I(P,N) = I(9,5) = (-9/9+5) log $_2$(9/9+5) – (5/9+5) log$_2$ (5/9+5)

= 0.940

2. Index:1

Income	Pi	Ni	I(Pi,Ni)
Low	3	1	0.8111
Med	4	2	0.9234
High	2	2	1

E(Income) = 4/14 I(3,1) + 6/14 I(4,2) + 4/14 I(2,2)------------(eq.2)

I(3,1) = 0.8111 (Using equation 1)

I(4,2) = 0.9234 (Using equation 1)

I(2,2) = 1

Contd…..

Information gain calculation for Index 1 contd:

Substituting the values in eq.2 we get,

E(Income) = 0.2317 + 0.3957 + 0.2857 = 0.9131

Gain (Income) = I(P,N) – E(Income)

= 0.940 – 0.9131 = 0.027

2. Index 2

Credit	Pi	Ni	I(Pi,Ni)
Fair	2	1	0.913
Exc	2	1	0.913

I(P,N) = I(4,2) = 0.9234 (Using equation 1)

E(Credit) = 3/6 I(2,1) + 3/6 I(2,1) -----(3)

I(2,1) = 0.913 (Using equation 1)

E(Credit) = 0.913 (Substituting value of I(2,1) in (3)

Gain(Credit) = I(P,N) – E(Credit) = 0.9234 – 0.913

= 0.01

Similarly we can calculate Information gain of tables at each stage.

Exercise

EXERCISE:

Construct a **correct tree** of your choice of attributes and **evaluate:**

1. correctness of your rules, i.e.

the predictive accuracy with respect to the TRAINING data

2. **predictive accuracy** with respect to test data from **Exercise 2**

- Remember
- **The TERMINATION CONDITIONS!**