Segmentation

- Divide or partition into homogeneous pieces: intensity, texture, etc. Dual problem is finding the edges.

- Many Techniques

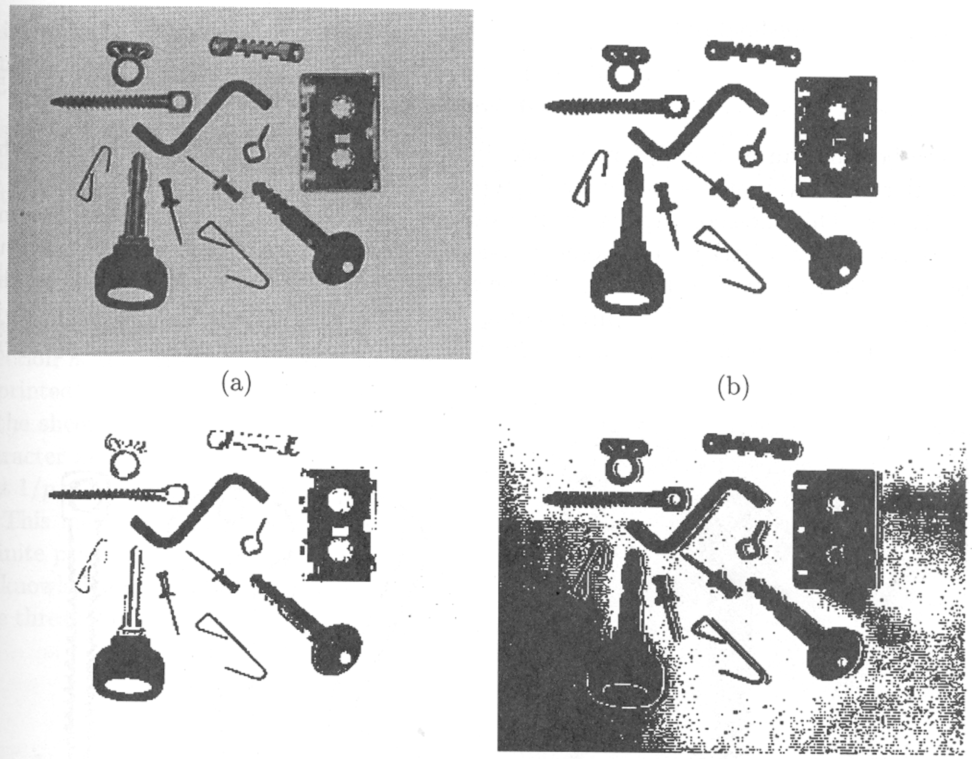
Segmentation

- Thresholding
- Edge-based techniques
- Knowledge-Based Bayesian techniques
- Partial Differential Equations (PDEs) based techniques

Motivation

- Image Segmentation and Image Registration are core components of medical imaging

  - 2005:
    - The word "Segmentation" appears 100 times at MICCAI'05 program ~ 50%
    - The word "Registration" appears 53 times at MICCAI'05 program ~ 25%
  - 2006:
    - The word "Segmentation" appears 120 times at MICCAI'06 program ~ 55%
    - The word "Registration" appears 67 times at MICCAI'04 program ~ 30%



(a) (b)

(c) (d)

Figure 5.1: *Image thresholding: (a) original image; (b) threshold segmentation; (c) threshold too low; (d) threshold too high.*

Edge Detection Via Laplace of Gaussian

We smooth to minimize noise effects and then we enhance.
This leads to the Laplacian-of-the-Gaussian (LOG) operator.

Basic Idea:
Smooth image through convolution with a Gaussian-shaped kernel to minimize noise.

Following Gaussian smoothing, Laplacian operator is applied.

One Dimensional Example

(a) Take 1D function of the form $f(x) = -2u_{-1}(x) + 1$

Unit step

(b) Convolve this with a 1D Gaussian smoothing function, $g(x)$, of the form

$$g(x) = \left(1/\sigma\sqrt{2\pi}\right) \ \exp\left(-x^2/2\sigma^2\right)$$

which yields the blurred 1D image $f_2(x)$,

$$f_2(x) = 1 - 2\int_{-\infty}^{x} \left(\sigma\sqrt{2\pi}\right)^{-1} \ \exp\left(-t^2/2\sigma^2\right) dt$$

(c) Take the second derivative of $f_2(x)$:

$$f_2''(x) = \left(2x/\sigma^3\sqrt{2\pi}\right) \ \exp\left(-x^2/2\sigma^2\right)$$

This has zero crossing at $x = 0$.

$f_2''(x)$ is the step response of LOG operator.

$$\xrightarrow{\text{step}} \boxed{\textbf{LOG}} \xrightarrow{f_2''(x)}$$

One Dimensional-Continued

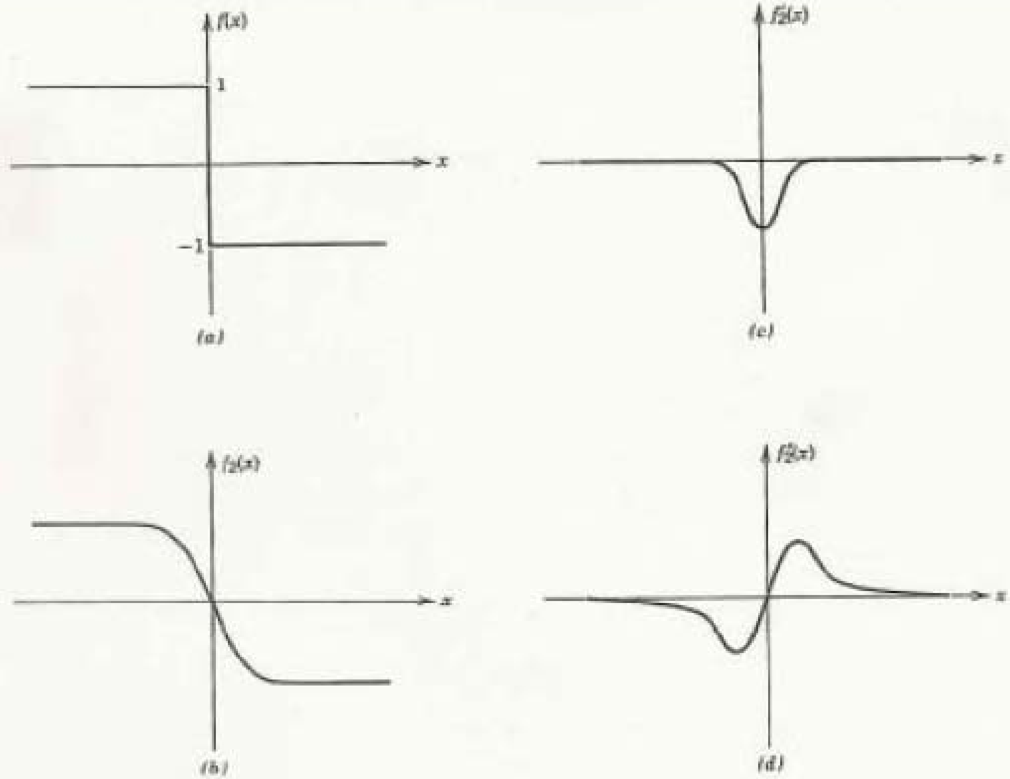


FIGURE 4.24 LOG processing sequence (1–D)
(a) Ideal step (eq. 4–56)
(b) Smoothed 1-D step
(c) First derivative
(d) Second derivative (note: this represents the overall step response of the LOG operator)

LOG Operator

2D Gaussian smoothing operator in (x, y) coordinates is

$$G_\sigma(x,y) = \frac{1}{\sqrt{2\pi}\sigma^2}\exp\left(\frac{-\left(x^2+y^2\right)}{2\sigma^2}\right)$$

$G_\vartheta(x,y)$ is circularly symmetric and the smoothing effect may be controlled through σ.
$\sigma \to \infty$, more smoothing.

Set $f_\sigma(x,y,\vartheta) = f(x,y) * G_\sigma(x,y)$.

Since operator is circulary symmetric many times use a polar representation $G_\sigma(r)$ where $r = \left(x^2+y^2\right)^{1/2}$

Note spatial extent of operator varies with σ. Now we mimic one-variable case:

$$f_{edge}(x,y) = \Delta\left(G_\sigma(x,y) * f(x,y)\right)$$

edge-
enhanced
image

$$= \underbrace{\left(\Delta G_\sigma(x,y)\right)}_{LOG\ operator} * f(x,y)$$

$$\boxed{\Delta G_\sigma(r) = \frac{1}{\pi\sigma^4}\left(\frac{r^2}{2\sigma^2} - 1\right)\exp\left(\frac{-r^2}{2\sigma^2}\right)}$$

To find edges, set

$$f_{edge}(x,y) = 0\,.$$

$\Delta G_\sigma(r)$ looks like an inverted Mexican sombrero. Called the ***Sombrero function***.