cse547/ams547 PRACTICE Final Spring 2010
(15 extra points. We will correct ONE problem)

NAME **ID:** **ams/cs**

Test has similar FORMAT to your real FINAL. It has (and FINAL will have) two parts.
PART ONE covers problems from homeworks 1-3 AND Lecture notes (Concrete Mathematics). This is Part 1 and 2 as described in the syllabus.
PART TWO covers problems from Homework 4 (Discrete Mathematics). This is Part 3 as described in the syllabus.
ATTENTION: The REAL FINAL will contain more problems, as you will have more TIME then one class period.
ATTENTION: REAL FINAL is worth 200pts. Points distribution over parts is: PART ONE -150 points, PART TWO -50 points.

1 PART ONE

QUESTION 1 Evaluate the following sum by using Multiple sum method (Method 5).

$$S_n = \sum_{k=0}^{n} k8^k$$

QUESTION 2 Use summation by parts to evaluate

$$\sum_{0 \le k < n} \frac{H_k}{(k+1)(k+2)}.$$

QUESTION 3 Show that The Harmonic series

$$H = \Sigma_{n=1}^{\infty} \frac{1}{n}$$

does not react on **D'Alambert's Criterium**.

QUESTION 4 Assuming that 'n' is a non-negative integer, $lgk = log_2 k$, find a closed form for the sum:

$$\sum_{1<k<2^{2^n}} \left[\frac{1}{2^{\lfloor \lg k \rfloor} 4^{\lfloor \lg \lg k \rfloor}} \right]$$

QUESTION 5 Prove that

$$\binom{r}{m}\binom{m}{k} = \binom{r}{k}\binom{r-k}{m-k}$$

holds for all $m, k \in Z$ and $r \in R$.

2 PART TWO

1. DEFINITIONS

The definitions listed below are correct, or have small mistakes.
Circle YES if the definition listed is correct.
Circle NOT and CORRECT it, if the definition is not correct.

Inverse function Let $f: A \longrightarrow B$ and $g: B \longrightarrow A$.
g is called an INVERSE function to f iff $\forall a \in A (f \circ g)(a) = g(f(a) = a)$.

y **n**

Equivalence relation $R \subseteq A \times A$ is an equivalence relation in A iff it is reflexive, antisymmetric and transitive.

y **n**

Partition A family of sets $\mathbf{P} \subseteq \mathcal{P}(A)$ is called a partition of the set A iff the following conditions hold.

1. $\forall\ X \in \mathbf{P}\ (X\ = \emptyset)$
2. $\forall\ X, Y \in \mathbf{P}\ (X \cup Y = \emptyset)$
3. $\bigcup \mathbf{P} = A$

y **n**

Greatest (largest) $a_0 \in A$ is a greatest (largest) element in the poset $(A, \preceq)$ iff $\forall a \in A\ (a \preceq a_0)$.

y **n**

Maximal $a_0 \in A$ is a maximal element in the poset $(A, \preceq)$ iff $\neg\ \forall a \in A\ (a_0 \preceq a\ \cap\ a_0 \neq a)$.

y **n**

Upper Bound Let $B \subseteq A$ and $(A, \preceq)$ is a poset. $a_0 \in A$ is an upper bound of a set B iff $\forall b \in B\ (b \preceq a_0)$.

y **n**

Least upper bound of B (lub B) Given: a set $B \subseteq A$ and $(A, \preceq)$ a poset.

y **n**

An element $x_0 \in B$ is a least upper bound of B, $x_0 = lubB$ iff x_0 is (if exists) the least (smallest) element in the set of all upper bounds of B, ordered by the poset order $\preceq$.

y **n**

Lattice A poset $(A, \preceq)$ is a lattice iff For all $a, b \in A$ $lub\{a,b\}$ or $glb\{a,b\}$ exist.

y **n**

Lattice orderings Let the $(A, \cup, \cap)$ be a lattice. The relations:

$a \preceq b$ iff $a \cup b = b$, $a \preceq b$ iff $a \cap b = a$
are order relations in A and are called a lattice orderings.

y **n**

Distributive lattice A lattice $(A, \cup, \cap)$ is called a distributive lattice iff for all $a, b, c \in A$ the following conditions hold

l4 $a \cup (b \cap c) = (a \cup b) \cap (a \cup c)$

l5 $a \cap (b \cup c) = (a \cap b) \cup (a \cap c)$.

y **n**

Complement axioms Let $(A, \cup, \cap, 1, 0)$ be a lattice with unit and zero. The complement of $a \in A$ is usually denoted by $-a$ and the above conditions that define the complement above are called complement axioms. The complement axioms are usually written as follows.

c1 $a \cup -a = 0$

c2 $a \cap -a = 1$.

y **n**

Boolean Algebra A distributive lattice with zero and unit such that each element has a complement is called a Boolean Algebra.

y **n**

Countable A set A is countable iff $|A| = \aleph_0$.

y **n**

Uncountable A set A is uncountable iff A is NOT countable.

y **n**

Union 3 $\aleph_0 + \mathcal{C} = \mathcal{C}$.
Union of an infinitely countable set and an uncountable set is an uncountable set.

y **n**

Cartesian Product 1 $\aleph_0 \cdot \aleph_0 = \aleph_0$.
Cartesian Product of two countable sets is a countable set.

y **n**

2. QUESTIONS

Circle proper answer. WRITE a short JUSTIFICATION. NO JUSTIFICATION, NO CREDIT.

1. There is an equivalence relation on Z with infinitely countably many equivalence classes.

JUSTIFY: **y** **n**

2. A is uncountable iff $|A| = |R|$ where R is the set of real numbers.

JUSTIFY: **y** **n**

3. A is infinite iff some subsets of A are infinite.

JUSTIFY: **y** **n**

4. A is finite iff some subsets of A are finite.

JUSTIFY:: **y** **n**

5. $\mathcal{P}(A) = \{B : B \subset A\}$

JUSTIFY:

y **n**

6. $|Q \cup N| = \aleph_0$

JUSTIFY:

y **n**

7. $|R \times Q| = \mathcal{C}$

JUSTIFY: **y** **n**

8. $|N| \leq \aleph_0$

JUSTIFY:

y **n**

9. Any non empty POSET has at least one MAX element.

JUSTIFY: **y** **n**

10. If $(A, \preceq)$ is a finite poset (i.e. A is a finite set), then a unique maximal is the largest element and a unique minimal is the least element.

JUSTIFY: **y** **n**

11. There is a poset $(A, \preceq)$ and a set $B \subseteq A$ and that B has none infinite number of lower bounds.

JUSTIFY: **y** **n**

12. If $(A, \cup, \cap)$ is a finite lattice (i.e. A is a finite set), then 1 and 0 always exist.

JUSTIFY: **y** **n**

13. Any finite lattice is distributive.

JUSTIFY: **y** **n**

14. If $(A, \cup, \cap)$ is a finite lattice (i.e. A is a finite set), then 1 and 0 always exist.

JUSTIFY: **y** **n**

15. Any finite lattice is distributive.

JUSTIFY: **y** **n**

16. Every Boolean algebra is a lattice.

JUSTIFY: **y** **n**

17. Any infinite Boolean algebra has unit (greatest) and zero (smallest) elements.

JUSTIFY: **y** **n**

3 Properties

$$\lfloor x \rfloor = x \iff x \in Z, \qquad \lceil x \rceil = x \iff x \in Z$$

$$x - 1 < \lfloor x \rfloor \le x \le \lceil x \rceil < x + 1$$

$$\lfloor -x \rfloor = -\lceil x \rceil, \qquad \lceil -x \rceil = -\lfloor x \rfloor$$

$$\lceil x \rceil - \lfloor x \rfloor = 0 \ if \ x \in Z, \qquad \lceil x \rceil - \lfloor x \rfloor = 1 \ if \ x \notin Z$$

$$\lfloor x \rfloor = n \iff n \le x < n + 1$$

$$\lfloor x \rfloor = n \iff x - 1 < n \le x$$

$$\lceil x \rceil = n \iff n - 1 < x \le n$$

$$\lceil x \rceil = n \iff x \le n < x + 1$$

$$\lfloor x + n \rfloor = \lfloor x \rfloor + n$$

$$x < n \iff \lfloor x \rfloor < n$$

$$n < x \iff n < \lceil x \rceil$$

$$x \le n \iff \lceil x \rceil \le n$$

$$n \le x \iff n \le \lfloor x \rfloor$$

$$[\alpha \ldots \beta] \ contains \ \lfloor \beta \rfloor - \lceil \alpha \rceil + 1 \ integers, \ for \ \alpha \le \beta$$

$$[\alpha \ldots \beta) \ contains \ \lceil \beta \rceil - \lceil \alpha \rceil \ integers, \ for \ \alpha \le \beta$$

$$(\alpha \ldots \beta] \ contains \ \lfloor \beta \rfloor - \lfloor \alpha \rfloor \ integers, \ for \ \alpha \le \beta$$

$$(\alpha \ldots \beta) \ contains \ \lceil \beta \rceil - \lfloor \alpha \rfloor - 1 \ intergers, \ for \ \alpha < \beta$$

$$x = y \left\lfloor \frac{x}{y} \right\rfloor + x \bmod y$$

Chapter 5

1. $\sum_k \binom{r}{m+k}\binom{s}{n-k} = \binom{r+s}{m+n}$, s.t. int m, n.
2. $\sum_k \binom{l}{m+k}\binom{s}{n+k} = \binom{l+s}{l-m+n}$, s.t. $l \geq 0$, int m, n.
3. $\sum_k \binom{l}{m+k}\binom{s+k}{n}(-1)^k = (-1)^{l+m}\binom{s-m}{n-l}$, s.t. $l \geq 0$, int m, n.
4. $\sum_{k \leq 1} \binom{l-k}{m}\binom{s}{k-n}(-1)^k = (-1)^{l+m}\binom{s-m-l}{l-m-n}$, int l, m, n ≥ 0.
5. $\sum_{0 \leq k \leq 1} \binom{l-k}{m}\binom{q+k}{n} = \binom{l+q+1}{m+n+1}$, int l, m ≥ 0, int $n \geq q \geq 0$.
6. $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ int $n \geq k \geq 0$. factorial expansion.
7. $\binom{n}{k} = \binom{n}{n-k}$ int $n \geq 0$, k. symmetry.
8. $\binom{r}{k} = \frac{r}{k}\binom{r-1}{k-1}$ int $k \neq 0$. absorption/extraction.
9. $\binom{r}{k} = \binom{r-1}{k} + \binom{r-1}{k-1}$ int k. addition/induction.
10. $\binom{r}{k} = (-1)^k\binom{k-r-1}{k}$ int k. upper negation.
11. $\binom{r}{m}\binom{m}{k} = \binom{r}{k}\binom{r-k}{m-k}$ int m, k, real r.
12. $\sum_k \binom{r}{k} x^k y^{r-k} = (x+y)^r$ int $r \geq 0$, or $|x/y| < 1$. binomial theorem.
13. $\sum_{k \leq n} \binom{r+k}{k} = \binom{r+n+1}{n}$ int n. parallel summation.
14. $\sum_{0 \leq k \leq n} \binom{k}{m} = \binom{n+1}{m+1}$ int m, $n \geq 0$. upper summation.
15. $\sum_k \binom{r}{k}\binom{s}{n-k} = \binom{r+s}{n}$ int n. Vandermonde convolution.