EVALUATE: (Book page 39)

\[ S_m = \sum_{1 \leq \delta < k \leq m} \frac{1}{k-\delta} \]

\[ S_m = \sum_{P(j,k)} a_{k,j} \]

\[ a_{k,j} = \frac{1}{k-j} \]

\[ S_2 = \sum_{1 \leq j < k \leq 2} a_{k,j} \]

\[ S_2 = a_{2,1} = \frac{1}{2-1} = 1 \]

\[ S_2 = 1 \]

\[ S_i = \sum_{1 \leq j < k \leq 2} a_{k,j} \]

\[ S_i \text{ is undefined} \]

Book defines \[ S_i = 0 \]

\[ S_i = 0 \text{ only when} \]
\[ S_n = \sum_{1 \leq j < k \leq n} a_{k,j} \]

\[ S_3 = \sum_{1 \leq j < k \leq 3} a_{k,j} \]

\[ S_3 = a_{3,2} + a_{3,1} + a_{2,1} \]

\[ = \frac{1}{3-2} + \frac{1}{3-1} + \frac{1}{2-1} = \frac{1}{1} + \frac{1}{2} + 1 = \frac{3}{2} \]

\[ S_3 = \frac{3}{2} \]

\[ P(k, j) = \]

\[ \sum_{1 \leq j < k \leq 3} \frac{1}{k-j} = 3 \]

Now we want to express \( P(k, j) \)

\[ P(k, j) = P_1(k) \cap P_2(j) \]

\[ \sum a_{k,j} \quad \text{def.} \quad \sum_{P(k,j)} a_{k,j} = \sum_{P_1(k)} \sum_{P_2(j)} a_{k,j} \]
Obviously:

\[ 1 \leq j < k \leq n \implies 1 < k \leq m \land 1 \leq j < k \]

For \( n = 1 \):

\[ X = F \]

For \( n > 1 \):

\[ T = T \]

We get from (x):

\[
S_n = \sum_{k=1}^{n} \sum_{1 \leq j < k} \frac{1}{k-j}
\]

\[
= \sum_{1 \leq k \leq n} \sum_{1 \leq k-j < k} \frac{1}{k-j}
\]

\[
= \sum_{1 \leq k \leq n} \frac{1}{k-j}
\]

\[
= \sum_{1 \leq k \leq n} H_{k-1}
\]

\[
= \sum_{k=1}^{n} H_k
\]

\[
S_m = \sum_{k=1}^{m} H_k
\]

\[
H_m = \sum_{1 \leq k \leq m} \frac{1}{k-j}
\]

Put \( j = k - \delta \)

Evaluate boundaries:

\[ 0 < \delta \leq k - 1 \]

\[ 1 \leq k - j < k \]

\[ 1 - k \leq -\delta < 0 \]

\[ k - 1 > \delta > 0 \]

Put \( k' = k + 1 \)

Boundaries

\[ 1 < k' \leq n \]

\[ 0 < k' \leq n - 1 \]

Use

\[ H_m = \sum_{1 \leq k \leq m} \frac{1}{k-j} \]

Not closed
Check our result:

\[ S_m = \sum_{k=1}^{n-1} \frac{H}{k} \]

\[ \sum_{1 \leq j \leq k \leq n} = 1 \]

\[ S_1 = 0 \]

UNDEFINED

\[ S_3 = H_1 + H_2 \]

\[ S_3 = 1 + \frac{\sqrt{2}}{2} \]

we define

\[ S_m = \sum_{k=0}^{m} H_k \]

\[ S_2 = 1 \]

\[ S_3 = 5/2 \]

\[ S_1 = 1 \]

\[ S_3 = 5/2 \]

\[ H_m = \sum_{j=1}^{m} \frac{1}{n} \]

\[ H_1 = 1 \]

\[ H_2 = \sum_{j=1}^{2} \frac{1}{n} = \frac{1}{2} + \frac{1}{2} \]

\[ H_2 = 1 + \frac{1}{2} \]
\[ S_m = \sum_{1 \leq j < k \leq n} \frac{1}{k-j} \]

Express boundaries differently:

\[ P(k, j) = R_1(k) \cap R_2(j) \]

\[ S_m = \sum_{P(k, j)} a_{k, j} = \sum_{R_2(j)} \sum_{R_1(k)} a_{k, j} \]

We write:

\[ 1 \leq j \leq k \leq n \equiv (1 \leq j < n) \cap (j \leq k \leq n) \]

\[ S_m = \sum_{1 \leq j < k \leq n} \frac{1}{k-j} = \sum_{1 \leq j < n} \sum_{j+1 \leq k \leq n} \frac{1}{k-j} \]

\[ = \sum_{1 \leq j < n} \sum_{1 \leq k \leq n-j} \frac{1}{k} \]

\[ = \sum_{1 \leq j < n} H_{n-j} \]

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\[ S_m = \sum_{j=0}^{n-1} H_{n-j} \]

\[ S_n = \sum_{j=1}^{n-1} H_j \]

Bounds on \( j \):
- \( 1 \leq j < n \)
- \( 1 \leq n-j < n \)
- \( 1-n \leq -j < 0 \)
- \( n-1 \geq j > 0 \)
- \( 0 < j \leq n-1 \)
- \( 1 \leq j \leq n-1 \)

Similar formula as in step 1:
\[ S_n = \sum_{j=1}^{n-1} \frac{1}{k-1} = \frac{S_m}{n-j+1} \]

- \( S_1 \) undefined
- \( S_2 = H_1 = 1 \)
- \( S_2 = 1 \)
- \( S_3 = \sum_{j=1}^{n-1} H_j = H_1 + H_2 = 1 + \frac{1}{2} = \frac{3}{2} \)

Put \( j = n-j \)

Book wrong:
\[ S_n = \sum_{j=0}^{m-1} H_j \]

\( H_0 \) not exist

Not closed

\( \sum_{j=1}^{n-1} \frac{1}{k-1} = S_m \)

\( S_1 \) undefined

\( S_2 = 1 \)

\( S_3 = \frac{5}{2} \)