There are 5 Problems. Each problem is worth 20pts. There is one extra credit problem (10pts). If needed, use extra pages attached.

PROBLEM 1 Use a summation factor to solve the recurrence

\[ T_0 = 5; \]
\[ 2T_n = nT_{n-1} + 3n!, \quad n > 0. \]

Write carefully all steps of solution.
PROBLEM 2 Use the Perturbation Method to evaluate a closed formula for:

Part 1 \[ S_n = \sum_{k=0}^{n} (-1)^{n-k}. \]

Part 2 \[ S_n = \sum_{k=0}^{n} (-1)^{n-k}k. \]
PROBLEM 3  Show that

1. $\sum_{k=0}^{n} k^2 = \sum_{1 \leq j \leq k \leq n} k$

2. $\sum_{k=0}^{n} k^2 + \sum_{k=0}^{n} k^3 = 2\sum_{1 \leq j \leq k \leq n} jk$. 
PROBLEM 4 Use summation by parts to prove that

\[ \sum_{k=0}^{n-1} \frac{H_k}{(k+1)(k+2)} = \frac{n - H_n}{n+1}. \]
PROBLEM 5  Use the formula

$$\Delta (c^x) = \frac{c^{x+2}}{(c-x)}$$

to prove that

$$\sum_{k=1}^{n} \frac{(-2)^k}{k} = (-1) + (-1)^n n!$$
EXTRA CREDIT  Show that the infinite sum $\sum_{k=0}^{\infty} \frac{1}{(k+1)(k+2)}$ CONVERGES to 1; i.e.

$$\sum_{k=1}^{\infty} \frac{1}{(k+1)(k+2)} = 1.$$
1 Useful Formulas

\[ T_n = \frac{1}{sn} \left( s_1 b_1 T_0 + \sum_{k=1}^{n} s_k c_k \right), \quad s_n = \frac{s_{n-1}a_{n-1}}{b_n} = \frac{a_{n-1}a_{n-2} \ldots a_1}{b_n b_{n-1} \ldots b_2} \]

\[ x^m = x(x-1) \ldots (x-m+1), \quad \text{integer } m \geq 0 \]

\[ x^m = x(x+1) \ldots (x+m-1), \quad \text{integer } m \geq 0 \]

\[ x^{-m} = \frac{1}{(x+1)(x+2) \ldots (x+m)}, \quad \text{for } m > 0 \]

\[ x^{m+n} = x^m x^n, \quad \text{integers } m \text{ and } n \]

\[ \Box_n = \sum_{0 \leq k \leq n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \text{for } n \geq 0 \]

\[ H_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} = \sum_{k=1}^{n} \frac{1}{k}, \quad \text{for } n \geq 1 \]

\[ Df(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, \quad \Delta f(x) = f(x + 1) - f(x) \]

\[ D(x^m) = mx^{m-1}, \quad \Delta(x^m) = mx^{m-1} \]

DEFINITION 1
If the limit of the sequence \( \{S_n = \sum_{k=1}^{n} a_k\} \) exists we call it an INFINITE SUM write it as

\[ \sum_{n=1}^{\infty} a_n = \lim_{n \to \infty} S_n = \lim_{n \to \infty} \sum_{k=1}^{n} a_k. \]

The sequence \( \{S_n\} \) is called its sequence of partial sums.

DEFINITION 2
If the limit \( \lim_{n \to \infty} S_n \) exists and is finite, i.e.

\[ \lim_{n \to \infty} S_n = S, \]

then we say that the infinite sum \( \sum_{n=1}^{\infty} a_n \) CONVERGES to S otherwise the infinite sum DIVERGES.

In a case that \( \lim_{n \to \infty} S_n \) exists and is infinite, then we say that the infinite sum \( \sum_{n=1}^{\infty} a_n \) DIVERGES to \( \infty \) and we write

\[ \sum_{n=1}^{\infty} a_n = \infty. \]

In a case that \( \lim_{n \to \infty} S_n \) does not exist we say that the infinite sum \( \sum_{n=1}^{\infty} a_n \) DIVERGES.
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