cse547,mat547 DISCRETE MATHEMATICS Lectures Content For Midterm 2 and Final Infinite Series, Chapter 3 and Chapter 4

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CHAPTER 2 PART 5: INFINITE SUMS (SERIES)

Here are Definitions, Basic Theorems and Examples you must know

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Series

Definitions, Theorems, Simple Examples

Must Know STATEMENTS- do not need to PROVE the Theorems

Definition

If the limit $\lim_{n\to\infty} S_n$ exists and is finite, i.e.

 $\lim_{n\to\infty}S_n=S,$

then we say that the infinite sum $\sum_{n=1}^{\infty} a_n$ converges to S and we write

$$\Sigma_{n=1}^{\infty} a_n = \lim_{n \to \infty} \Sigma_{k=1}^n a_k = S,$$

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otherwise the infinite sum diverges

Show

The infinite sum $\sum_{n=1}^{\infty} (-1)^n$ diverges

The infinite sum $\sum_{n=0}^{\infty} \frac{1}{(k+1)(k+2)}$ converges to 1

Theorem 1

If the infinite sum

$$\sum_{n=1}^{\infty} a_n$$
 converges, then $\lim_{n \to \infty} a_n = 0$

Definition 5

An infinite sum

$$\Sigma_{n=1}^{\infty}(-1)^{n+1}a_n$$
, for $a_n \geq 0$

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is called **alternating infinite sum** (alternating series)

Theorem 6 Comparing the series Let $\sum_{n=1}^{\infty} a_n$ be an infinite sum and $\{b_n\}$ be a sequence such that

 $0 \le b_n \le a_n$ for all n

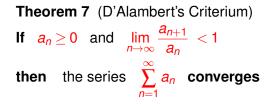
If the infinite sum $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} b_n$ also converges and

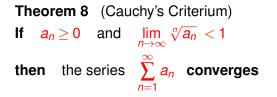
 $\Sigma_{n=1}^{\infty}b_n \leq \Sigma_{n=1}^{\infty}a_n$

Use Theorem 6 to prove that the series,

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)^2}$$

converges





Theorem 9 (Divergence Criteria)

If $a_n \ge 0$ and $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} > 1$ or $\lim_{n \to \infty} \sqrt[n]{a_n} > 1$ then the series $\sum_{n=1}^{\infty} a_n$ diverges Prove The series $\sum_{n=1}^{\infty} \frac{1}{(n+1)^2}$ does not react on D'Alambert's Criterium (Theorem 7)

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STUDY ALL EXAMPLES for Lecture 10

CHAPTER 3 INTEGER FUNCTIONS

Here is the **proofs** in course material you need to know for **Midterm 2** and **Final** Plus the regular Homeworks Problems

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Prove the following Fact 3 For any $x, y \in R$ $\lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$ when $0 \le \{x\} + \{y\} < 1$ and

 $\lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor + 1 \quad \text{when} \quad 1 \le \{x\} + \{y\} < 2$ Fact 5 For any $x \in \mathbb{R}, x \ge 0$ the following property holds $\left| \sqrt{\lfloor x \rfloor} \right| = \lfloor \sqrt{x} \rfloor$

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Prove the following properties of characteristic functions **F1** For any predicates P(k), Q(k)

 $[P(k) \cap Q(k)] = [P(k)][Q(k)]$

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F2 For any predicates P(k), Q(k) $[P(k) \cup Q(k)] = [P(k)] + [Q(k)] - [P(k) \cap Q(k)]$

Prove the Combined Domains Property **Property 4**

$$\sum_{Q(k)\cup R(k)}a_k=\sum_{Q(k)}a_k+\sum_{R(k)}a_k-\sum_{Q(k)\cap R(k)}a_k$$

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where, as before,

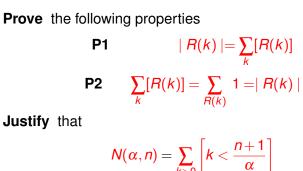
 $k \in K$ and $K = K_1 \times K_2 \cdots \times K_i$ for $1 \le i \le n$ and the above formula represents single (i = 1) and multiple (i > 1) sums

Study all 7 steps of our explanations to BOOK solution I will give you ONE to write in full on the test

1 W =
$$\sum_{n=1}^{1000} [n \text{ is a winner }] = \sum_{n=1}^{1000} [\lfloor \sqrt[3]{n} \rfloor \mid n]$$

2 W = $\sum_{k,n} [k = \lfloor \sqrt[3]{n} \rfloor] [k|n] [1 \le n \le 1000]$
3 W = $\sum_{k,n,m} [k^3 \le n < (k+1)^3] [n = km] [1 \le n \le 1000]$
4 W = 1 + $\sum_{k,m} [k^3 \le km < (k+1)^3] [1 \le k < 10]$
5 W = 1 + $\sum_{k,m} [m \in [k^2 \dots \frac{(k+1)^3}{k})] [1 \le k < 10]$
6 W = 1 + $\sum_{k,m} [m ([k^2 + 3k + 3 + \frac{1}{k}] - [k^2]))$
7 W = 1 + $\sum_{1 \le k < 10} (3k + 4) = 1 + \frac{7 + 31}{2}9 = 172$

PART2: Spectrum Partitions



Write a detailed proof of

$$N(\alpha, n) = \left\lceil \frac{n+1}{\alpha} \right\rceil - 1$$

Write a detailed proof of Finite Fact

 $|A_n| + |B_n| = n$ for any $n \in N - \{0\}$

PART2: Spectrum Partitions

Prove the following **Fact P2** If |A| + |B| = |X| and $A \neq \emptyset$, $B \neq \emptyset$ and $A \cap B = \emptyset$ then the sets A, B form a finite partition of XSpectrum Fact

$$Spec(\sqrt{2}) \cap Spec(2+\sqrt{2}) = \emptyset$$

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Finite Spectrum Partition Theorem

- **1.** $A_n \neq \emptyset$ and $B_n \neq \emptyset$
- **2.** $A_n \cap B_n = \emptyset$
- **3.** $A_n \cup B_n = \{1, 2, \dots, n\}$

PART2: Spectrum Partitions

Prove - use your favorite proof out of the two I have provided

Spectrum Partition Theorem

- **1.** Spec($\sqrt{2}$) $\neq \emptyset$ and Spec($2 + \sqrt{2}$) $\neq \emptyset$
- **2.** Spec($\sqrt{2}$) \cap Spec($2 + \sqrt{2}$) = \emptyset
- 3. Spec $(\sqrt{2}) \cup$ Spec $(2 + \sqrt{2}) = N \{0\}$

PART3: Sums

Write detailed evaluation of

 $\sum_{0 \le k < n} \lfloor \sqrt{k} \rfloor$

Hint: use

$$\sum_{0 \le k < n} \lfloor \sqrt{k} \rfloor = \sum_{0 \le k < n} \sum_{m \ge 0, m = \lfloor \sqrt{k} \rfloor} m$$

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Chapter 4 Material in the Lecture 12

JUSTIFY correctness of the following example and be ready to do similar problems upwards or downwards Represent 19151 in a system with base 12 **Example**

 $19151 = 1595 \cdot 12 + 11$ $1595 = 132 \cdot 12 + 11$ $132 = 11 \cdot 12 + 0$ $a_0 = 11, \quad a_1 = 11, \quad a_2 = 0, \quad a_3 = 11$ So we get

 $19151 = (11, 0, 11, 11)_{12}$

Write a proof of Step 1 or Step 2 of the Proof of the Correctness of Euclid Algorithm

You can use Lecture OR BOOK formalization and proofs

Use Euclid Algorithms to prove

When a product ac of two natural numbers is divisible by a number b that is **relatively prime** to a, the factor c must be divisible by b

Use Euclid Algorithms to prove

 $gcd(ka,kb) = k \cdot gcd(a,b)$

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Prove:

Any common multiple of **a** and **b** is **divisible** by lcm(a,b) **Prove** the following

$$\forall_{p,q_1q_2\dots q_n \in P} (p \mid \prod_{k=1}^n q_k \Rightarrow \exists_{1 \leq i \leq n} (p = q_i))$$

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Write down a formal formulation (in all details) of the Main Factorization Theorem and its General Form

Prove that the representation given by Main Factorization Theorem is unique

Explain why and show that 18 = < 1, 2 >

Prove

 $k = gcd(m, n) \quad \text{if and only if} \quad k_p = min\{m_p, n_p\}$ $k = lcd(m, n) \quad \text{if and only if} \quad k_p = max\{m_p, n_p\}$ Let $m = 2^0 \cdot 3^3 \cdot 5^2 \cdot 7^0 \qquad n = 2^0 \cdot 3^1 \cdot 5^0 \cdot 7^3$ Evaluate gcd(m, n) and k = lcd(m, n)

Exercises

1. Use Facts 6-8 to prove

Theorem 5

For any $a, b \in Z^+$ such that lcm(a,b) and gcd(a, b) exist

 $lcm(a,b) \cdot gcd(a,b) = ab$

2. Use **Theorem 5** and the BOOK version of Euclid Algorithm to express $lcm(n \mod m, m)$ when $nmodm \neq 0$ This is Ch4 Problem 2

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