Edge Detection Via Smoothing and Laplacian

We smooth to minimize noise effects and then we enhance. This leads to the Lapacian-of-the-Gaussian (LOG) operator.

Basic Idea:
(1) Smooth image through convolution with a Gaussian-shaped kernel to minimize noise.
(2) Following Gaussian smoothing, Laplacian operator is applied.

(a) Take 1D function of the form $f(x) = -2u_{-1}(x) + 1$

Unit step

(b) Convolve this with a 1D Gaussian smoothing function, $g(x)$, of the form

$$g(x) = \left(1/\sigma\sqrt{2\pi}\right) \quad \exp\left(-x^2/2\sigma^2\right)$$

which yields the blurred 1D image $f_2(x)$,

$$f_2(x) = 1 - 2\int_{-\infty}^{x} \left(\sigma\sqrt{2\pi}\right)^{-1} \quad \exp\left(-t^2/2\sigma^2\right) dt$$

(c) Take the second derivative of $f_2(x)$:

$$f_2''(x) = \left(2x/\sigma^3\sqrt{2\pi}\right) \quad \exp\left(-x^2/2\sigma^2\right)$$

This has zero crossing at $x = 0$.

$f_2''(x)$ is the step response of LOG operator.

$$\xrightarrow{\text{step}} \boxed{\textbf{LOG}} \xrightarrow{f_2''(x)}$$

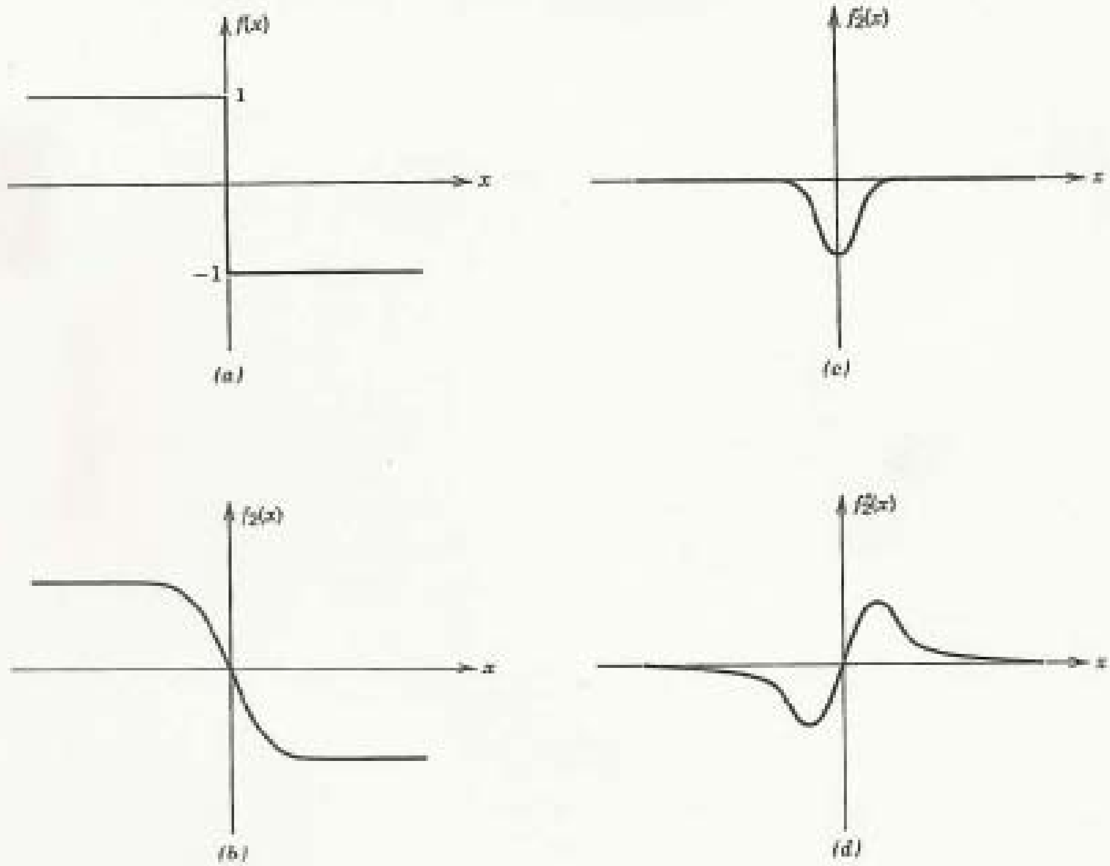

FIGURE 4.24 LOG processing sequence (1-D)
(a) Ideal step (eq. 4-56)
(b) Smoothed 1-D step
(c) First derivative
(d) Second derivative (note: this represents the overall step response of the LOG operator)

2D Gaussian smoothing operator in (x, y) coordinates is

$$G_\sigma(x, y) = \frac{1}{\sqrt{2\pi}\sigma^2}\exp\left(\frac{-\left(x^2+y^2\right)}{2\sigma^2}\right)$$

$G_\vartheta(x, y)$ is circularly symmetric and the smoothing effect may be controlled through σ. $\sigma \to \infty$, more smoothing.

Set $f_\sigma(x, y, \vartheta) = f(x, y) * G_\sigma(x, y)$.

Since operator is circulary symmetric many times use a polar representation $G_\sigma(r)$ where $r = \left(x^2 + y^2\right)^{1/2}$

Note spatial extent of operator varies with σ. Now we mimic one-variable case:

$$f_{edge}(x, y) = \Delta\left(G_\sigma(x, y) * f(x, y)\right)$$

edge-
enhanced
image

$$= \underbrace{\left(\Delta G_\sigma(x, y)\right)}_{LOG\ operator} * f(x, y)$$

$$\boxed{\Delta G_\sigma(r) = \frac{1}{\pi\sigma^4}\left(\frac{r^2}{2\sigma^2} - 1\right)\exp\left(\frac{-r^2}{2\sigma^2}\right)}$$

To find edges, set

$$f_{edge}(x, y) = 0\,.$$

$\Delta G_\sigma(r)$ looks like an inverted Mexican sombrero. Called the ***Sombrero function***.