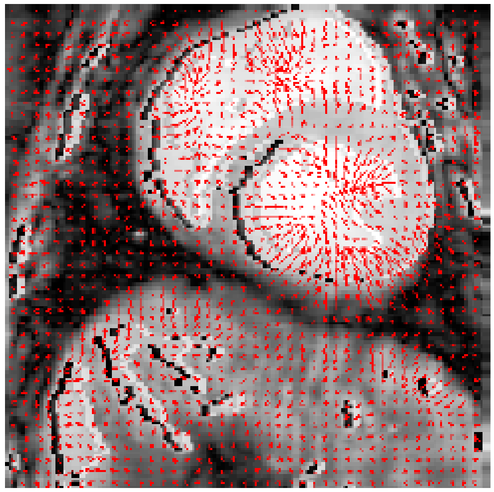
Optical Flow



.Optical Flow

There is a great deal of both spatial and temporal redundany in a sequence of images containing moving objects. Information contained in 512 $\times$ 512 pixel images sampled at 60-Hz rate may be transmitted via systems with far lens bandwidth than is required to transmit the raw data.

Optical flow research concerns the determination of the "motion" of the individual pixel locations by using intensity data in a sequence of images the resultant *optical flow field* is the field of 20 pixel "velocity" vectors. Optical flow field conveys valuable information concerning the characteristics (e.g., curvature and orientation) and depth of surfaces as well as the relative motion among scene objects and the sensor system.

There are two basic approaches:

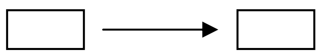
1. *Feature-based* – Here we select some features in the image frames and then match these features and calculate the disparities between frames. Problem I selecting features and establishing correspondence is not easy. Moreover, this method only produces velocity vectors at sparse points. Close to stereo disparity.

2. *Gradient based* – I will discuss this in detail.

Gradient – Based Methods for Optical Flow

Gradient-based methods exploit the relationship between the spatial and temporal gradients of the velocity. This relationship can be used to segment images based on the velocity of points.

Let $E(x, y, t) = \text{image intensity at point } (\mathrm{x}, \mathrm{y})$ at time t



intensity doesn't
change for small
motion

$$\frac{dE}{dt} = 0$$

Using chain rule: $$\frac{\partial E}{\partial x}\frac{dx}{dt} + \frac{\partial E}{\partial y}\frac{dy}{dt} + \frac{\partial E}{\partial t} = 0$$

Set $$u = \frac{dx}{dt}$$

$$v = \frac{dy}{dt}$$

Then relationship between spatial and temporal gradients is given by

$$\boxed{E_x u + E_y v + E_t = 0}$$

Optical Flow Constraint

Note E_x, E_y *and* E_t can be computed directly from the image.

At every point in an image, there are two unknowns, *u* and *v* and only one equation. Using information only at a point, optical flow cannot be determined. This is the *aperture problem*. Velocity components at a point cannot be determined using the information at only one point in the image without making further assumptions.

If one used a tube such that only one point is visible, motion of point cannot be determined. One can get sense of motion, not components of the motion vector.

Formally:

$$E_x \, u + E_y \, v + E_t = 0$$

$\Leftrightarrow$

$$(E_x, E_y)\cdot(u,v) = -E_t$$

$$\left(\frac{(E_x, E_y)}{\sqrt{E_x{}^2 + E_y{}^2}}\right)\cdot(u,v) = \frac{-E_t}{\sqrt{E_x{}^2 + E_y{}^2}}$$

Can only determine (u,v) in the direction $\left(\frac{(E_x, E_y)}{\sqrt{E_x{}^2 + E_y{}^2}}\right)$, i.e. where intensity spatially changes.

Variational Methods for Computation of Optical Flow

Problem is under-determined:

$$E_x u + E_y v + E_t = 0$$

Need more global information.

Typical trick: Demand that optical flow field is smooth. For this we need a variational formulation:

$$\min_{(u,v)} \iint \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial \upsilon}{\partial x}\right)^2 + \left(\frac{\partial \upsilon}{\partial y}\right)^2\right] dxdy$$

over all (u,v) which satisfy

$$E_x u + E_y v + E_t = 0$$

Notice this enforces smoothness: large derivatives (“non-smoothness”) make the integral large.

Calculus of Variations in Two Variables