DEFINITIONS

Check the LIST OF DEFINITIONS (in Downloads) to verify the mistakes in case of NO answer.

PART 1: GENERAL DEFINITIONS

Power Set \( \mathcal{P}(A) = \{ X : A \subseteq X \} \).

Relative Complement \( A - B = \{ a : a \in A \land a \notin B \} \).

(Cartesian) Product of two sets \( A \) and \( B \).
\( A \times B = \{ (a, b) : a \in A \land b \in B \} \).

Domain of \( R \) Let \( R \subseteq A \times A \), we define domain of \( R \): \( D_R = \{ a \in A : (a, b) \in R \} \).

ONTO function \( f : A \onto B \iff \forall b \in A \exists a \in B \; f(a) = b \).

Composition Let \( f : A \rightarrow B \) and \( g : B \rightarrow C \), we define a new function \( h : A \rightarrow C \), called a COMPOSITION of \( f \) and \( g \), as follows: for any \( x \in A \), \( h(x) = g(f(x)) \).

Inverse function Let \( f : A \rightarrow B \) and \( g : B \rightarrow A \).
\( g \) is called an INVERSE function to \( f \) iff \( \forall a \in A \; ((f \circ g)(a) = g(f(a)) = a) \).

Sequence of elements of a set \( A \) is any function \( f : \mathbb{N} \rightarrow A \) or \( f : \mathbb{N} - \{0\} \rightarrow A \).

Generalized Intersection of a sequence \( \{A_n\}_{n \in \mathbb{N}} \) of sets: \( \bigcap_{n \in \mathbb{N}} A_n = \{ x : \exists n \in \mathbb{N} \; x \in A_n \} \).

Equivalence relation \( R \subseteq A \times A \) is an equivalence relation in \( A \) iff it is reflexive, antisymmetric and transitive.

Partition A family of sets \( \mathcal{P} \subseteq \mathcal{P}(A) \) is called a partition of the set \( A \) iff the following conditions hold.

1. \( \forall X \in \mathcal{P} \; (X = \emptyset) \)
2. \( \forall X, Y \in P \ (X \cup Y = \emptyset) \)

3. \( \bigcup P = A \)

**Partition and Equivalence**  For any partition \( P \subseteq \mathcal{P}(A) \) of \( A \), there is an equivalence relation on \( A \) such that its equivalence classes are some sets of the partition \( P \).

**Mathematical Induction**  Let \( P(n) \) be any property (predicate) defined on a set \( N \) of all natural numbers such that:

- **Base Case**  \( n = 2 \)  \( P(2) \) is true.
- **Inductive Step**  The implication \( P(n) \Rightarrow P(n + 1) \) can be proved for any \( n \in N \)  \( \therefore P(n) \) is a true statement.

**PART 2: POSETS**

**Poset**  A set \( A \neq \emptyset \) ordered by a relation \( R \) is called a poset. We write it as a tuple: \( (A, R), (A, \leq) \) or \( (A, \preceq) \). Name poset stands for "partially ordered set".

**Smallest (least)**  \( a_0 \in A \) is a smallest (least) element in the poset \( (A, \leq) \)  \( \iff \exists a \in A \ (a_0 \leq a) \).

**Greatest (largest)**  \( a_0 \in A \) is a greatest (largest) element in the poset \( (A, \leq) \)  \( \iff \forall a \in A \ (a \leq a_0) \).

**Maximal**  \( a_0 \in A \) is a maximal element in the poset \( (A, \preceq) \)  \( \iff \neg \forall a \in A \ (a_0 \preceq a \land a_0 \neq a) \).

**Minimal**  \( a_0 \in A \) is a minimal element in the poset \( (A, \preceq) \)  \( \iff \neg \exists a \in A \ (a \preceq a_0 \land a_0 \neq a) \).

**Lower Bound**  Let \( B \subseteq A \) and \( (A, \preceq) \) is a poset. \( a_0 \in A \) is a lower bound of a set \( B \)  \( \iff \exists b \in B \ (a_0 \leq b) \).

**Upper Bound**  Let \( B \subseteq A \) and \( (A, \preceq) \) is a poset. \( a_0 \in A \) is an upper bound of a set \( B \)  \( \iff \forall b \in B \ (b \leq a_0) \).

**Least upper bound of \( B \) (lub \( B \))**  Given: a set \( B \subseteq A \) and \( (A, \preceq) \) a poset.

An element \( x_0 \in B \) is a least upper bound of \( B \), \( x_0 = \text{lub}B \) if \( x_0 \) is (if exists) the least (smallest) element in the set of all upper bounds of \( B \), ordered by the poset order \( \preceq \).

**Greatest lower bound of \( B \) (glb \( B \))**  Given: a set \( B \subseteq A \) of a poset \( (A, \preceq) \).

An element \( x_0 \in A \) is a greatest lower bound of \( B \), \( x_0 = \text{glb}B \) if \( x_0 \) is (if exists) the greatest element in the set of all lower bounds of \( B \), ordered by the poset order \( \preceq \).
PART 3: LATTICES and BOOLEAN ALGEBRAS

Lattice  A poset \((A, \preceq)\) is a lattice iff for all \(a, b \in A\) lub\{a, b\} or glb\{a, b\} exist.

Lattice notation  Observe that by definition elements lub\(B\) and glb\(B\) are always unique (if they exist). For \(B = \{a, b\}\) we denote:
lub\{a, b\} = a ∪ b and glb\{a, b\} = a ∩ b.

Lattice union (meet)  The element lub\{a, b\} = a ∩ b is called a lattice union (meet) of \(a\) and \(b\).
By lattice definition for any \(a, b \in A\) a ∩ b always exists.

Lattice intersection (joint)  The element glb\{a, b\} = a ∪ b is called a lattice intersection (joint) of \(a\) and \(b\).
By lattice definition for any \(a, b \in A\) a ∪ b always exists.

Lattice as an Algebra  An algebra \((A, \cup, \cap)\), where \(\cup, \cap\) are two argument operations on \(A\) is called a lattice iff the following conditions hold for any \(a, b, c \in A\) (they are called lattice AXIOMS):

\[
\begin{align*}
11 & \quad a \cup b = b \cup a \text{ and } a \cap b = b \cap a \\
12 & \quad (a \cup b) \cap c = a \cup (b \cap c) \text{ and } (a \cap b) \cap c = a \cap (b \cap c) \\
13 & \quad a \cap (a \cup b) = a \text{ and } a \cup (a \cap b) = a.
\end{align*}
\]

Lattice axioms  The conditions 11-13 from above definition are called lattice axioms.

Lattice orderings  Let the \((A, \cup, \cap)\) be a lattice. The relations:
\(a \preceq b\) iff \(a \cup b = b\), \(a \preceq b\) iff \(a \cap b = a\)
are order relations in \(A\) and are called a lattice orderings.

Distributive lattice Axioms  A lattice \((A, \cup, \cap)\) is called a distributive lattice iff for all \(a, b, c \in A\) the following conditions hold

\[
\begin{align*}
14 & \quad a \cup (b \cap c) = (a \cup b) \cap (a \cup c) \\
15 & \quad a \cap (b \cup c) = (a \cap b) \cup (a \cap c).
\end{align*}
\]

Conditions 14-15 from above are called a distributive lattice axioms.

Lattice special elements  The greatest element in a lattice (if exists) is denoted by 1 and is called a lattice UNIT. The least (smallest) element in \(A\) (if exists) is denoted by 0 and called a lattice zero.

Lattice with unit and zero  If 0 (lattice zero) and 1 (lattice unit) exist in a lattice, we will write the lattice as: \((A, \cup, \cap, 0, 1)\) and call it a lattice with zero and unit.

Lattice Unit Definition  Let \((A, \cup, \cap)\) be a lattice. An element \(x \in A\) is called a lattice unit iff for any \(a \in A\) \(x \cup a = a\) and \(x \cap a = x\).
Lattice Unit Axioms If lattice unit $x$ exists we denote it by 1 and we write the unit axioms as follows.

16 $\ 1 \cap a = a$
17 $\ 1 \cup a = 1$.

Lattice Zero Let $(A, \cup, \cap)$ be a lattice. An element $x \in A$ is called a lattice zero iff for any $a \in A$

$x \cup a = x$ and $x \cap a = a$.

Lattice Zero Axioms If lattice zero exists we denote it by 0 and write the zero axioms as follows.

18 $\ 0 \cup a = 0$
19 $\ 0 \cap a = a$.

Complement Definition Let $(A, \cup, \cap, 1, 0)$ be a lattice with unit and zero. An element $x \in A$ is called a complement of an element $a \in A$ iff

$a \cap x = 1$ and $a \cup x = 0$.

Complement axioms Let $(A, \cup, \cap, 1, 0)$ be a lattice with unit and zero. The complement of $a \in A$
is usually denoted by $-a$ and the above conditions that define the complement above are called complement axioms. The complement axioms are usually written as follows.

1 $a \cup -a = 0$
2 $a \cap -a = 1$.

Boolean Algebra A distributive lattice with zero and unit such that each element has a complement is called a Boolean Algebra.

Boolean Algebra Axioms A lattice $(A, \cup, \cap, 1, 0)$ is called a Boolean Algebra iff the operations $
\cap, \cup$ satisfy axioms 11 -15, 0 $\in A$ and 1 $\in A$ satisfy axioms 16 -19 and each element $a \in A$ has a complement $-a \in A$, i.e.

11o $\forall a \in A \exists -a \in A (\ (a \cup -a = 1) \cap (a \cap -a = 0))$.

PART 4: CARDINALITIES OF SETS, Finite and Infinite Sets.

Cardinality definition Sets $A$ and $B$ have the same cardinality iff $\exists f( f : A \xrightarrow{1 \text{1-to-1} \text{onto}} B)$.

Cardinality notations $|A| = |B|$ or $\text{card}A = \text{card}B$, or $A \sim B$ all denote that the sets $A$ and $B$
have the same cardinality.

Finite We say: a set $A$ is finite \ iff \ $\exists n \in N(|A| = n)$.

Infinite A set $A$ is infinite \ iff \ $A$ is NOT finite.

Cardinality Aleph zero We say that a set $A$ has a cardinality $\aleph_0$ ($|A| = \aleph_0$) \ iff \ $|A| = |N|$.

Countable A set $A$ is countable \ iff \ $|A| = \aleph_0$.  

4
Uncountable  A set $A$ is uncountable iff $A$ is NOT countable.

Cardinality Continuum  We say that a set $A$ has a cardinality $C$ ( $|A| = C$ ) iff $|A| = |R|$.

Cardinality $A \leq$ Cardinality $B$  $|A| \leq |B|$ iff $A \sim C$ and $C \subseteq B$.

Cardinality $A <$ Cardinality $B$  $|A| < |B|$ iff $|A| \leq |B|$ or $|A| \neq |B|$.

Cantor Theorem  For any set $A$, $|A| \leq \mathcal{P}(A)$.

PART 5: ARITHMETIC OF CARDINAL NUMBERS

Sum ( $\mathcal{N} + \mathcal{M}$)  We define:
$\mathcal{N} + \mathcal{M} = |A \cup B|$, where $A, B$ are such that $|A| = \mathcal{N}$, $|B| = \mathcal{M}$.

Multiplication ( $\mathcal{N} \cdot \mathcal{M}$)  We define:
$\mathcal{N} \cdot \mathcal{M} = |A \times B|$, where $A, B$ are such that $|A| = \mathcal{N}$, $|B| = \mathcal{M}$.

Power ( $\mathcal{M}^{\mathcal{N}}$)  $\mathcal{M}^{\mathcal{N}} = card\{ f : f : A \rightarrow B \}$, where $A, B$ are such that $|A| = \mathcal{M}$, $|B| = \mathcal{N}$.

Power $2^{\mathcal{N}}$  We define:
$2^{\mathcal{N}} = card\{ f : f : A \rightarrow \{0, 1\} \}$, where $|A| = \mathcal{N}$.

PART 4: ARITHMETIC OF $n$, $\aleph_0$, $C$

Union 1  $\aleph_0 + \aleph_0 = \aleph_0$.
Union of two countable sets is a countable set.

Union 2  $\aleph_0 + n = \aleph_0$.
Union of a finite ( cardinality $n$) and a countable set is an infinitely countable set.

Union 3  $\aleph_0 + C = C$.
Union of an infinitely countable set and an uncountable set is an uncountable set.

Cartesian Product 1  $\aleph_0 \cdot \aleph_0 = \aleph_0$.
Cartesian Product of two countable sets is a countable set.

Cartesian Product 2  $n \cdot \aleph_0 = \aleph_0$.
Cartesian Product of a finite set and an infinite set is an infinite set.
Cartesian Product 3  $\aleph_0 \cdot \mathcal{C} = \mathcal{C}$.
Cartesian Product of an infinitely countable set and a set of the same cardinality as Real numbers has the same cardinality as the set of Real numbers.

Cartesian Product 4  $\mathcal{C} \cdot \mathcal{C} = \mathcal{C}$.
Cartesian Product of two uncountable sets is an uncountable set.

Power 1  $2^{\aleph_0} = \mathcal{C}$.

Power 2  $\aleph_0^{\aleph_0} = \mathcal{C}$ means that
\[ \text{card}\{ f : f : N \rightarrow N \} = \mathcal{C}. \]

Power 3  $\mathcal{C}^\mathcal{C} = 2^\mathcal{C}$ means that there are $2^\mathcal{C}$ of all functions that map $\mathbb{R}$ into $\mathbb{R}$.

Inequalities  $n < \aleph_0 \leq \mathcal{C}$.

QUESTIONS
Circle proper answer. WRITE a short JUSTIFICATION. NO JUSTIFICATION, NO CREDIT.

Here are YES/NO answers with FEW JUSTIFICATIONS as examples

1. If $f : A \xrightarrow{1-1\text{onto}} B$ and $g : B \xrightarrow{1-1\text{onto}} A$, then $g$ is an inverse to $f$.
   JUSTIFY: The statement guarantee only that INVERSE function EXISTS.

2. Let $f : N \times N \rightarrow N$ be given by a formula $f(n, m) = n + m^2$. $f$ is a $1-1$ function.
   JUSTIFY: $f(1, 2) = 5 = f(4, 1)$

3. Let $A = \{a, \emptyset, \emptyset\}$, $B = \{\emptyset, \emptyset, \emptyset\}$. There is a function $f : A \xrightarrow{1-1\text{onto}} B$.
   JUSTIFY: $|A| = 3$, $|B| = 2$

4. If $f : A \xrightarrow{1-1} B$ and $g : B \xrightarrow{\text{onto}} A$, then $f \circ g$ and $g \circ f$ are onto.
   JUSTIFY: $g \circ f$ no; take $|A| = 2$, $|B| = 3$

5. $f : R - \{0\} \xrightarrow{1-1} R$ is given by a formula: $f(x) = \frac{1}{2}$ and $g : R - \{0\} \rightarrow R - \{0\}$ given by $g(x) = \frac{x}{2}$.
   g is inverse to f.
   JUSTIFY: $f$ is not ”onto”; inverse does not exist.
6. \{(1, 2), (a, 1)\} is a binary relation in \{1, 2, 3, \}.  

JUSTIFY: \(a \not\in \{1, 2, 3, \}\).  

7. The function \(f : N \rightarrow \mathcal{P}(N)\) given by formula: \(f(n) = \{m \in N : m \leq n\}\) is a 1–1 function.  

JUSTIFY: \(n_1 \neq n_2\) then obviously \(f(n_1) \neq f(n_2)\).  

8. The function \(f : N \times N \rightarrow \mathcal{P}(N)\) given by formula: \(f(n, m) = \{m \in N : m + n = 1\}\) is a sequence.  

JUSTIFY: Domain of \(f\) is not \(N\).  

9. The function \(f : N \times N \rightarrow \mathcal{P}(N)\) given by formula: \(f(n, m) = \{m \in N : m + n = 1\}\) is 1–1.  

JUSTIFY: \(f(n, m) = \emptyset\) for all \(n, m\) such that \(m + n \neq 1\).  

10. The \(f : N \rightarrow \mathcal{P}(N)\) given by formula: \(f(n) = \{m \in N : m + n = 1\}\) is a family of sets.  

JUSTIFY: Values of \(f\) are sets.  

11. Let \(P\) be a predicate. If \(P(0)\) is true and for all \(k \leq n\), \(P(k)\) is true implies \(P(n + 1)\) is true, then \(\forall n \in N \ P(n)\) is true.  

JUSTIFY: Principle of mathematical Induction.  

12. Let \(A_n = \{x \in R : n \leq x \leq n + 1\}\). Consider \(\{A_n\}_{n \in N}\). \(\bigcap_{n \in N} A_n = \emptyset\).  

JUSTIFY: \(A_0 = \{1\}\), \(A_1 = \{1, 2\}\), \(A_2 = \{1, 2\}\) and \(\bigcap_{n \in T} A_n = \emptyset\).  

13. Let \(A_n = \{x \in R : n + 1 \leq x \leq n + 2\}\). Consider \(\{A_n\}_{n \in N}\). \(\bigcup_{n \in N} A_n = R\).  

JUSTIFY: \(\bigcup_{n \in N} \{x \in R : n + 1 \leq x \leq n + 2\} = [1, \infty) \neq R\).  

14. \(x \in \bigcup_{t \in T} A_t\) iff \(\exists t \in T(x \in A_t)\)  

JUSTIFY: definition.  

15. Let \(A_n = \{x \in N : 0 < x < n\}\). The family \(\{A_n\}_{n \in N}\) form a partition of \(N\).  

JUSTIFY: \(A_0 = \{x \in N : 0 < x < 0\} = \emptyset\).  

16. Let \(A_t = \{x \in \{1, 2, 3\} : x > t\}\) for \(t \in \{0, 1, 2\}\). \(\bigcap_{t \in T} A_t = \{1\}\).  

JUSTIFY: \(A_0 = \{1, 2, 3\}, A_1 = \{2, 3\}, A_2 = \{3\}\) and \(\bigcap_{t \in T} A_t = \emptyset\).  

17. There is an equivalence relation on \(N\) with infinite number of equivalence classes.  

JUSTIFY: Equality on \(N\).
18. There is an equivalence relation on \( A = \{ x \in R : 1 \leq x < 4 \} \) with equivalence classes: 
\([1] = \{ x \in R : 1 \leq x < 2 \}, \ [2] = \{ x \in R : 2 \leq x < 3 \}, \text{and} \ [3] = \{ x \in R : 3 \leq x < 4 \}.

JUSTIFY: \{[1], [2], [3] \} is a partition of \( A \).

19. Each element of a partition of a set \( A = \{ 1,2,3 \} \) is an equivalence class of a certain equivalence relation.

JUSTIFY: True for any set \( A \neq \).

20. Set of all equivalence classes of a given equivalence relation is a partition.

JUSTIFY: Partition Theorem.

21. Let \( R \subseteq A \times A \) The set \( [a] = \{ b \in A : (a, b) \in R \} \) is an equivalence class with a representative \( a \).

JUSTIFY: ONLY when \( R \) is an equivalence relation.

22. Let \( A = \{ a, b, c, d \} \). There are \( 4^3 \) words of length 3 in \( A^* \).

JUSTIFY: Counting the functions theorem.

23. If a set \( A \) has \( n \) elements \( (n \in N) \), then every subset of \( A \) is finite.

JUSTIFY: Any subset of a finite set is a finite set.

24. Let \( \sum \) be an alphabet \( \sum = \{ \%, $, & \} \). Denote \( \sum^k = \{ w \in \sum^* : \text{length}(w) = k \} \).
The set \( \sum^3 \) has 29 elements.

JUSTIFY: \( 3^3 = 27 \)

25. There is an order relation that is also an equivalence relation and a function.

JUSTIFY: Equality on any set.

26. \( R = \{(N, \{1, 2, 3\}), (Z, \{1, 2, 3\}), (1, N), (-1, N), (3, Z)\} \) is a function defined on a set \( \{N, Z, 1, -1, 3\} \) with values in the set \( Z \).

JUSTIFY: Elements of the range (values) of \( R \) are SUBSETS, not elements of \( Z \).

27. If \( f : R \to R \) and \( g : R \to R \), then \( g \circ f \) and \( f \circ g \) exists.

JUSTIFY: Corresponding domains and ranges agree.

28. \( \{(1, 2), (a, 1), (a, a)\} \) is a transitive binary relation defined in \( A = \{1, 2, a\} \).

JUSTIFY: \( (a, 2) \notin R \).

29. \( f : N \to \mathcal{P}(R) \) is given by the formula: \( f(n) = \{ x \in R : x \leq \frac{-n^3 + 1}{\sqrt{n^3 + 3 + 6}} \} \) is a sequence.

JUSTIFY: Domain of \( f \) is \( N \).
30. There is an order relation $R$ defined in $A \neq \emptyset$ such that $(A, R)$ is a poset.
   
   **JUSTIFY:** Definition of Poset.

31. Let $A = \{\emptyset, N, \{1\}, \{a, b, 3\}\}$. There are no more then 50 words of length 4 in $A^*$.
   
   **JUSTIFY:** $|A| = 4$, $3^4 > 50$.

32. There is an equivalence relation on $Z$ with infinitely countably many equivalence classes.
   
   **JUSTIFY:** Equality on $Z$.

33. $A$ is uncountable iff $|A| = |R|$ where $R$ is the set of real numbers.
   
   **JUSTIFY:** $A = \mathcal{P}(R)$ is uncountable and by Cantor theorem $|R| < |\mathcal{P}(R)|$.

34. $A$ is infinite iff some subsets of $A$ are infinite.
   
   **JUSTIFY:** All subsets of a finite set are finite.

35. There exists an equivalence relation on $N$ with $\aleph_0$ equivalence classes.
   
   **JUSTIFY:** Equality; $[n] = \{n\}$.

36. $A$ is finite iff some subsets of $A$ are finite.
   
   **JUSTIFY:** all subsets are finite; $\{1\} \subseteq N$ and $N$ is infinite.

37. If $A$ is a countable set, then any subset of $A$ is countable.
   
   **JUSTIFY:** Theorem

38. If $A$ is uncountable set, then any subset of $A$ is uncountable.
   
   **JUSTIFY:** $N \subseteq R$.

39. $\{x \in Q : 1 \leq x \leq 2\}$ has the same cardinality as $\{x \in Q : 5 \leq x \leq 10\}$.
   
   **JUSTIFY:** both sets are of cardinality $\aleph_0$.

40. If $A$ is infinite set and $B$ is a finite set, then $((A \cup B) \cap A)$ is infinite set.
   
   **JUSTIFY:** $((A \cup B) \cap A) = A$.

41. The set of all squares centered in the origin has the same cardinality as $R$.
   
   **JUSTIFY:** All such circles are uniquely defined by the radius $r$ and $r \in R$.

42. If $A, B$ are infinitely countable sets, then $A \cap B$ is a countable set.
   
   **JUSTIFY:** $A \cap B$ is finite or infinitely countable.
43. A is uncountable iff there is a subset $B$ of $A$ such that $|B| = |A|$.  

JUSTIFY: $N \subseteq Q$, $|N| = |Q|$ and $Q$ is NOT uncountable.  

44. A is uncountable iff $|A| = \mathcal{C}$.  

JUSTIFY: $\mathcal{P}(\mathbb{R})$ is uncountable and $|\mathcal{P}(\mathbb{R})| \neq \mathcal{C}$.  

45. $\aleph_0 + \aleph_0 = \aleph_0$ means that the union of two infinitely countable sets is an infinitely countable set.  

JUSTIFY: The fact that the union of two infinitely countable sets is an infinitely countable set is true (theorem), but does not reflect the definition of sum of cardinal numbers; two DISJOINT infinitely countable sets.  

46. $|\mathcal{P}(\mathbb{N})| = \aleph_0$  

JUSTIFY: $|\mathcal{P}(\mathbb{N})| = \mathcal{C}$.  

47. $\text{card}(N \cap \{1, 3\}) = \text{card}(Q \cap \{1, 2\})$  

JUSTIFY: both sets have 2 elements.  

48. A relation in $\mathbb{N}$ defined as follows: $n \approx m$ iff $n + m \in \text{EVEN}$ has $\aleph_0$ equivalence classes. in $\mathbb{N}$.  

JUSTIFY: two equivalence classes.  

49. $\text{card}A < \text{card}\mathcal{P}(A)$  

JUSTIFY: Cantor Theorem  

50. $A$ is infinite set iff there is $f : \mathbb{N} \longrightarrow \overset{1}{1}_{\text{onto}} A$.  

JUSTIFY: this is definition of the infinitely countable set.  

51. $\mathcal{P}(A) = \{B : B \subseteq A\}$  

JUSTIFY: $B \subseteq A$  

52. $|Q \cup N| = \aleph_0$  

JUSTIFY: $Q \cup N = Q$.  

53. $|\mathbb{R} \times \mathbb{Q}| = \mathcal{C}$  

JUSTIFY: $\mathcal{C} \cdot \aleph_0 = \mathcal{C}$.  


54. \(|N| \leq \aleph_0\)

JUSTIFY: \(|A| \leq |A|\).

y

55. Any non empty POSET has at least one MAX element.

JUSTIFY: \((N, \leq)\) has no max element for \(\leq\) natural order.

n

56. If \((A, \leq)\) is a finite poset (i.e. \(A\) is a finite set), then a unique maximal is the largest element and a unique minimal is the least element.

JUSTIFY: Theorem

y

57. There is a non empty POSET that has no Max element.

JUSTIFY: \((N, \leq)\) has no max element for \((\leq\) natural order.

y

58. Any lattice is a POSET.

JUSTIFY: definition

y

59. It is possible to order \(N\) in such a way that \((N, \leq)\) has \(\aleph_0\) MAX elements and no MIN elements.

JUSTIFY: diagram (lecture)

y

60. In any poset \((A, \leq)\), the greatest and least elements are unique.

JUSTIFY: Theorem

y

61. If a non empty poset is finite, then unique MAX element is the smallest.

JUSTIFY: in a finite poset unique MAX element is the greatest.

n

62. Each non empty lattice has 0 and 1.

JUSTIFY: \((Z, \leq)\)

n

63. In any poset \((A, \leq)\), if a greatest and a least elements exist, then they are unique.

JUSTIFY: Theorem

y

64. Each distributive lattice has zero and unit elements.

JUSTIFY: diagram

n

65. It is possible to order the set of Natural numbers \(N\) in such a way that the poset \((N, \leq)\) has a unique maximal element (minimal element) and no largest element (least element).

JUSTIFY: diagram

n
66. It is possible to order the set of rational numbers \( \mathbb{Q} \) in such a way that the poset \( (\mathbb{Q}, \preceq) \) has a unique minimal element and no smallest (least) element.

JUSTIFY: diagram

67. In any poset, the largest element is a unique maximal element and the least element is the unique minimal element.

JUSTIFY: Theorem

68. If \( (A, \cup, \cap) \) is an infinite lattice (i.e. the set \( A \) is infinite), then 1 or 0 might or might not exist.

JUSTIFY: always true

69. There is a poset \( (A, \preceq) \) and a set \( B \subseteq A \) and that \( B \) has none upper bounds.

JUSTIFY: \( (\mathbb{N}, \preceq), B = \mathbb{N} - \{0\} \).

70. There is a poset \( (A, \preceq) \) and a set \( B \subseteq A \) and that \( B \) has infinite number of lower bounds.

JUSTIFY: \( (\mathbb{N}, \succeq), B = \{0, 1\} \).

71. If \( (A, \cup, \cap) \) is a finite lattice (i.e. \( A \) is a finite set), then 1 and 0 always exist.

JUSTIFY: Theorem

72. Any finite lattice is distributive.

JUSTIFY: example in the lecture of 5element non-distributive lattice

73. Every Boolean algebra is a lattice.

JUSTIFY: definition

74. Any infinite Boolean algebra has unit (greatest) and zero (smallest) elements.

JUSTIFY: by definition every Boolean algebra has unit (greatest) and zero (smallest) elements

75. A non-generate Finite Boolean Algebras always have \( 2^n \) elements \( (n \geq 1) \).

JUSTIFY: Theorem

76. Sets \( A \) and \( B \) have the same cardinality iff \( \exists f ( f : A \xrightarrow{1-1} B ) \).

JUSTIFY: \( f \) must be also "onto."

77. We say: a set \( A \) is finite iff \( \exists n \in \mathbb{N} (|A| = n) \).

JUSTIFY: definition

78. A set \( A \) is infinite iff \( A \) is NOT finite.

JUSTIFY: definition
79. $\aleph_0$ (Aleph zero) is a cardinality of only $\mathbb{N}$ (Natural numbers).

    JUSTIFY: definition

80. A set $A$ is countable iff $|A| = \aleph_0$.

    JUSTIFY: A set $A$ is countable iff is FINITE or $|A| = \aleph_0$.

81. $\mathcal{C}$ (Continuum) is a cardinality of Real Numbers, i.e. $\mathcal{C} = |\mathbb{R}|$.

    JUSTIFY: definition

82. For any set $A$, $|A| < |\mathcal{P}(A)|$.

    JUSTIFY: Cantor Theorem

83. $\mathcal{M}^\mathbb{N}$ is the cardinality of all functions that map a set $A$ (of cardinality $\mathbb{N}$) into a set $B$ (of cardinality $\mathcal{M}$).

    JUSTIFY: definition