cse547/ams547 Midterm 1 Spring 2010

100pts + 10 extra credit

NAME **ID:** **ams/cs**

There are 5 Problems. Each problem is worth 20pts. There is one extra credit problem (10pts). If needed, use extra pages attached.

PROBLEM 1 Use a summation factor to solve the recurrence

$$T_0 = 5;$$

$$2T_n = nT_{n-1} + 3n!,\ n > 0.$$

Write carefully all steps of solution.

PROBLEM 2 Use the Perturbation Method to evaluate a closed formula for:

Part 1 $S_n = \sum_{k=0}^{n} (-1)^{n-k}$.

Part 2 $S_n = \sum_{k=0}^{n} (-1)^{n-k} k$.

PROBLEM 3 Show that

1. $\sum_{k=0}^{n} k^2 = \sum_{1 \le j \le k \le n} k$

2. $\sum_{k=0}^{n} k^2 + \sum_{k=0}^{n} k^3 = 2 \sum_{1 \le j \le k \le n} jk.$

PROBLEM 4 Use summation by parts to prove that

$$\sum_{k=0}^{n-1} \frac{H_k}{(k+1)(k+2)} = \frac{n - H_n}{n+1}.$$

PROBLEM 5 Use the formula

$$\Delta(c^{\underline{x}}) = \frac{c^{\underline{x+2}}}{(c-x)}$$

to prove that

$$\sum_{k=1}^{n} \frac{(-2)^{\underline{k}}}{k} = (-1) + (-1)^n n!$$

EXTRA CREDIT Show that the infinite sum $\Sigma_{k=0}^{\infty} \frac{1}{(k+1)(k+2)}$ CONVERGES to 1; i.e.

$$\Sigma_{k=1}^{\infty} \frac{1}{(k+1)(k+2)} = 1.$$

1 Useful Formulas

$$T_n = \frac{1}{s_n a_n}\left(s_1 b_1 T_0 + \sum_{k=1}^{n} s_k c_k\right), \qquad s_n = \frac{s_{n-1}a_{n-1}}{b_n} = \frac{a_{n-1}a_{n-2}\dots a_1}{b_n b_{n-1}\dots b_2}$$

$$x^{\underline{m}} = \overbrace{x(x-1)\dots(x-m+1)}^{\text{m factors}}, \qquad \textit{integer } m \geq 0$$

$$x^{\overline{m}} = \overbrace{x(x+1)\dots(x+m-1)}^{\text{m factors}}, \qquad \textit{integer } m \geq 0$$

$$x^{\underline{-m}} = \frac{1}{(x+1)(x+2)\dots(x+m)}, \qquad for\ m > 0$$

$$x^{\underline{m+n}} = x^{\underline{m}}(x-m)^{\underline{n}}, \qquad integers\ m\ and\ n$$

$$\Box_n = \sum_{0\leq k\leq n} k^2 = \frac{n(n+1)(2n+1)}{6}, \qquad for\ n \geq 0$$

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} = \sum_{k=1}^{n}\frac{1}{k}, \qquad for\ n \geq 1$$

$$\mathrm{D}f(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}, \qquad \Delta f(x) = f(x+1) - f(x)$$

$$\mathrm{D}(x^m) = mx^{m-1}, \qquad \Delta(x^{\underline{m}}) = mx^{\underline{m-1}}$$

DEFINITION 1
If the limit of the sequence $\{S_n = \Sigma_{k=1}^n\ a_k\}$ exists we call it an INFINITE SUM write it as

$$\Sigma_{n=1}^{\infty}\ a_n = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\Sigma_{k=1}^{n}\ a_k.$$

The sequence $\{S_n\}$ is called its sequence of partial sums.

DEFINITION 2
If the limit $\lim_{n\to\infty} S_n$ exists and is finite, i.e.

$$\lim_{n\to\infty} S_n = S,$$

then we say that the infinite sum $\Sigma_{n=1}^{\infty}\ a_n$ CONVERGES to S otherwise the infinite sum DIVERGES.

In a case that $\lim_{n\to\infty} S_n$ exists and is infinite, then we say that the infinite sum $\Sigma_{n=1}^{\infty}\ a_n$ DIVERGES to ∞ and we write

$$\Sigma_{n=1}^{\infty}\ a_n = \infty.$$

In a case that $\lim_{n\to\infty} S_n$ does not exist we say that the infinite sum $\Sigma_{n=1}^{\infty}\ a_n$ DIVERGES.

Extra space

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