

cse547/ams547 PRACTICE MIDTERM Spring 2010  
(10 extra points)

NAME

ID:

ams/cs

**THE TEST IS WORTH 10 EXTRA PTS.** You get 1pt for each of attempted problems. We will correct ONE problem of OUR choice (same for all students) - for up to 5 points

**QUESTION 1** Prove

$$S_n + a_{n+1} = a_0 + \sum_{k=0}^n a_{k+1},$$

for  $S_n = \sum_{k=0}^n a_k$

**QUESTION 2** Evaluate the following sum by using 3 methods.

$$S_n = \sum_{k=0}^n k3^k$$

1. Perturbation method

**2.** Multiple sum method (Method 5)

HINT:  $k = \sum_{j=1}^k 1$ .

**3.** Summation by Parts method. Write all details.

**QUESTION 3** We know that  $x^{m+n} = x^m(x - m)^n$ .

Prove that the following property hold for all integer  $m$  and  $n$ ,  $x \in R$ , unless one of the denominators is zero.

$$\frac{x^m}{(x - n)^m} = \frac{x^n}{(x - m)^n}$$

**QUESTION 4** Compute  $\Delta(c^x)$ , for  $x, c \in R$  and use it to evaluate the sum

$$\sum_{k=1}^n \frac{(-2)^k}{k}.$$

Explain all your steps.

**QUESTION 5** Use the repertoire method to evaluate a closed formula for

$$S_n = \sum_{k=0}^n (-1)^k k^2.$$

HINT: Generalize it and use functions  $R_n = 1$ ,  $R_n = (-1)^n$ ,  $R_n = n(-1)^n$ , and  $R_n = n^2(-1)^n$ . Write CAREFULLY all steps of computations.

Extra space