Problem 1

Outline only.

Compute visit ratios first. For CPU, disk A and disk B they are 25, 20 and 4 respectively. The service demands are then 1, 0.6, 0.1 sec respectively.

Part (a)

$$U_A = X_A \cdot S_A = V_A \cdot X \cdot S_A \Rightarrow \underset{\substack{system \\ throughput}}{X} = \frac{U_A}{V_A \cdot S_A} = \frac{U_A}{D_A} = 1 \text{ job/sec}$$

$$U_B = V_B \cdot X \cdot S_B = 0.1$$

$$U_{CPU} = V_{CPU} \cdot X \cdot S_{CPU} = 1$$

Part (b)

$R = \frac{N}{X} - Z$. Observe that utilization of disk B is already 0.1. Use X = 1, N = 20, Z = 5 to get R = 15 sec.

Part (c)

CPU is the bottleneck device (with the highest service demand).

Part (d)

Sum up all service demands. You will get minimum response time as 1.7 sec.

Part (e)

The CPU is already in saturation. So, the max possible disk A utilization is 0.6.

Part (f)

For a similar reason, X is 1 job/sec.

Part (g)

N = 15, Z = 5, R = 10 gives, X = 1.667. If we consider that the CPU is at saturation (utilization 1), service demand for CPU = 1/X = 0.6. Since its visit ratio is 25, CPU service time must be at least 0.6/25 sec = 24 ms. Thus we will need a 40% faster CPU. We still need to check whether now something else becomes the bottleneck. But the service demand of the next possible candidate (disk A) is also 0.6. Thus, disks do not need to be faster.

Problem 12.6 in R.J.

$$F(x) = 1 - x^{-a}, 1 \le x \le \infty$$

$$pdf = f(x) = \frac{dF(x)}{dx} = a \cdot x^{-a-1}$$

$$mean = \int_{-\infty}^{\infty} x \cdot f(x) = a\int_{1}^{\infty} x \cdot x^{-a-1} dx = a\int_{1}^{\infty} x^{-a} dx = a\left[\frac{1}{-a+1} \cdot x^{-a+1}\right]_1^{\infty} = \begin{cases} a\left(0 - \frac{1}{-a+1}\right) = \frac{a}{a-1}, a > 1 \\ \text{infinite, otherwise} \end{cases}$$

Problem 12.7 in R.J.

$$N(x;\mu,\sigma) = N(x;5,1) \sim N(z;0,1), z = \frac{x-\mu}{\sigma}$$

12.7 a.

$$P(X > 8) = 1 - P(X \le 8) = 1 - P\left(Z \le \frac{8-5}{1}\right) = 1 - P(Z \le 3) = 1 - (0.5 + 4987) = 0.0013$$

12.7 b.

$$P(X \le 6) = P\left(Z \le \frac{6-5}{1}\right) = P(Z \le 1) = 0.8413$$

12.7 c.

$$P(4 \le X \le 7) = P(X \le 7) - P(X \le 4) = P\left(Z \le \frac{7-5}{1}\right) - P\left(Z \le \frac{4-5}{1}\right) =$$

$$= P(Z \le 2) - P(Z \le -1) = P(Z \le 2) - (1 - P(Z \le 1)) = 0.9772 - (1 - 0.8413) = 0.8185$$

12.7 d.

$$z_p = \frac{x-\mu}{\sigma}$$

$$z_p = 1.645 \text{ sec (from table, area } 0.45 + 0.5 = 0.95)$$

$$\Rightarrow 1.645 = \frac{\text{x - 5}}{1}$$

$$\Rightarrow x = 5 + 1.645 = 6.645 \text{ sec}$$